Chapter 5

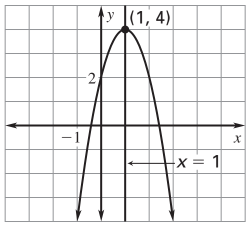
Prerequisite Skills (p. 328)

1. -3 and 1

2. 4

3. $y = ax^2 + bx + c$

4.



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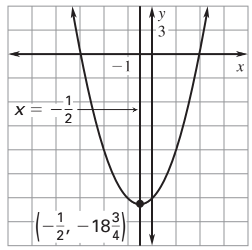

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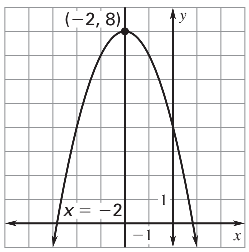


7. $x^2 + 9x + 20 = (x + 4)(x + 5)$

8. $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

9. $9x^2 - 64 = (3x - 8)(3x + 8)$

10. $2x^2 + x + 6 = 0$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(6)}}{2(2)} = \frac{-1 \pm \sqrt{-47}}{4} = \frac{-1 \pm i\sqrt{47}}{4}$$

11.
$$10x^2 + 13x = 3$$
$$10x^2 + 13x - 3 = 0$$
$$(5x - 1)(2x + 3) = 0$$
$$5x - 1 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$5x = 1 \quad \text{or} \quad 2x = -3$$
$$x = \frac{1}{5} \quad \text{or} \quad x = -\frac{3}{2}$$

12.
$$x^2 + 6x + 2 = 20$$
$$x^2 + 6x - 18 = 0$$
$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-18)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{108}}{2}$$
$$= \frac{-6 \pm 6\sqrt{3}}{2}$$
$$= -3 \pm 3\sqrt{3}$$

Lesson 5.1

5.1 Guided Practice (pp. 331–333)

1. $(4^2)^3 = 4^6$ Power of a power property

$= 4096$

2. $(-8)(-8)^3 = (-8)^{1+3}$ Product of powers property

$= (-8)^4$

$= 4096$

3. $\left(\frac{2}{9}\right)^3 = \frac{2^3}{9^3}$ Power of a quotient property

$= \frac{8}{729}$

4. $\frac{6 \cdot 10^{-4}}{9 \cdot 10^7} = \frac{6}{9} \cdot 10^{-4-7}$ Quotient of powers property

$= \frac{6}{9} \cdot 10^{-11}$

$= \frac{2}{3 \cdot 10^{11}}$ Negative exponent property

5. $x^{-6}x^5x^3 = x^{-6+5+3}$ Product of powers property

$= x^2$

6. $(7y^2z^5)(y^{-4}z^{-1}) = 7y^{2+(-4)}z^{5+(-1)}$ Product of powers property

$= 7y^{-2}z^4$

$= \frac{7z^4}{y^2}$ Negative exponent property

7. $\left(\frac{s^3}{t^{-4}}\right)^2 = \frac{(s^3)^2}{(t^{-4})^2}$ Power of a quotient property

$= \frac{s^6}{t^{-8}}$ Power of a power property

$= s^6t^8$ Negative exponent property

8. $\left(\frac{x^4y^{-2}}{x^3y^6}\right)^3 = \frac{(x^4y^{-2})^3}{(x^3y^6)^3}$ Power of a quotient property

$= \frac{x^{4 \cdot 3}y^{-2 \cdot 3}}{x^{3 \cdot 3}y^{6 \cdot 3}}$ Power of a product property

$= \frac{x^{12}y^{-6}}{x^9y^{18}}$

$= x^{12-9}y^{-6-18}$ Quotient of powers property

$= \frac{x^3}{y^{24}}$ Negative exponent property

5.1 Exercises (pp. 333–335)

Skill Practice

1. a. Product of powers property

b. Negative exponent property

c. Power of a product property

2. No, 25.2 is not less then 10; the number should be 2.52×10^{-2}.

3. $3^3 \cdot 3^2 = 3^{(3+2)}$ Product of powers property

$= 3^5$

$= 243$

4. $(4^{-2})^3 = 4^{-6}$ Power of a power property

$= \frac{1}{4^6}$ Negative exponent property

$= \frac{1}{4096}$

5. $(-5)(-5)^4 = (-5)^{(1+4)}$ Product of powers property
$= (-5)^5$
$= -3125$

6. $(2^4)^2 = 2^8$ Power of a power property
$= 256$

7. $\frac{5^2}{5^5} = 5^{(2-5)}$ Quotient of powers property
$= 5^{-3}$
$= \frac{1}{5^3}$ Negative exponent property
$= \frac{1}{125}$

8. $\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4}$ Power of a quotient property
$= \frac{81}{625}$

9. $\left(\frac{2}{7}\right)^{-3} = \frac{2^{-3}}{7^{-3}}$ Power of a quotient property
$= \frac{7^3}{2^3}$ Negative exponent property
$= \frac{343}{8}$

10. $9^3 \cdot 9^{-1} = 9^{(3+(-1))}$ Product of powers property
$= 9^2$
$= 81$

11. $\frac{3^4}{3^{-2}} = 3^{4-(-2)}$ Quotient of powers property
$= 3^6$
$= 729$

12. $\left(\frac{2}{3}\right)^{-5}\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{-5+4}$ Product of powers property
$= \left(\frac{2}{3}\right)^{-1}$
$= \frac{3}{2}$ Negative exponent property

13. $6^3 \cdot 6^0 \cdot 6^{-5} = 6^{3+0-5}$ Product of powers property
$= 6^{-2}$
$= \frac{1}{6^2}$ Negative exponent property
$= \frac{1}{36}$

14. $\left(\left(\frac{1}{2}\right)^{-5}\right)^2 = \left(\frac{1}{2}\right)^{-10}$ Power of a power property
$= \frac{1^{-10}}{2^{-10}}$ Power of a quotient property
$= 2^{10}$ Negative exponent property
$= 1024$

15. $(4.2 \times 10^3)(1.5 \times 10^6) = (4.2 \times 1.5)(10^3 \times 10^6)$
$= 6.3 \times 10^9$

16. $(1.2 \times 10^{-3})(6.7 \times 10^{-7}) = (1.2 \times 6.7)(10^{-3} \times 10^{-7})$
$= 8.04 \times 10^{-10}$

17. $(6.3 \times 10^5)(8.9 \times 10^{-12}) = (6.3 \times 8.9)(10^5 \times 10^{-12})$
$= 56.07 \times 10^{-7}$
$= 5.607 \times 10^1 \times 10^{-7}$
$= 5.607 \times 10^{-6}$

18. $(7.2 \times 10^9)(9.4 \times 10^8) = (7.2 \times 9.4)(10^9 \times 10^8)$
$= 67.68 \times 10^{17}$
$= 6.768 \times 10^1 \times 10^{17}$
$= 6.768 \times 10^{18}$

19. $(2.1 \times 10^{-4})^3 = 2.1^3 \times (10^{-4})^3$
$= 9.261 \times 10^{-12}$

20. $(4.0 \times 10^3)^4 = 4.0^4 \times (10^3)^4$
$= 256 \times 10^{12}$
$= 2.56 \times 10^2 \times 10^{12}$
$= 2.56 \times 10^{14}$

21. $\frac{8.1 \times 10^{12}}{5.4 \times 10^9} = \frac{8.1}{5.4} \times \frac{10^{12}}{10^9}$
$= 1.5 \times 10^3$

22. $\frac{1.1 \times 10^{-3}}{5.5 \times 10^{-8}} = \frac{1.1}{5.5} \times \frac{10^{-3}}{10^{-8}}$
$= 0.2 \times 10^5$
$= 2.0 \times 10^{-1} \times 10^5$
$= 2.0 \times 10^4$

23. $\frac{(7.5 \times 10^8)(4.5 \times 10^{-4})}{1.5 \times 10^7} = \frac{(7.5 \times 4.5)(10^8 \times 10^{-4})}{1.5 \times 10^7}$
$= \frac{33.75 \times 10^4}{1.5 \times 10^7}$
$= \frac{3.375 \times 10^1 \times 10^4}{1.5 \times 10^7}$
$= \frac{3.375 \times 10^5}{1.5 \times 10^7}$
$= \frac{3.375}{1.5} \times \frac{10^5}{10^7}$
$= 2.25 \times 10^{-2}$

24. $\frac{w^{-2}}{w^6} = w^{-2-6}$ Quotient of powers property
$= w^{-8}$
$= \frac{1}{w^8}$ Negative exponent property

25. $(2^2y^3)^5 = (2^2)^5(y^3)^5$ Power of a product property
$= 2^{10}y^{15}$ Power of a power property
$= 1024y^{15}$

26. $(p^3q^2)^{-1} = p^{-3}q^{-2}$ Power of a product property
$= \frac{1}{p^3q^2}$ Negative exponent property

27. $(w^3x^{-2})(w^6x^{-1}) = w^{3+6}x^{-2+(-1)}$ Product of powers property
$= w^9x^{-3}$
$= \frac{w^9}{x^3}$ Negative exponent property

28. $(5s^{-2}t^4)^{-3} = 5^{-3}(s^{-2})^{-3}(t^4)^{-3}$ Power of a product property

$= \frac{1}{125} \cdot s^{-2(-3)} \cdot t^{4(-3)}$ Power of a power property

$= \frac{1}{125} \cdot s^6 \cdot t^{-12}$

$= \frac{s^6}{125t^{12}}$ Negative exponent property

29. $(3a^3b^5)^{-3} = (3)^{-3}(a^3)^{-3}(b^5)^{-3}$ Power of a product property

$= \frac{1}{27}a^{3(-3)}b^{5(-3)}$ Power of a power property

$= \frac{1}{27}a^{-9}b^{-15}$

$= \frac{1}{27a^9b^{15}}$ Negative exponent property

30. $\frac{x^{-1}y^2}{x^2y^{-1}} = x^{-1-2}y^{2-(-1)}$ Quotient of powers property

$= x^{-3}y^3$

$= \frac{y^3}{x^3}$ Negative exponent property

31. $\frac{3c^3d}{9cd^{-1}} = \frac{3}{9}c^{3-1}d^{1-(-1)}$ Quotient of powers property

$= \frac{c^2d^2}{3}$

32. $\frac{4r^4s^5}{24r^4s^{-5}} = \frac{4}{24}r^{4-4}s^{5-(-5)}$ Quotient of powers property

$= \frac{1}{6}r^0s^{10}$

$= \frac{1}{6} \cdot 1 \cdot s^{10}$ Zero exponent property

$= \frac{s^{10}}{6}$

33. $\frac{2a^3b^{-4}}{3a^5b^{-2}} = \frac{2}{3}a^{3-5}b^{-4-(-2)}$ Quotient of powers property

$= \frac{2}{3}a^{-2}b^{-2}$

$= \frac{2}{3a^2b^2}$ Negative exponent property

34. $\frac{y^{11}}{4z^3} \cdot \frac{8z^7}{y^7} = \frac{8y^{11}z^7}{4y^7z^3}$

$= 2y^{11-7}z^{7-3}$ Quotient of powers property

$= 2y^4z^4$

35. $\frac{x^2y^{-3}}{3y^2} \cdot \frac{y^2}{x^{-4}} = \frac{x^2y^{-1}}{3x^{-4}y^2}$ Product of powers property

$= \frac{x^{2-(-4)}y^{-1-2}}{3}$ Quotient of powers property

$= \frac{x^6y^{-3}}{3}$

$= \frac{x^6}{3y^3}$ Negative exponent property

36. B;

$\frac{2x^2y}{6xy^{-1}} = \frac{x^{2-1}y^{1-(-1)}}{3}$

$= \frac{xy^2}{3}$

37. The error is that the exponents were divided and not subtracted.

$\frac{x^{10}}{x^2} = x^{10-2}$

$= x^8$

38. The error is that the exponents were multiplied instead of added.

$x^5 \cdot x^3 = x^{5+3}$

$= x^8$

39. The error is that the bases were multiplied.

$(-3)^2 \cdot (-3)^4 = (-3)^6$

40. $A = \frac{\sqrt{3}}{4}s^2$ $\qquad s = \frac{x}{3}$

$= \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

$= \frac{\sqrt{3}}{4}\left(\frac{x^2}{3^2}\right)$

$= \frac{\sqrt{3}x^2}{4 \cdot 9}$

$= \frac{\sqrt{3}x^2}{36}$

41. $v = \pi r^2h$ $\qquad r = x \qquad h = \frac{x}{2}$

$= \pi(x)^2\left(\frac{x}{2}\right)$

$= \pi x^2 \cdot \frac{x}{2}$

$= \frac{\pi x^3}{2}$

42. $v = \ell \cdot w \cdot h$ $\qquad \ell = 2x, w = \frac{5x}{3}, h = x$

$= (2x)\left(\frac{5x}{3}\right)(x)$

$= \frac{10x^3}{3}$

43. $x^{15}y^{12}z^8 = x^4y^7z^{11} \cdot ?$

$\frac{x^{15}y^{12}z^8}{x^4y^7z^{11}} = ?$

$x^{15-4}y^{12-7}z^{8-11} = ?$

$x^{11}y^5z^{-3} = \frac{x^{11}y^5}{z^3} = ?$

44. $3x^3y^2 = \frac{12x^2y^5}{?}$

$? = \frac{12x^2y^5}{3x^3y^2}$

$= 4x^{2-3}y^{5-2}$

$= 4x^{-1}y^3$

$= \frac{4y^3}{x}$

45. $(a^5b^4)^2 = a^{14}b^{-1} \cdot ?$

$a^{5 \cdot 2}b^{4 \cdot 2} = a^{14}b^{-1} \cdot ?$

$a^{10}b^8 = a^{14}b^{-1} \cdot ?$

$\dfrac{a^{10}b^8}{a^{14}b^{-1}} = ?$

$a^{10-14} \cdot b^{8-(-1)} = a^{-4}b^9 = \dfrac{b^9}{a^4} = ?$

46. *Sample answer:*

$x^{12}x^{16} = (x^4y^4)(x^8y^{12}) = (x^2y^{15})(x^{10}y^1) = (x^{-5}y^7)(x^{17}y^9)$

47. $\dfrac{1}{a^m} = \dfrac{a^0}{a^m} = a^{0-m} = a^{-m}$

48. $\dfrac{a^m}{a^n} = a^m \cdot \dfrac{1}{a^n}$

$a^{-n} \cdot a^m = a^{-n+m} = a^{m-n}$

Problem Solving

49. Pacific: $V = (1.56 \times 10^{14})(4.03 \times 10^3)$

$= (1.56 \times 4.03)(10^{14} \times 10^3)$

$= 6.2868 \times 10^{17}$ meters^3

Atlantic: $V = (7.68 \times 10^{13})(3.93 \times 10^3)$

$= (7.68 \times 3.93)(10^{13} \times 10^3)$

$= 30.1824 \times 10^{16}$

$= 3.01824 \times 10^1 \times 10^{16}$

$= 3.01824 \times 10^{17}$ meters^3

Indian: $V = (6.86 \times 10^{13})(3.96 \times 10^3)$

$= (6.86 \times 3.96)(10^{13} \times 10^3)$

$= 27.1656 \times 10^{16}$

$= 2.71656 \times 10^1 \times 10^{16}$

$= 2.71656 \times 10^{17}$ meters^3

Arctic: $V = (1.41 \times 10^{13})(1.21 \times 10^3)$

$= (1.41 \times 1.21)(10^{13} \times 10^3)$

$= 1.7061 \times 10^{16}$ meters^3

50. $(0.000022)(125{,}000{,}000) = (2.2 \times 10^{-5})(1.25 \times 10^8)$

$= (2.2 \times 1.25)(10^{-5} \times 10^8)$

$= 2.75 \times 10^3$

The continent has moved 2.75×10^3 miles.

51. Bead: $d = 6$ mm

$r = 3$ mm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi$

Pearl : $d = 9$ mm

$r = \frac{9}{2}$ mm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{9}{2}\right)^3 = \frac{243\pi}{2}$

$\dfrac{\text{Volume of pearl}}{\text{Volume of bead}} = \dfrac{\frac{243\pi}{2}}{36\pi} = \dfrac{243}{72} = 3.375$

The volume of the pearl is about 3.375 times greater than the volume of the bead.

52. a. $V = 3\left(\frac{4}{3}\pi r^3\right) = 4\pi r^3$

b. $V = \pi r^2 h$

c. $h = 3(2r) = 6r$

d. Volume of cylinder $= \pi r^2(6r) = 6\pi r^3$

Occupied volume $= \dfrac{4\pi r^3}{6\pi r^3} = \dfrac{2}{3}$

The tennis balls take up $\frac{2}{3}$ of the can's volume.

53. a. $V = \pi r^2 h = \pi(9.53)^2(1.55) \approx 442\text{ mm}^3 = 4.42 \times 10^{-7}\text{ m}^3$

b. Answers will vary.

c. pennies $= \dfrac{\text{Volume of classroom}}{\text{Volume of 1 penny}}$

The answer is an overestimate because of the shape of the pennies. There will be gaps between them.

54. a. Volume of inner core $= \frac{4}{3}\pi r^3$

Earth's total volume $= \frac{4}{3}\pi(5r)^3$

$= \frac{4}{3}\pi(125r^3)$

$= 125 \cdot \frac{4}{3}\pi r^3$

$= 125 \cdot$ Volume of inner core

ratio $= 125 : 1$

b. Volume of outer core $= \frac{4}{3}\pi\left(\frac{11}{6}r + r\right)^3 - \frac{4}{3}\pi r^3$

$= \frac{4913}{216}\left(\frac{4}{3}\pi r^3\right) - \frac{4}{3}\pi r^3$

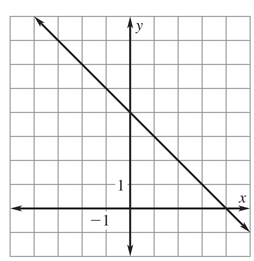
$= \frac{4697}{216}\left(\frac{4}{3}\pi r^3\right)$

ratio $= \dfrac{\frac{4697}{216} \cdot \frac{4}{3}\pi r^3}{\frac{4}{3}\pi r^3}$

$= \dfrac{4697}{216}$

Mixed Review

55.



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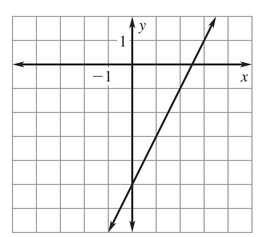

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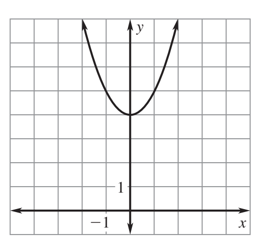

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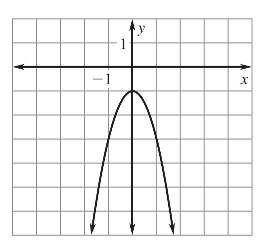

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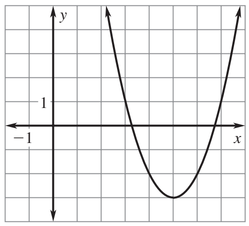

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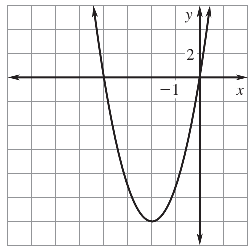


61. $x + y = 2$

$7x + 8y = 21$

$$\overset{A}{\begin{bmatrix} 1 & 1 \\ 7 & 8 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 2 \\ 21 \end{bmatrix}}$$

$$A^{-1} = \frac{1}{8-7}\begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 21 \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution is $(-5, 7)$.

62. $-x - 2y = 3$

$2x + 8y = 1$

$$\overset{A}{\begin{bmatrix} -1 & -2 \\ 2 & 8 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 3 \\ 1 \end{bmatrix}}$$

$$A^{-1} = \frac{1}{-8-(-4)}\begin{bmatrix} 8 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -2 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{13}{2} \\ \frac{7}{4} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution is $\left(-\frac{13}{2}, \frac{7}{4}\right)$.

63. $4x + 3y = 6$

$6x - 2y = 10$

$$\overset{A}{\begin{bmatrix} 4 & 3 \\ 6 & -2 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 6 \\ 10 \end{bmatrix}}$$

$$A^{-1} = \frac{1}{-8-18}\begin{bmatrix} -2 & -3 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{26} \\ \frac{3}{13} & \frac{2}{13} \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} \frac{1}{13} & \frac{3}{26} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix}\begin{bmatrix} 6 \\ 10 \end{bmatrix} = \begin{bmatrix} \frac{21}{13} \\ -\frac{2}{13} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution is $\left(\frac{21}{13}, -\frac{2}{13}\right)$.

64. $(8 + 3i) - (7 + 4i) = 8 + 3i - 7 - 4i = 1 - i$

65. $(5 - 2i) - (-9 + 6i) = 5 - 2i + 9 - 6i = 14 - 8i$

66. $i(3 + i) = 3i + i^2 = 3i + (-1) = -1 + 3i$

67. $(12 + 5i) - (7 - 8i) = 12 + 5i - 7 + 8i = 5 + 13i$

68. $(5 + 4i)(2 + 3i) = 10 + 15i + 8i + 12i^2$

$= 10 + 23i + 12(-1)$

$= -2 + 23i$

69. $(8 - 4i)(1 + 6i) = 8 + 48i - 4i - 24i^2$

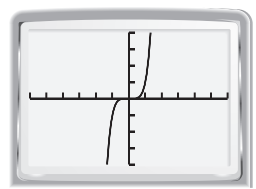
$= 8 + 44i - 24(-1)$

$= 32 + 44i$

Lesson 5.2

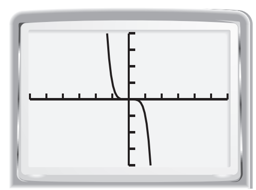
Investigating Algebra Activity 5.2 (p. 336)

1.



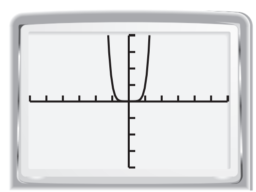
As x approaches $-\infty$, $f(x)$ approaches $-\infty$. As x approaches $+\infty$, $f(x)$ approaches $+\infty$.

2.



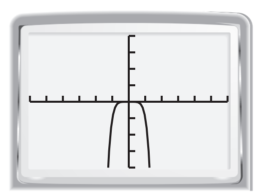
As x approaches $-\infty$, $f(x)$ approaches $+\infty$. As x approaches $+\infty$, $f(x)$ approaches $-\infty$.

3.



As x approaches $-\infty$, $f(x)$ approaches $+\infty$. As x approaches $+\infty$, $f(x)$ approaches $+\infty$.

4.



As x approaches $-\infty$, $f(x)$ approaches $-\infty$. As x approaches $+\infty$, $f(x)$ approaches $-\infty$.

5. **a.** $f(x) = x^n$, where x is odd: As x approaches $+\infty$, $f(x)$ approaches $+\infty$. As x approaches $-\infty$, $f(x)$ approaches $-\infty$.

b. $f(x) = -x^n$, where n is odd: As x approaches $-\infty$, $f(x)$ approaches $+\infty$. As x approaches $+\infty$, $f(x)$ approaches $-\infty$.

c. $f(x) = x^n$, where n is even : As x approaches $-\infty$, $f(x)$ approaches $+\infty$. As x approaches $+\infty$, $f(x)$ approaches $+\infty$.

d. $f(x) = -x^n$, where x is odd : As x approaches $-\infty$, $f(x)$ approaches $-\infty$. As x approaches $+\infty$, $f(x)$ approaches $-\infty$.

6. As x approaches $-\infty$, $f(x)$ approaches $+\infty$. As x approaches $+\infty$, $f(x)$ approaches $+\infty$. $f(x)$ is going to resemble its highest power. Because x^6 is the highest power, the end behavior will be the same as $f(x) = x^6$.

5.2 Guided Practice (pp. 338–341)

1. The function is a polynomial function written as $f(x) = -2x + 13$ in standard form. It has degree 1 (linear) and a leading coefficient of -2.

2. The function is not a polynomial function because the term $5x^{-2}$ has an exponent that is not a whole number.

3. The function is a polynomial function written as $h(x) = 6x^2 - 3x + \pi$ in standard form. It has a degree of 2 (quadratic) and a leading coefficient of 6.

4. $f(x) = x^4 + 2x^3 + 3x^2 - 7$

$$\begin{aligned} f(-2) &= (-2)^4 + 2(-2)^3 + 3(-2)^2 - 7 \\ &= 16 - 16 + 12 - 7 \\ &= 5 \end{aligned}$$

5. $g(x) = x^3 - 5x^2 + 6x + 1$

$$\begin{aligned} g(4) &= (4)^3 - 5(4)^2 + 6(4) + 1 \\ &= 64 - 80 + 24 + 1 \\ &= 9 \end{aligned}$$

6.
$$\begin{array}{r|rrrr} 2 & 5 & 3 & -1 & 7 \\ & & 10 & 26 & 50 \\ \hline & 5 & 13 & 25 & 57 \end{array}$$

$f(2) = 57$

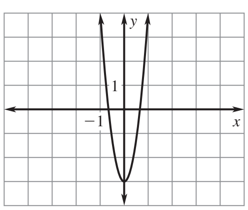
7.
$$\begin{array}{r|rrrrr} -1 & -2 & -1 & 0 & 4 & -5 \\ & & 2 & -1 & 1 & -5 \\ \hline & -2 & 1 & -1 & 5 & -10 \end{array}$$

$f(-1) = -10$

8. The degree is odd and the leading coefficient is negative.

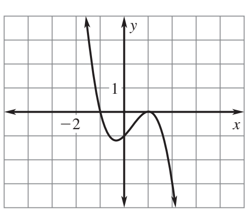
9.

x	-3	-2	-1	0	1	2	3
y	132	37	4	-3	4	37	132



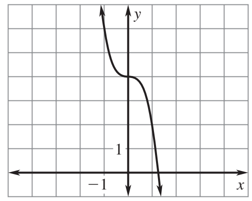
10.

x	-3	-2	-1	0	1	2	3
y	32	9	0	-1	0	-3	-16



11.

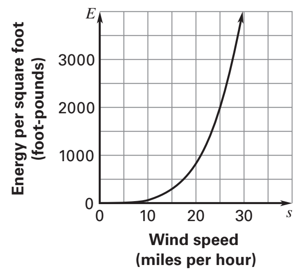
x	-3	-2	-1	0	1	2	3
y	58	20	6	4	2	-12	-50



12.

s	0	5	10	15	20	25	30
E	0	3.2	51	258.2	816	1992.2	4131

The wind speed needed to generate the wave is about 25 miles per hour.



5.2 Exercises (pp. 341–344)

Skill Practice

1. $f(x)$ is a degree 4 (quartic) polynomial function. The leading coefficient is -5 while the constant term is 6.

2. The end behavior of a polynomial is the behavior the function demonstrates as it approaches $\pm\infty$.

3. The function is a polynomial function written as $f(x) = -x^2 + 8$ in standard form. It has a degree 2 (quadratic) and a leading coefficient of -1.

4. The function is a polynomial function written as $f(x) = 8x^4 + 6x - 3$ in standard form. It has a degree 4 (quartic) and a leading coefficient of 8.

5. The function is a polynomial that is already written in standard form. It has degree 4 (quartic) and a leading coefficient of π.

6. The function is not a polynomial function because the term $5x^{-2}$ has an exponent that is not a whole number.

7. The function is a polynomial function already written in standard form. It has degree 3 (cubic) and a leading coefficient of $-\frac{5}{2}$.

8. The function is not a polynomial function because the term $\frac{2}{x}$ has an exponent that is not a whole number.

9. $f(x) = 5x^3 - 2x^2 + 10x - 15$

$$\begin{aligned} f(-1) &= 5(-1)^3 - 2(-1)^2 + 10(-1) - 15 \\ &= -5 - 2 - 10 - 15 \\ &= -32 \end{aligned}$$

10. $f(x) = 5x^4 - x^3 - 3x^2 + 8x$

$f(2) = 5(2)^4 - (2)^3 - 3(2)^2 + 8(2)$

$= 80 - 8 - 12 + 16$

$= 76$

11. $g(x) = -2x^5 + 4x^3$

$g(-3) = -2(-3)^5 + 4(-3)^3 = 486 - 108 = 378$

12. $h(x) = 6x^3 - 25x + 20$

$h(5) = 6(5)^3 - 25(5) + 20 = 750 - 125 + 20 = 645$

13. $h(x) = \frac{1}{2}x^4 - \frac{3}{4}x^3 + x + 10$

$h(-4) = \frac{1}{2}(-4)^4 - \frac{3}{4}(-4)^3 + (-4) + 10$

$= 128 + 48 - 4 + 10$

$= 182$

14. $g(x) = 4x^5 + 6x^3 + x^2 - 10x + 5$

$g(-2) = 4(-2)^5 + 6(-2)^3 + (-2)^2 - 10(-2) + 5$

$= -128 - 48 + 4 + 20 + 5$

$= -147$

15.
$$\begin{array}{r|rrrr} 3 & 5 & -2 & -8 & 16 \\ & & 15 & 39 & 93 \\ \hline & 5 & 13 & 31 & 109 \end{array}$$

$f(3) = 109$

16.
$$\begin{array}{r|rrrrr} -2 & 8 & 12 & 6 & -5 & 9 \\ & & -16 & 8 & -28 & 66 \\ \hline & 8 & -4 & 14 & -33 & 75 \end{array}$$

$f(-2) = 75$

17.
$$\begin{array}{r|rrrr} -6 & 1 & 8 & -7 & 35 \\ & & -6 & -12 & 114 \\ \hline & 1 & 2 & -19 & 149 \end{array}$$

$f(-6) = 149$

18.
$$\begin{array}{r|rrrr} 4 & -8 & 0 & 14 & -35 \\ & & -32 & -128 & -456 \\ \hline & -8 & -32 & -114 & -491 \end{array}$$

$f(4) = -491$

19.
$$\begin{array}{r|rrrrr} 2 & -2 & 3 & 0 & -8 & 13 \\ & & -4 & -2 & -4 & -24 \\ \hline & -2 & -1 & -2 & -12 & -11 \end{array}$$

$f(2) = -11$

20.
$$\begin{array}{r|rrrrrr} -3 & 6 & 0 & 10 & 0 & 0 & -27 \\ & & -18 & 54 & -192 & 576 & -1728 \\ \hline & 6 & -18 & 64 & -192 & 576 & -1755 \end{array}$$

$f(-3) = -1755$

21.
$$\begin{array}{r|rrrr} 3 & -7 & 11 & 4 & 0 \\ & & -21 & -30 & -78 \\ \hline & -7 & -10 & -26 & -78 \end{array}$$

$f(3) = -78$

22.
$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & 3 & -20 \\ & & 4 & 16 & 64 & 268 \\ \hline & 1 & 4 & 16 & 67 & 248 \end{array}$$

$f(4) = 248$

23. The error is that a zero was not placed as the coefficient of the missing x^3 term.

$$\begin{array}{r|rrrrr} -2 & -4 & 0 & 9 & -21 & 7 \\ & & 8 & -16 & 14 & 14 \\ \hline & -4 & 8 & -7 & -7 & 21 \end{array}$$

$f(-2) = 21 \neq 117$

24. A; The ends approach opposite directions so the function must be odd. It imitates an odd function whose coefficient is positive.

25. The degree is even and the leading coefficient is positive.

26. The degree is odd and the leading coefficient is negative.

27. The degree is even and the leading coefficient is negative.

28. $f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

29. $f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

30. $f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

31. $f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

32. $f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

33. $f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

34. $f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

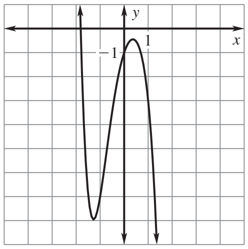
35. $f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

36. $f(x) \to +\infty$ as $x \to -\infty$

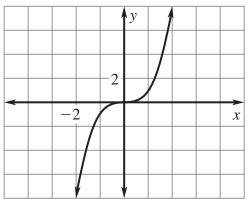
$f(x) \to -\infty$ as $x \to +\infty$

37. *Sample answer:* $f(x) = -x^5 - 4x^2 + 3x - 1$



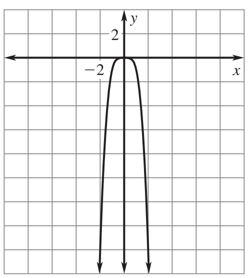
38.

x	-3	-2	-1	0	1	2	3
y	-27	-8	-1	0	1	8	27



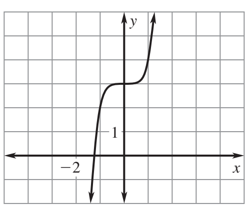
39.

x	-3	-2	-1	0	1	2	3
y	-81	-16	-1	0	-1	-16	-81



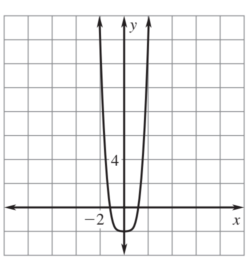
40.

x	-3	-2	-1	0	1	2	3
y	-240	-29	2	3	4	35	246



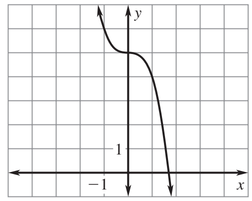
41.

x	-3	-2	-1	0	1	2	3
y	79	14	-1	-2	-1	14	79



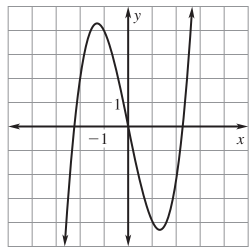
42.

x	-3	-2	-1	0	1	2	3
y	32	13	6	5	4	-3	-22



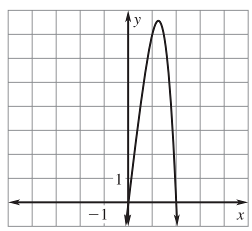
43.

x	-3	-2	-1	0	1	2	3
y	-12	2	4	0	-4	-2	12



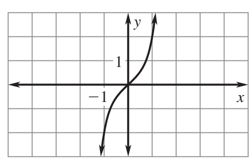
44.

x	-3	-2	-1	0	1	2	3
y	-105	-32	-9	0	7	0	-57



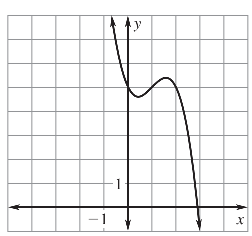
45.

x	-3	-2	-1	0	1	2	3
y	-246	-34	-2	0	2	34	246



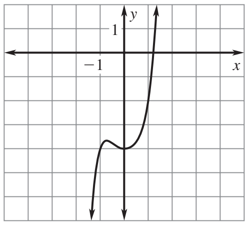
46.

x	-3	-2	-1	0	1	2	3
y	65	29	11	5	5	5	-1



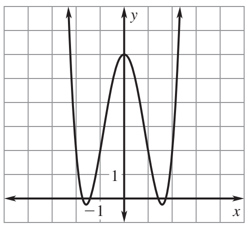
47.

x	-3	-2	-1	0	1	2	3
y	-238	-32	-4	-4	-2	32	248



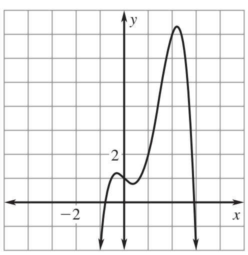
48.

x	-3	-2	-1	0	1	2	3
y	42	2	2	6	2	2	42



49.

x	-3	-2	-1	0	1	2	3
y	-158	-37	-2	1	2	7	-2



50. B; $f(x) = -\frac{1}{3}x^3 + 1$

From the graph, $f(x) \to +\infty$ as $x \to -\infty$; and $f(x) \to -\infty$ as $x \to +\infty$. So, the degree is odd and the leading coefficient is negative. The constant term $= f(0) = 1$.

51. When $g(x) = -f(x)$:

$g(x) \to -\infty$ as $x \to -\infty$ and $g(x) \to +\infty$ as $x \to +\infty$.

52. $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

When $a_3 = 2$ and $a_0 = -5$:

$f(x) = 2x^3 + a_2x^2 + a_1x - 5$

When $f(1) = 0$:

$2(1)^3 + a_2(1)^2 + a_1(1) - 5 = 0$

$a_2 + a_1 = 3$

When $f(2) = 3$:

$2(2)^3 + a_2(2)^2 + a_1(2) - 5 = 3$

$4a_2 + 2a_1 = -8$

Solve the system of equations:

$a_2 + a_1 = 3 \xrightarrow{\times -4} -4a_2 - 4a_1 = -12$

$4a_2 + 2a_1 = -8 \longrightarrow 4a_2 + 2a_1 = -8$

$-2a_1 = -20$

$a_1 = 10$

Substitute a_1 into one of the original equations. Solve for a_2.

$a_2 + 10 = 3 \longrightarrow a_2 = -7$

The cubic function is $f(x) = 2x^3 - 7x^2 + 10x - 5$.

So, $f(-5) = 2(-5)^3 - 7(-5)^2 + 10(-5) - 5 = -480$.

53. a.

x	**f(x)**	**g(x)**	$\frac{f(x)}{g(x)}$
10	1000	840	1.19
20	8000	7280	1.10
50	125,000	120,200	1.04
100	1,000,000	980,400	1.02
200	8,000,000	7,920,800	1.01

b. As $x \to +\infty$, $\frac{f(x)}{g(x)} \to 1$.

c. For the graph of a polynomial function, the end behavior is determined by the function's degree and the sign of its leading coefficient.

Problem Solving

54. $w = 0.00071d^3 - 0.090d^2 + 0.48d$

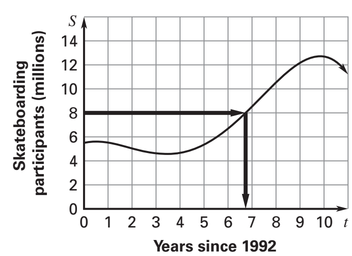
$w = 0.0071(15)^3 - 0.090(15)^2 + 0.48(15)$

$= 23.9625 - 20.25 + 7.2$

$= 10.9125$ carats

55.

t	0	2	4	6	8
S	5.5	5.1	4.7	6.7	10.5



There were 8 million skateboarders 6 years after 1992 (1998).

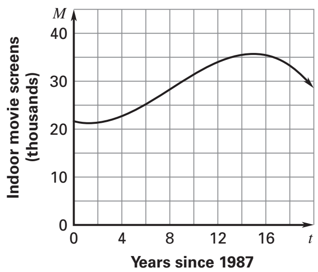
56. a. The function is degree 3 (cubic).

b.

t	0	2	4	6	8
M	21,600	21,396	22,800	25,284	28,320

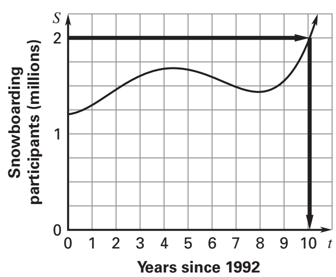
t	10	12	14	16
M	31,380	33,936	35,460	35,424

c.



57.

t	0	2	4	6	8	10	12
s	1.2	1.46	1.68	1.59	1.44	1.97	4.41

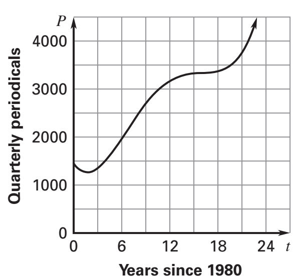


The number of snowboarders was more than 2 million in 2002.

58. a. As $t \to +\infty$, $P(x) \to +\infty$.

As $t \to -\infty$, $P(x) \to +\infty$.

b.



c. $p(t) = 0.138t^4 - 6.24t^3 + 86.8t^2 - 239t + 1450$

$$p(30) = 0.138(30)^4 - 6.24(30)^3 + 86.8(30)^2 - 239(30) + 1450$$

$$= 111{,}780 - 168{,}480 + 78{,}120 - 7170 + 1450$$

$$= 15{,}700 \text{ periodicals}$$

It is not appropriate to use this model to predict the number of periodicals in 2010 beacuse the model was made for 1980 to 2002.

59. a. $S = -0.122t^3 + 3.49t^2 - 14.6t + 136$

$$S(5) = -0.122(5)^3 + 3.49(5)^2 - 14.6(5) + 136$$

$$= -15.25 + 87.25 - 73 + 136$$

$$= 135 \text{ grams}$$

$$H = -0.115t^3 + 3.71t^2 - 20.6t + 124$$

$$H(5) = -0.115(5)^3 + 3.71(5)^2 - 20.6(5) + 124$$

$$= -14.375 + 92.75 - 103 + 124$$

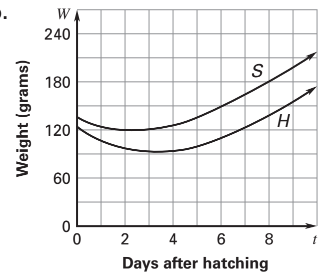
$$= 99.375 \text{ grams}$$

$$\text{Difference} = S(5) - H(5)$$

$$= 135 - 99.375$$

$$= 35.625 \text{ grams}$$

b.



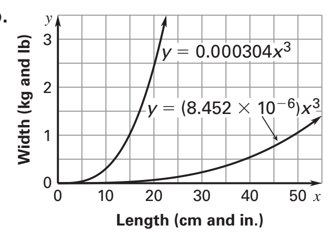
c. The chick is more likely to be a sarus chick than a hooded chick. At 3 days the sarus chick's weight is about 120 grams, higher than the hooded chicks. Even though neither chick's average weight at 3 days equals 130 grams, the sarus chick is closer.

60. a. $y = 0.000304x^3$

$$2.20y = 0.000304(0.394x)^3$$

$$y = \frac{0.000304(0.394x)^3}{2.2} \approx (8.452 \times 10^{-6})x^3$$

b.



The function from part (a) is a vertical shrink of the original function.

Mixed Review

61. $2b + 11 = 15 - 6b$

$$2b = 4 - 6b$$

$$8b = 4$$

$$b = \frac{1}{2}$$

62. $2.7n + 4.3 = 12.94$

$$2.7n = 8.64$$

$$n = 3.2$$

63. $-7 < 6y - 1 < 5$

$$-6 < 6y < 6$$

$$-1 < y < 1$$

64. $x^2 - 14x + 48 = 0$

$(x - 6)(x - 8) = 0$

$x - 6 = 0$ or $x - 8 = 0$

$x = 6$ or $x = 8$

65. $-24q^2 - 90q = 21$

$-24q^2 - 90q - 21 = 0$

$8q^2 + 30q + 7 = 0$

$(4q + 1)(2q + 7) = 0$

$4q + 1 = 0$ or $2q + 7 = 0$

$4q = -1$ or $2q = -7$

$q = -\frac{1}{4}$ or $q = -\frac{7}{2}$

66. $z^2 + 5z < 36$

$z^2 + 5z - 36 < 0$

$z^2 + 5z - 36 = 0$

$(z + 9)(z - 4) = 0$

$z + 9 = 0$ $(z - 4) = 0$

$z = -9$ $z = 4$

Part of z-axis	Chosen pt., x	$z^2 + 5z - 36$
$z < -9$	-10	$(-10)^2 + 5(-10) - 36 = 14$
$-9 < z < 4$	0	$(0)^2 + 5(0) - 36 = -36$
$z > 4$	5	$(5)^2 + 5(5) - 36 = 14$

Is $z^2 + 5z - 36 < 0$?
no
yes
no

Solution: $-9 < z < 4$

67. $y = ax$

$12 = 4a$

$3 = a$

$y = 3x$

$y = -3$: $-3 = 3x$

$x = \frac{-3}{3} = -1$

68. $y = ax$

$-21 = 3a$

$-7 = a$

$y = -7x$

$y = -3$: $-3 = -7x$

$x = \frac{-3}{-7} = \frac{3}{7}$

69. $y = ax$

$-4 = 10a$

$-\frac{2}{5} = a$

$y = -\frac{2}{5}x$

$y = -3$:$-3 = -\frac{2}{5}x$

$x = \frac{15}{2}$

70. $y = ax$

$0.2 = 0.8a$

$0.25 = a$

$y = 0.25x$

$y = -3$: $-3 = 0.25x$

$x = -3 \cdot \frac{1}{0.25} = -12$

71. $y = ax$

$-0.35 = -0.45a$

$\frac{7}{9} = a$

$y = \frac{7}{9}x$

$y = -3$: $-3 = \frac{7}{9}x$

$x = \frac{-3}{\frac{7}{9}} = -3\frac{6}{7}$

72. $y = ax$

$3.9 = -6.5a$

$-0.6 = a$

$y = -0.6x$

$y = -3$: $-3 = -0.6x$

$x = \frac{-3}{-0.6} = 5$

73. $y = (x + 3)(x - 7) = x^2 - 7x + 3x - 21$

$= x^2 - 4x - 21$

74. $y = 8(x - 4)(x + 2)$

$= 8(x^2 + 2x - 4x - 8)$

$= 8(x^2 - 2x - 8)$

$= 8x^2 - 16x - 64$

75. $y = -3(x - 5)^2 - 25$

$= -3(x^2 - 10x + 25) - 25$

$= -3x^2 + 30x - 75 - 25$

$= -3x^2 + 30x - 100$

76. $y = 2.5(x - 6)^2 + 9.3$

$= 2.5(x^2 - 12x + 36) + 9.3$

$= 2.5x^2 - 30x + 90 + 9.3$

$= 2.5x^2 - 30x + 99.3$

77. $y = \frac{1}{2}(x - 4)^2$

$= \frac{1}{2}(x^2 - 8x + 16)$

$= \frac{1}{2}x^2 - 4x + 8$

78. $y = -\frac{5}{3}(x + 4)(x + 9)$

$= -\frac{5}{3}(x^2 + 9x + 4x + 36)$

$= -\frac{5}{3}(x^2 + 13x + 36)$

$= -\frac{5}{3}x^2 - \frac{65}{3}x - 60$

Graphing Calculator Activity 5.2 (p. 345)

1. $-10 \le x \le 10,\ -10 \le y \le 50$
2. $-20 \le x \le 50,\ -2000 \le y \le 8000$
3. $-10 \le x \le 10,\ -10 \le x \le 10$
4. $-5 \le x \le 5,\ -10 \le x \le 30$
5. $-5 \le x \le 10,\ -10 \le x \le 50$
6. $-5 \le x \le 10,\ -40 \le x \le 40$
7. $-5 \le x \le 10,\ -20 \le x \le 50$
8. $-5 \le x \le 5,\ -20 \le x \le 20$
9. *Sample answer:* The window for $g(x)$ should be the same x-interval, but the y-interval will be shifted by c units.
10. *Sample answer:* $-10 \le x \le 20,\ 0 \le y \le 3000$

Lesson 5.3

5.3 Guided Practice (pp. 346–348)

1. $(t^2 - 6t + 2) + (5t^2 - t - 8)$
$= t^2 + 5t^2 - 6t - t + 2 - 8$
$= 6t^2 - 7t - 6$

2. $(8d - 3 + 9d^3) - (d^3 - 13d^2 - 4)$
$= 8d - 3 + 9d^3 - d^3 + 13d^2 + 4$
$= 9d^3 - d^3 + 13d^2 + 8d - 3 + 4$
$= 8d^3 + 13d^2 + 8d + 1$

3. $(x + 2)(3x^2 - x - 5)$
$= (x + 2)3x^2 - (x + 2)(x) - (x + 2)(5)$
$= 3x^3 + 6x^2 - x^2 - 2x - 5x - 10$
$= 3x^3 + 5x^2 - 7x - 10$

4. $(a - 5)(a + 2)(a + 6) = (a^2 - 3a - 10)(a + 6)$
$= (a^2 - 3a - 10)a + (a^2 - 3a - 10)6$
$= a^3 - 3a^2 - 10a + 6a^2 - 18a - 60$
$= a^3 + 3a^2 - 28a - 60$

5. $(xy - 4)^3 = (xy)^3 - 3(xy)^2(4) + 3(xy)(4)^2 - (4)^3$
$= x^3y^3 - 12x^2y^2 + 48xy - 64$

6. $T = C \cdot D$

$$
\begin{array}{rrrr}
= & 0.542t^2 & -\ 7.16t & +\ 79.4 \\
\times & & 109t & +\ 4010 \\
\hline
 & 2173.42t^2 & -\ 28711.6t & +\ 318{,}394 \\
5.9078t^3 & -\ 780.44t^2 & +\ 8654.6t & \\
\hline
59.078t^3 & +\ 1392.98t^2 & -\ 20{,}057t & +\ 318{,}394
\end{array}
$$

$T = 59.078t^3 + 1392.98t^2 - 20{,}057t + 318{,}394$

5.3 Exercises (pp. 349–352)

Skill Practice

1. When you add or subtract polynomials, you add or subtract the coefficients of *like terms*.
2. Subtracting two polynomials is simply adding the opposite of the subtracted polynomial to the first polynomial.
3. $(3x^2 - 5) + (7x^2 - 3) = 3x^2 + 7x^2 - 5 - 3$
$= 10x^2 - 8$

4. $(x^2 - 3x + 5) - (-4x^2 + 8x + 9)$
$= x^2 - 3x + 5 + 4x^2 - 8x - 9$
$= x^2 + 4x^2 - 3x - 8x + 5 - 9$
$= 5x^2 - 11x - 4$

5. $(4y^2 + 9y - 5) - (4y^2 - 5y + 3)$
$= 4y^2 + 9y - 5 - 4y^2 + 5y - 3$
$= 4y^2 - 4y^2 + 9y + 5y - 5 - 3$
$= 14y - 8$

6. $(z^2 + 5z - 7) + (5z^2 - 11z - 6)$
$= z^2 + 5z^2 + 5z - 11z - 7 - 6$
$= 6z^2 - 6z - 13$

7. $(3s^3 + s) + (4s^3 - 2s^2 + 7s + 10)$
$= 3s^3 + 4s^3 - 2s^2 + s + 7s + 10$
$= 7s^3 - 2s^2 + 8s + 10$

8. $(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4)$
$= 2a^2 - 8 - a^3 - 4a^2 + 12a - 4$
$= -a^3 + 2a^2 - 4a^2 + 12a - 8 - 4$
$= -a^3 - 2a^2 + 12a - 12$

9. $(5c^2 + 7c + 1) + (2c^3 - 6c + 8)$
$= 2c^3 + 5c^2 + 7c - 6c + 1 + 8$
$= 2c^3 + 5c^2 + c + 9$

10. $(4t^3 - 11t^2 + 4t) - (-\ 7t^2 - 5t + 8)$
$= 4t^3 - 11t^2 + 4t + 7t^2 + 5t - 8$
$= 4t^3 - 11t^2 + 7t^2 + 4t + 5t - 8$
$= 4t^3 - 4t^2 + 9t - 8$

11. $(5b - 6b^3 + 2b^4) - (9b^3 + 4b^4 - 7)$
$= 5b - 6b^3 + 2b^4 - 9b^3 - 4b^4 + 7$
$= 2b^4 - 4b^4 - 6b^3 - 9b^3 + 5b + 7$
$= -2b^4 - 15b^3 + 5b + 7$

12. $(3y^2 - 6y^4 + 5 - 6y) + (5y^4 - 6y^3 + 4y)$
$= -6y^4 + 5y^4 - 6y^3 + 3y^2 - 6y + 4y + 5$
$= -y^4 - 6y^3 + 3y^2 - 2y + 5$

13. $(x^4 - x^3 + x^2 - x + 1) + (x + x^4 - 1 - x^2)$
$= x^4 + x^4 - x^3 + x^2 - x^2 - x + x + 1 - 1$
$= 2x^4 - x^3$

14. $(8v^4 - 2v^2 + v - 4) - (3v^3 - 12v^2 + 8v)$
$= 8v^4 - 2v^2 + v - 4 - 3v^3 + 12v^2 - 8v$
$= 8v^4 - 3v^3 - 2v^2 + 12v^2 + v - 8v - 4$
$= 8v^4 - 3v^3 + 10v^2 - 7v - 4$

15. B;
$(8x^4 - 4x^3 - x + 2) - (2x^4 - 8x^2 - x + 10)$
$= 8x^4 - 4x^3 - x + 2 - 2x^4 + 8x^2 + x - 10$
$= 8x^4 - 2x^4 - 4x^3 + 8x^2 - x + x + 2 - 10$
$= 6x^4 - 4x^3 + 8x^2 - 8$

16. $x(2x^2 - 5x + 7) = 2x^2(x) - 5x(x) + 7(x)$
$= 2x^3 - 5x^2 + 7x$

17. $5x^2(6x + 2) = 6x(5x^2) + 2(5x^2)$
$= 30x^3 + 10x^2$

18. $(y - 7)(y + 6) = (y - 7)(y) + (y - 7)(6)$
$= y^2 - 7y + 6y - 42$
$= y^2 - y - 42$

19. $(3z + 1)(z - 3) = (3z + 1)(z) - (3z + 1)(3)$
$= 3z^2 + z - 9z - 3$
$= 3z^2 - 8z - 3$

20. $(w + 4)(w^2 + 6w - 11)$
$= (w + 4)(w^2) + (w + 4)(6w) - (w + 4)(11)$
$= w^3 + 4w^2 + 6w^2 + 24w - 11w - 44$
$= w^3 + 10w^2 + 13w - 44$

21. $(2a - 3)(a^2 - 10a - 2)$
$= (2a - 3)(a^2) - (2a - 3)(10a) - (2a - 3)(2)$
$= 2a^3 - 3a^2 - 20a^2 + 30a - 4a + 6$
$= 2a^3 - 23a^2 + 26a + 6$

22. $(5c^2 - 4)(2c^2 + c - 3)$
$= (5c^2 - 4)(2c^2) + (5c^2 - 4)(c) - (5c^2 - 4)(3)$
$= 10c^4 - 8c^2 + 5c^3 - 4c - 15c^2 + 12$
$= 10c^4 + 5c^3 - 23c^2 - 4c + 12$

23. $(-x^2 + 4x + 1)(x^2 - 8x + 3)$
$= (-x^2 + 4x + 1)(x^2) - (-x^2 + 4x + 1)(8x)$
$+ (-x^2 + 4x + 1)(3)$
$= -x^4 + 4x^3 + x^2 + 8x^3 - 32x^2 - 8x - 3x^2$
$+ 12x + 3$
$= -x^4 + 12x^3 - 34x^2 + 4x + 3$

24. $(-d^2 + 4d + 3)(3d^2 - 7d + 6)$
$= (-d^2 + 4d + 3)(3d^2) - (-d^2 + 4d + 3)(7d)$
$+ (-d^2 + 4d + 3)(6)$
$= -3d^4 + 12d^3 + 9d^2 + 7d^3 - 28d^2 - 21d$
$- 6d^2 + 24d + 18$
$= -3d^4 + 19d^3 - 25d^2 + 3d + 18$

25. $(3y^2 + 6y - 1)(4y^2 - 11y - 5)$
$= (3y^2 + 6y - 1)(4y^2) - (3y^2 + 6y - 1)(11y)$
$- (3y^2 + 6y - 1)(5)$
$= 12y^4 + 24y^3 - 4y^2 - 33y^3 - 66y^2 + 11y$
$- 15y^2 - 30y + 5$
$= 12y^4 - 9y^3 - 85y^2 - 19y + 5$

26. The error is that the negative sign was not carried through the subtracted polynomial $(x^3 + 7x - 2)$.
$(x^2 - 3x + 4) - (x^3 + 7x - 2)$
$= x^2 - 3x + 4 - x^3 - 7x + 2$
$= -x^3 + x^2 - 10x + 6$

27. $(2x - 7)^3$ is not the difference of the cubes of $(2x)$ and (7). It is the quantity $(2x - 7)$ multiplied together three times.
$(2x - 7)^3 = (2x)^3 - 3(2x)^2(7) + 3(2x)(7)^2 - (7)^3$
$= 8x^3 - 21(4x^2) + 6x(49) - 343$
$= 8x^3 - 84x^2 + 294x - 343$

28. $(x + 4)(x - 6)(x - 5) = (x^2 - 2x - 24)(x - 5)$
$= (x^2 - 2x - 24)(x) - (x^2 - 2x - 24)(5)$
$= x^3 - 2x^2 - 24x - 5x^2 + 10x + 120$
$= x^3 - 7x^2 - 14x + 120$

29. $(x + 1)(x - 7)(x + 3) = (x^2 - 6x - 7)(x + 3)$
$= (x^2 - 6x - 7)(x) - (x^2 - 6x - 7)(3)$
$= x^3 - 6x^2 - 7x + 3x^2 - 18x - 21$
$= x^3 - 3x^2 - 25x - 21$

30. $(z - 4)(-z + 2)(z + 8) = (-z^2 + 6z - 8)(z + 8)$
$= (-z^2 + 6z - 8)(z) + (-z^2 + 6z - 8)(8)$
$= -z^3 + 6z^2 - 8z - 8z^2 + 48z - 64$
$= -z^3 - 2z^2 + 40z - 64$

31. $(a - 6)(2a + 5)(a + 1) = (2a^2 - 7a - 30)(a + 1)$
$= (2a^2 - 7a - 30)(a) + (2a^2 - 7a - 30)(1)$
$= 2a^3 - 7a^2 - 30a + 2a^2 - 7a - 30$
$= 2a^3 - 5a^2 - 37a - 30$

32. $(3p + 1)(p + 3)(p + 1) = (3p^2 + 10p + 3)(p + 1)$
$= (3p^2 + 10p + 3)(p) + (3p^2 + 10p + 3)(1)$
$= 3p^3 + 10p^2 + 3p + 3p^2 + 10p + 3$
$= 3p^3 + 13p^2 + 13p + 3$

33. $(b - 2)(2b - 1)(-b + 1) = (2b^2 - 5b + 2)(-b + 1)$
$= -(2b^2 - 5b + 2)(b) + (2b^2 - 5b + 2)(1)$
$= -2b^3 + 5b^2 - 2b + 2b^2 - 5b + 2$
$= -2b^3 + 7b^2 - 7b + 2$

34. $(2s + 1)(3s - 2)(4s - 3) = (6s^2 - s - 2)(4s - 3)$
$= (6s^2 - s - 2)(4s) - (6s^2 - s - 2)(3)$
$= 24s^3 - 4s^2 - 8s - 18s^2 + 3s + 6$
$= 24s^3 - 22s^2 - 5s + 6$

35. $(w - 6)(4w - 1)(-3w + 5)$
$= (4w^2 - 25w + 6)(-3w + 5)$
$= -(4w^2 - 25w + 6)(3w) + (4w^2 - 25w + 6)(5)$
$= -12w^3 + 75w^2 - 18w + 20w^2 - 125w + 30$
$= -12w^3 + 95w^2 - 143w + 30$

36. $(4x - 1)(-2x - 7)(-5x - 4)$
$= (-8x^2 - 26x + 7)(-5x - 4)$
$= -(-8x^2 - 26x + 7)(5x) - (-8x^2 - 26x + 7)(4)$
$= 40x^3 + 130x^2 - 35x + 32x^2 + 104x - 28$
$= 40x^3 + 162x^2 + 69x - 28$

37. $(3q - 8)(-9q + 2)(q - 2)$
$= (-27q^2 + 78q - 16)(q - 2)$
$= (-27q^2 + 78q - 16)(q) - (-27q^2 + 78q - 16)(2)$
$= -27q^3 + 78q^2 - 16q + 54q^2 - 156q + 32$
$= -27q^3 + 132q^2 - 172q + 32$

38. $(x + 5)(x - 5) = (x)^2 - (5)^2 = x^2 - 25$

39. $(w - 9)^2 = (w)^2 - 2(w \cdot 9) + (9)^2$
$= w^2 - 18w + 81$

40. $(y + 4)^3 = (y)^3 + 3(y)^2(4) + 3(y)(4)^2 + (4)^3$
$= y^3 + 12y^2 + 48y + 64$

41. $(2c + 5)^2 = (2c)^2 + 2(2c \cdot 5) + (5)^2$
$= 4c^2 + 20c + 25$

42. $(3t - 4)^3 = (3t)^3 - 3(3t)^2(4) + 3(3t)(4)^2 - (4)^3$
$= 27t^3 - 108t^2 + 144t - 64$

43. $(5p - 3)(5p + 3) = (5p)^2 - (3)^2$
$= 25p^2 - 9$

44. $(7x - y)^3 = (7x)^3 - 3(7x)^2(y) + 3(7x)(y)^2 - (y)^3$
$= 343x^3 - 147x^2y + 21xy^2 - y^3$

45. $(2a + 9b)(2a - 9b) = (2a)^2 - (9b)^2$
$= 4a^2 - 81b^2$

46. $(3z + 7y)^3 = (3z)^3 + 3(3z)^2(7y) + 3(3z)(7y)^2 + (7y)^3$
$= 27z^3 + 189z^2y + 441zy^2 + 343y^3$

47. D;
$(3x - 2y)^2 = (3x)^2 - 2(3x \cdot 2y) + (2y)^2$
$= 9x^2 - 12xy + 4y^2$

48. $V = \ell wh$ $\qquad \ell = 3x + 1; w = x; h = x + 3$
$= (3x + 1)(x)(x + 3)$
$= (3x^2 + x)(x + 3)$
$= (3x^2 + x)(x) + (3x^2 + x)(3)$
$= 3x^3 + x^2 + 9x^2 + 3x$
$= 3x^3 + 10x^2 + 3x$

49. $V = \pi r^2 h$ $\qquad r = x - 4 ; h = 2x + 3$
$= \pi(x - 4)^2(2x + 3)$
$= \pi(x^2 - 8x + 16)(2x + 3)$
$= \pi((x^2 - 8x + 16)(2x) + (x^2 - 8x + 16)(3))$
$= \pi(2x^3 - 16x^2 + 32x + 3x^2 - 24x + 48)$
$= \pi(2x^3 - 13x^2 + 8x + 48)$
$= 2\pi x^3 - 13\pi x^2 + 8\pi x + 48\pi$

50. $V = s^3$ $\qquad s = x - 5$
$= (x - 5)^3$
$= (x)^3 - 3(x)^2(5) + 3(x)(5)^2 - (5)^3$
$= x^3 - 15x^2 + 75x - 125$

51. $V = \frac{1}{3}Bh$ $\qquad B = (2x - 3)^2 ; h = (3x + 4)$
$= \frac{1}{3}(2x - 3)^2(3x + 4)$
$= \frac{1}{3}(4x^2 - 12x + 9)(3x + 4)$
$= \frac{1}{3}((4x^2 - 12x + 9)(3x) + (4x^2 - 12x + 9)(4))$
$= \frac{1}{3}(12x^3 - 36x^2 + 27x + 16x^2 - 48x + 36)$
$= \frac{1}{3}(12x^3 - 20x^2 - 21x + 36)$
$= 4x^3 - \frac{20}{3}x^2 - 7x + 12$

52. $(a + b)(a - b) \stackrel{?}{=} a^2 - b^2$
$(a + b)(a) - (a + b)(b) \stackrel{?}{=} a^2 - b^2$
$a^2 + ab - ab - b^2 \stackrel{?}{=} a^2 - b^2$
$a^2 - b^2 = a^2 - b^2$ ✓

53. $(a + b)^2 \stackrel{?}{=} a^2 + 2ab + b^2$
$(a + b)(a + b) \stackrel{?}{=} a^2 + 2ab + b^2$
$(a + b)(a) + (a + b)(b) \stackrel{?}{=} a^2 + 2ab + b^2$
$a^2 + ab + ab + b^2 \stackrel{?}{=} a^2 + 2ab + b^2$
$a^2 + 2ab + b^2 = a^2 + 2ab + b^2$ ✓

54. $(a + b)^3 \stackrel{?}{=} a^3 + 3a^2b + 3ab^2 + b^3$
$(a + b)(a + b)(a + b) \stackrel{?}{=} a^3 + 3a^2b + 3ab^2 + b^3$
$(a^2 + 2ab + b^2)(a + b) \stackrel{?}{=} a^3 + 3a^2b + 3ab^2 + b^3$
$(a^2 + 2ab + b^2)(a) + (a^2 + 2ab + b^2)(b)$
$\stackrel{?}{=} a^3 + 3a^2b + 3ab^2 + b^3$
$a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$
$\stackrel{?}{=} a^3 + 3a^2b + 3ab^2 + b^3$
$a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$ ✓

55. $(a - b)^3 \stackrel{?}{=} a^3 - 3a^2b + 3ab^2 - b^3$
$(a - b)(a - b)(a - b) \stackrel{?}{=} a^3 - 3a^2b + 3ab^2 - b^3$
$(a^2 - 2ab + b^2)(a - b) \stackrel{?}{=} a^3 - 3a^2b + 3ab^2 - b^3$
$(a^2 - 2ab + b^2)(a) - (a^2 - 2ab + b^2)(b)$
$\stackrel{?}{=} a^3 - 3a^2b + 3ab^2 - b^3$
$a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 + b^3$
$\stackrel{?}{=} a^3 - 3a^2b + 3ab^2 - b^3$
$a^3 - 3a^2b + 3ab^2 + b^3 = a^3 - 3a^2b + 3ab^2 - b^3$ ✓

56. **a.** $p(x) + q(x) = (x^4 - 7x + 14) + (x^2 - 5)$
$= x^4 + x^2 - 7x + 9$
degree 4

b. $p(x) - q(x) = (x^4 - 7x + 14) - (x^2 - 5)$
$= x^4 - x^2 - 7x + 19$
degree 4

c. $p(x) \cdot q(x) = (x^4 - 7x + 14)(x^2 - 5)$
$= (x^4 - 7x + 14)(x^2) - (x^4 - 7x + 14)(5)$
$= x^6 - 7x^3 + 14x^2 - 5x^4 + 35x - 70$
$= x^6 - 5x^4 - 7x^3 + 14x^2 + 35x - 70$
degree 6

d. $p(x) + q(x)$: degree m

$p(x) - q(x)$: degree m

$p(x) \cdot q(x)$: degree $(m + n)$

57. a. $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

Check: $$\begin{aligned}&(x^4 + x^3 + x^2 + x + 1)(x) \\ &\quad - (x^4 + x^3 + x^2 + x + 1) \\ &= x^5 + x^4 + x^3 + x^2 + x - x^4 - x^3 \\ &\quad - x^2 - x - 1 \\ &= x^5 - 1\end{aligned}$$

$x^6 - 1 = (x - 1)(x^5 + x^4 + x^3 + x^2 + x + 1)$

Check: $$\begin{aligned}&(x^5 + x^4 + x^3 + x^2 + x + 1)(x) \\ &\quad - (x^5 + x^4 + x^3 + x^2 + x + 1) \\ &= x^6 + x^5 + x^4 + x^3 + x^2 + x \\ &\quad - x^5 - x^4 - x^3 - x^2 - x - 1 \\ &= x^6 - 1\end{aligned}$$

b. $$\begin{aligned}x^n - 1 &= (x - 1)(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + x^{n-n}) \\ &= (x - 1)(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + 1)\end{aligned}$$

Check: $$\begin{aligned}&(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + 1)(x) \\ &\quad - (x^{n-1} + x^{n-2} + x^{n-3} + \cdots + 1) \\ &= x^{n-1+1} + x^{n-2+1} + x^{n-3+1} + \cdots + x \\ &\quad - x^{n-1} - x^{n-2} - x^{n-3} - \cdots - 1 \\ &= x^n + x^{n-1} + x^{n-2} + \cdots x^{n-(n-1)} \\ &\quad - x^{n-1} - x^{n-2} - x^{n-3} \cdots -1 \\ &= x^n - 1\end{aligned}$$

58. $$\begin{aligned}f(x) &= (x + a)(x + b)(x + c)(x + d) \\ &= (x^2 + ax + bx + ab)(x + c)(x + d) \\ &= ((x^2 + ax + bx + ab)(x) \\ &\quad + (x^2 + ax + bx + ab)(c))(x + d) \\ &= (x^3 + ax^2 + bx^2 + abx + cx^2 \\ &\quad + acx + bcx + abc)(x + d) \\ &= (x^3 + ax^2 + bx^2 + abx + cx^2 + acx \\ &\quad + bcx + abc)(x) + (x^3 + ax^2 + bx^2 + abx \\ &\quad + cx^2 + acx + bcx + abc)(d) \\ &= x^4 + ax^3 + bx^3 + abx^2 + cx^3 + acx^2 \\ &\quad + bcx^2 + abcx + dx^3 + adx^2 + bdx^2 + abdx \\ &\quad + cdx^2 + acdx + bcdx + abcd \\ &= x^4 + (a + b + c + d)x^3 \\ &\quad + (ab + ac + bc + ad + bd + cd)x^2 \\ &\quad + (abc + abd + acd + bcd)x + abcd\end{aligned}$$

coefficient of $x^3 = (a + b + c + d)$

constant term $= abcd$

Problem Solving

59. $$\begin{aligned}T &= M + F \\ &= (0.091t^3 - 4.8t^2 + 110t + 5000) + (0.19t^3 - 12t^2 \\ &\quad + 350t + 3600) \\ &= 0.281t^3 - 16.8t^2 + 460t + 8600\end{aligned}$$

60. $$\begin{aligned}R &= P \cdot D \\ &= (6.82t^2 - 61.7t + 265)(4.11t + 4.44) \\ &= (6.82t^2 - 61.7t + 265)(4.11t) + (6.82t^2 - 61.7t \\ &\quad + 265)(4.44) \\ &= 28.0302t^3 - 253.587t^2 + 1089.15t + 30.2808t^2 \\ &\quad - 273.948t + 1176.6 \\ &= 28.0302t^3 - 223.3062t^2 + 815.202t + 1176.6 \\ R(3) &= 28.0302(3)^3 - 223.3062(3)^2 + 815.202(3) \\ &\quad + 1176.6 \\ &= 756.8154 - 2009.7558 + 2445.606 + 1176.6 \\ &= 2369.2656\end{aligned}$$

The total revenue in 2002 from DVD sales was 2369.2656 million dollars, or $2,369,265,600.

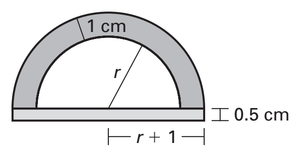
61. $$\begin{aligned}F &= 0.0116s^2 + 0.789 \\ P &= 0.00267sF \\ &= 0.00267s(0.0116s^2 + 0.789) \\ &= 0.00003s^3 + 0.00211s\end{aligned}$$

The model is $P = 0.00003s^3 + 0.00211s$.

$$\begin{aligned}P(10) &= 0.00003(10)^3 + 0.00211(10) \\ &= 0.03 + 0.0211 \\ &= 0.0511\end{aligned}$$

So, you need 0.0511 horsepower to keep the bicycle moving 10 miles per hour.

62.



$$\begin{aligned}M(r) &= \text{volume of cookie} + \text{volume of marshmallow} \\ &= \pi(r + 1)^2h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) \\ &= \pi(r + 1)^2\left(\frac{1}{2}\right) + \frac{\pi}{2} \cdot \frac{4}{3}r^3 \\ &= \frac{\pi}{2}(r^2 + 2r + 1) + \frac{\pi}{2} \cdot \frac{4}{3}r^3 \\ &= \frac{\pi}{2}\left(\frac{4}{3}r^3 + r^2 + 2r + 1\right) \\ &= \frac{2}{3}\pi r^3 + \frac{\pi}{2}r^2 + \pi r + \frac{\pi}{2}\end{aligned}$$

$$\begin{aligned}D(r) &= \frac{1}{2}\left(\frac{4}{3}\pi(r + 1)^3\right) + \pi(r + 1)^2\left(\frac{1}{2}\right) \\ &= \frac{\pi}{2}\left(\frac{4}{3}(r + 1)^3\right) + \frac{\pi}{2}(r + 1)^2 \\ &= \frac{\pi}{2} \cdot \frac{4}{3}(r^3 + 3r^2 + 3r + 1) + \frac{\pi}{2}(r^2 + 2r + 1) \\ &= \frac{\pi}{2}\left(\frac{4}{3}r^3 + 4r^2 + 4r + \frac{4}{3} + r^2 + 2r + 1\right) \\ &= \frac{2}{3}\pi r^3 + \frac{5}{2}\pi r^2 + 3\pi r + \frac{7}{6}\pi\end{aligned}$$

$C(r) = D(r) - M(r)$

$= \frac{2}{3}\pi r^3 + \frac{5}{2}\pi r^2 + 3\pi r + \frac{7}{6}\pi$

$- \left(\frac{2}{3}\pi r^3 + \frac{\pi}{2}r^2 + \pi r + \frac{\pi}{2}\right)$

$= \frac{2}{3}\pi r^3 + \frac{5}{2}\pi r^2 + 3\pi r + \frac{7}{6}\pi - \frac{2}{3}\pi r^3$

$- \frac{\pi}{2}r^2 - \pi r - \frac{\pi}{2}$

$= 2\pi r^2 + 2\pi r + \frac{2}{3}\pi$

63. $N =$ total men's players + total women's players

$= (L_m \cdot S_m) + (L_w \cdot S_w)$

$L_m \cdot S_m = (-0.127t^3 + 0.822t^2 - 1.02t + 31.5) \cdot (5.57t + 182)$

$= (-0.127t^3 + 0.822t^2 - 1.02t + 31.5)(5.57t)$

$+ (-0.127t^3 + 0.822t^2 - 1.02t + 31.5)(182)$

$= -0.70739t^4 - 18.53546t^3 + 143.9226t^2$

$- 10.185t + 5733$

$L_w \cdot S_w = (-0.0662t^3 + 0.437t^2 - 0.725t + 22.3) \cdot (12.2t + 185)$

$= (-0.0662t^3 + 0.437t^2 - 0.725t + 22.3) \cdot (12.2t) + (-0.0662t^3 + 0.437t^2 - 0.725t + 22.3)(185)$

$= -0.80764t^4 - 6.9156t^3 + 72t^2 + 137.935t$

$+ 4125.5$

$N = -0.70739t^4 - 18.53546t^3 + 143.9226t^2$

$- 10.185t + 5733 - 0.80764t^4$

$- 6.9156t^3 + 72t^2 + 137.935t + 4125.5$

$= -1.51503t^4 - 25.45106t^3 + 215.9226t^2$

$+ 127.75 + 9858.5$

The model for the total number of people is written by adding the number of men players to the number of women players. The number of men and women players is written by multiplying the number of teams by the average team size for each group.

64. $s = t - 5$

$t = s + 5$

$C = -0.00105t^3 + 0.0281t^2 + 0.465t + 48.8$

$C = -0.00105(s + 5)^3 + 0.0281(s + 5)^2 + 0.465(s + 5) + 48.8$

$= -0.00105(s^3 + 3s^2(5) + 3s(25) + 125)$

$+ 0.0281(s^2 + 10s + 25) + 0.465s + 2.325 + 48.8$

$= -0.00105s^3 - 0.01575s^2 - 0.07875s - 0.13125$

$+ 0.0281s^2 + 0.281s + 0.7025 + 0.465s + 2.325$

$+ 48.8$

$= -0.00105s^3 + 0.01235s^2 + 0.66725s + 51.69625$

Mixed Review

65. $2x - 7 = 11$

$2x = 18$

$x = 9$

66. $10 - 3x = 25$

$-3x = 15$

$x = -5$

67. $4t - 7 = 2t$

$-7 = -2t$

$\frac{7}{2} = t$

68. $y^2 - 2y - 48 = 0$

$(y + 6)(y - 8) = 0$

$y + 6 = 0$ or $y - 8 = 0$

$y = -6$ or $y = 8$

69. $w^2 - 15w + 54 = 0$

$(w - 6)(w - 9) = 0$

$w - 6 = 0$ or $w - 9 = 0$

$w = 6$ or $w = 9$

70. $x^2 + 9x + 14 = 0$

$(x + 2)(x + 7) = 0$

$x + 2 = 0$ or $x + 7 = 0$

$x = -2$ or $x = -7$

71. $4z^2 + 21z - 18 = 0$

$(4z - 3)(z + 6) = 0$

$4z - 3 = 0$ or $z + 6 = 0$

$4z = 3$ or $z = -6$

$z = \frac{3}{4}$

72. $9a^2 - 30a + 25 = 0$

$(3a - 5)^2 = 0$

$3a - 5 = 0$

$3a = 5$

$a = \frac{5}{3}$

73. $x + y - 2z = -4 \rightarrow y = -x + 2z - 4$

$3x - y + z = 22$

$-x + 2y + 3z = -9$

$3x - (-x + 2z - 4) + z = 22$

$3x + x - 2z + 4 + z = 22$

$4x - z = 18$

$-x + 2(-x + 2z - 4) + 3z = -9$

$-x - 2x + 4z - 8 + 3z = -9$

$-3x + 7z = -1$

$4x - z = 18 \xrightarrow{\times 3} 12x - 3z = 54$

$-3x + 7z = -1 \xrightarrow{\times 4} -12x + 28z = -4$

$25z = 50$

$z = 2$

$-3x + 7(2) = -1 \rightarrow x = 5$

$y = -5 + 2(2) - 4 = -5$

The solution is $(5, -5, 2)$.

74. $x - 2y + z = -13 \rightarrow x = 2y - z - 13$

$-x + 4y + z = 35$

$3x + 2y + 4z = 28$

$-(2y - z - 13) + 4y + z = 35$

$-2y + z + 13 + 4y + z = 35$

$2y + 2z = 22$

$y + z = 11$

$3(2y - z - 13) + 2y + 4z = 28$

$6y - 3z - 39 + 2y + 4z = 28$

$8y + z = 67$

$$\begin{array}{lcrcr} y + z = 11 & \xrightarrow{\times -1} & -y - z &=& -11 \\ 8y + z = 67 & \longrightarrow & 8y + z &=& 67 \\ \hline & & 7y &=& 56 \\ & & y &=& 8 \end{array}$$

$8 + z = 11 \rightarrow z = 3$

$x = 2(8) - (3) - 13 = 0$

The solution is (0, 8, 3).

75. $3x - y - 2z = 20 \rightarrow y = 3x - 2z - 20$

$-x + 3y - z = -16$

$-2x - y + 3z = -5$

$-x + 3(3x - 2z - 20) - z = -16$

$-x + 9x - 6z - 60 - z = -16$

$8x - 7z = 44$

$-2x - (3x - 2z - 20) + 3z = -5$

$-2x - 3x + 2z + 20 + 3z = -5$

$-5x + 5z = -25$

$-x + z = -5$

$$\begin{array}{lcr} 8x - 7z = 44 & \longrightarrow & 8x - 7z = 44 \\ -x + z = -5 & \xrightarrow{\times 8} & -8x + 8z = -40 \\ \hline & & z = 4 \end{array}$$

$-x + 4 = -5 \rightarrow x = 9$

$y = 3(9) - 2(4) - 20 = -1$

The solution is (9, −1, 4).

76. $\det\begin{bmatrix} 3 & -4 \\ 3 & 1 \end{bmatrix} = 3(1) - 3(-4) = 3 + 12 = 15$

77. $\det\begin{bmatrix} 5 & 7 \\ -4 & 9 \end{bmatrix} = 5(9) - (-4)(7) = 45 + 28 = 73$

78. $\det\begin{bmatrix} -1 & 8 & 0 \\ 3 & 4 & -3 \\ -5 & 2 & 1 \end{bmatrix} = (-4 + 120 + 0) - (0 + 6 + 24)$

$= 116 - 30 = 86$

79. $\det\begin{bmatrix} 2 & 3 & -4 \\ -6 & 1 & 5 \\ -3 & -1 & -2 \end{bmatrix} = (-4 - 45 - 24) - (12 - 10 + 36)$

$= -73 - 38 = -111$

Quiz 5.1–5.3 (p. 352)

1. $3^5 \cdot 3^{-1} = 3^{5 + (-1)} = 3^4 = 81$

2. $(2^4)^2 = 2^{4 \cdot 2} = 2^8 = 256$

3. $\left(\frac{2}{3^{-2}}\right)^2 = \frac{2^2}{(3^{-2})^2} = \frac{2^2}{3^{-4}} = 2^2 \cdot 3^4 = 4 \cdot 81 = 324$

4. $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$

5. $(x^4y^{-2})(x^{-3}y^8) = x^{4 + (-3)}y^{(-2) + 8} = xy^6$

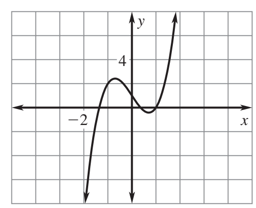
6. $(a^2b^{-5})^{-3} = (a^2)^{-3}(b^{-5})^{-3} = a^{-6} \cdot b^{15} = \frac{b^{15}}{a^6}$

7. $\frac{x^3y^7}{x^{-4}y^0} = x^{3 - (-4)}y^{7 - 0} = x^7y^7$

8. $\frac{c^3d^{-2}}{c^5d^{-1}} = c^{3 - 5} \cdot d^{-2 - (-1)} = c^{-2}d^{-1} = \frac{1}{c^2d}$

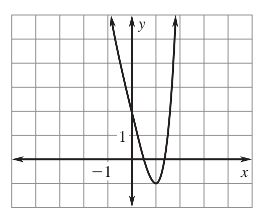
9.

x	−3	−2	−1	0	1	2	3
y	−44	−9	2	1	0	11	46



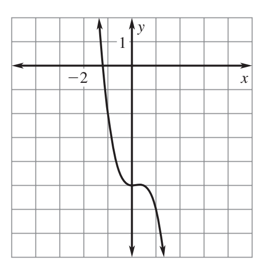
10.

x	−3	−2	−1	0	1	2	3
y	95	26	7	2	−1	10	71



11.

x	−3	−2	−1	0	1	2	3
y	58	15	−2	−5	−6	−17	−50



12. $(x^3 + x^2 - 6) - (2x^2 + 4x - 8)$
$= x^3 + x^2 - 6 - 2x^2 - 4x + 8$
$= x^3 - x^2 - 4x + 2$

13. $(-3x^2 + 4x - 10) + (x^2 - 9x + 15)$
$= -3x^2 + x^2 + 4x - 9x - 10 + 15$
$= -2x^2 - 5x + 5$

14. $(x - 5)(x^2 - 5x + 7)$
$= (x^2 - 5x + 7)(x) - (x^2 - 5x + 7)(5)$
$= x^3 - 5x^2 + 7x - 5x^2 + 25x - 35$
$= x^3 - 10x^2 + 32x - 35$

15. $(x + 3)(x - 6)(3x - 1) = (x^2 - 3x - 18)(3x - 1)$
$= (x^2 - 3x - 18)(3x) - (x^2 - 3x - 18)$
$= 3x^3 - 9x^2 - 54x - x^2 + 3x + 18$
$= 3x^3 - 10x^2 - 51x + 18$

16. $(7{,}282{,}000{,}000{,}000) \div (294{,}000{,}000) =$
$\frac{7.282 \times 10^{12}}{2.94 \times 10^8} = \frac{7.282}{2.94} \times \frac{10^2}{10^8}$
$\approx 2.48 \times 10^4$
$= \$24{,}800$

Lesson 5.4

5.4 Guided Practice (pp. 354-356)

1. $x^3 - 7x^2 + 10x = x(x^2 - 7x + 10) = x(x - 5)(x - 2)$

2. $3y^5 - 75y^3 = 3y^3(y^2 - 25) = 3y^3(y - 5)(y + 5)$

3. $16b^5 + 686b^2 = 2b^2(8b^3 + 343)$
$= 2b^2(2b + 7)(4b^2 - 14b + 49)$

4. $w^3 - 27 = (w - 3)(w^2 + 3w + 9)$

5. $x^3 + 7x^2 - 9x - 63 = x^2(x + 7) - 9(x + 7)$
$= (x + 7)(x^2 - 9)$
$= (x + 7)(x - 3)(x + 3)$

6. $16g^4 - 625 = (4g^2)^2 - (25)^2$
$= (4g^2 - 25)(4g^2 + 25)$
$= (2g - 5)(2g + 5)(4g^2 + 25)$

7. $4t^6 - 20t^4 + 24t^2 = 4t^2(t^4 - 5t^2 + 6)$
$= 4t^2(t^2 - 2)(t^2 - 3)$

8. $4x^5 - 40x^3 + 36x = 0$
$4x(x^4 - 10x^2 + 9) = 0$
$4x(x^2 - 9)(x^2 - 1) = 0$
$4x(x - 3)(x + 3)(x - 1)(x + 1) = 0$
$x = 0, x = 3, x = -3, x = 1$, or $x = -1$

9. $2x^5 + 24x = 14x^3$
$2x^5 - 14x^3 + 24x = 0$
$2x(x^4 - 7x^2 + 12) = 0$
$2x(x^2 - 3)(x^2 - 4) = 0$
$2x(x^2 - 3)(x - 2)(x + 2) = 0$
$x = 0, x = -\sqrt{3}, x = \sqrt{3}, x = 2$, or $x = -2$

10. $-27x^3 + 15x^2 = -6x^4$
$6x^4 - 27x^3 + 15x^2 = 0$
$3x^2(2x^2 - 9x + 5) = 0$
$3x^2 = 0$ or $2x^2 - 9x + 5 = 0$
$x = 0$ or $x = \frac{9 \pm \sqrt{9^2 - 4 \cdot 2 \cdot 5}}{2 \cdot 2} = \frac{9 \pm \sqrt{41}}{4}$

11.

Volume (ft^3)	=	Interior length (ft)	•	Interior width (ft)	•	Interior height (ft)
40	=	$(6x - 2)$	•	$(3x - 2)$	•	$(x - 1)$

$40 = 18x^3 - 36x^2 + 22x - 4$
$0 = 18x^3 - 36x^2 + 22x - 44$
$0 = 18x^2(x - 2) + 22(x - 2)$
$0 = (18x^2 + 22)(x - 2)$

The only real solutions is $x = 2$. The basin is 12 ft long, 6 ft wide, and 2 ft high.

5.4 Exercises (pp. 356–359)

Skill Practice

1. The expression $8x^6 + 10x^3 - 3$ is in *quadratic* form because it can be written as $2u^2 + 5u - 3$ where $u = 2x^3$.

2. The factorization of a polynomial is factored completely if it is written as a product of unfactorable polynomials with integer coefficients.

3. $14x^2 - 21x = 7x(2x - 3)$

4. $30b^3 - 54b^2 = 6b^2(5b - 9)$

5. $c^3 + 9c^2 + 18c = c(c^2 + 9c + 18) = c(c + 3)(c + 6)$

6. $z^3 - 6z^2 - 72z = z(z^2 - 6z - 72) = z(z - 12)(z + 6)$

7. $3y^5 - 48y^3 = 3y^3(y^2 - 16) = 3y^3(y - 4)(y + 4)$

8. $54m^5 + 18m^4 + 9m^3 = 9m^3(6m^2 + 2m + 1)$

9. A;
$(2x^7 - 32x^3) = 2x^3(x^4 - 16)$
$= 2x^3(x^2 - 4)(x^2 + 4)$
$= 2x^3(x + 2)(x - 2)(x^2 + 4)$

10. $x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$

11. $y^3 - 64 = y^3 - 4^3 = (y - 4)(y^2 + 4y + 16)$

12. $27m^3 + 1 = (3m)^3 + 1^3 = (3m + 1)(9m^2 - 3m + 1)$

13. $125n^3 + 216 = (5n)^3 + 6^3 = (5n + 6)(25n^2 - 30n + 36)$

14. $27a^3 - 1000 = (3a)^3 - 10^3$
$= (3a - 10)(9a^2 + 30a + 100)$

15. $8c^3 + 343 = (2c)^3 + 7^3 = (2c + 7)(4c^2 - 14c + 49)$

16. $192w^3 - 3 = 3(64w^3 - 1)$
$= 3((4w)^3 - 1^3)$
$= 3(4w - 1)(16w^2 + 4w + 1)$

17. $-5z^3 + 320 = -5(z^3 - 64)$
$= -5(z^3 - 4^3)$
$= -5(z - 4)(z^2 + 4z + 16)$

18. $x^3 + x^2 + x + 1 = x^2(x + 1) + (x + 1) = (x + 1)(x^2 + 1)$

19. $y^3 - 7y^2 + 4y - 28 = y^2(y - 7) + 4(y - 7)$

$= (y - 7)(y^2 + 4)$

20. $n^3 + 5n^2 - 9n - 45 = n^2(n + 5) - 9(n + 5)$

$= (n + 5)(n^2 - 9)$

$= (n + 5)(n - 3)(n + 3)$

21. $3m^3 - m^2 + 9m - 3 = 3m^3 + 9m - m^2 - 3$

$= 3m(m^2 + 3) - (m^2 + 3)$

$= (m^2 + 3)(3m - 1)$

22. $25s^3 - 100s^2 - s + 4 = 25s^2(s - 4) - (s - 4)$

$= (s - 4)(25s^2 - 1)$

$= (s - 4)(5s - 1)(5s + 1)$

23. $4c^3 + 8c^2 - 9c - 18 = 4c^2(c + 2) - 9(c + 2)$

$= (c + 2)(4c^2 - 9)$

$= (c + 2)(2c - 3)(2c + 3)$

24. $x^4 - 25 = (x^2)^2 - 5^2 = (x^2 - 5)(x^2 + 5)$

25. $a^4 + 7a^2 + 6 = (a^2 + 1)(a^2 + 6)$

26. $3s^4 - s^2 - 24 = (3s^2 + 8)(s^2 - 3)$

27. $32z^5 - 2z = 2z(16z^4 - 1)$

$= 2z(4z^2 + 1)(4z^2 - 1)$

$= 2z(4z^2 + 1)(2z + 1)(2z - 1)$

28. $36m^6 + 12m^4 + m^2 = m^2(36m^4 + 12m^2 + 1)$

$= m^2(6m^2 + 1)(6m^2 + 1)$

$= m^2(6m^2 + 1)^2$

29. $15x^5 - 72x^3 - 108x = 3x(5x^4 - 24x^2 - 36)$

$= 3x(5x^2 + 6)(x^2 - 6)$

30. The error is that when the binomial was factored, it was not factored properly.

$8x^3 - 27 = 0$

$(2x - 3)(4x^2 + 6x + 9) = 0$

$2x - 3 = 0$

$x = \frac{3}{2} \neq -\frac{3}{2}$

31. The error is that $x = 0$ is not included in the solutions, and $x^2 - 16$ was not factored completely before finding the zeros.

$3x(x^2 - 16) = 0$

$3x(x - 4)(x + 4) = 0$

$x = 0, x = 4, \text{ or } x = -4$

32. $y^3 - 5y^2 = 0$

$y^2(y - 5) = 0$

$y = 0 \text{ or } y = 5$

33. $18s^3 = 50s$

$18s^3 - 50s = 0$

$2s(9s^2 - 25) = 0$

$2s(3s - 5)(3s + 5) = 0$

$s = 0, s = \frac{5}{3}, \text{ or } s = -\frac{5}{3}$

34. $g^3 + 3g^2 - g - 3 = 0$

$g^2(g + 3) - (g + 3) = 0$

$(g + 3)(g^2 - 1) = 0$

$(g + 3)(g + 1)(g - 1) = 0$

$g = -3, g = -1, \text{ or } g = 1$

35. $m^3 + 6m^2 - 4m - 24 = 0$

$m^2(m + 6) - 4(m + 6) = 0$

$(m + 6)(m^2 - 4) = 0$

$(m + 6)(m + 2)(m - 2) = 0$

$m = -6, m = -2, \text{ or } m = 2$

36. $4w^4 + 40w^2 - 44 = 0$

$4(w^4 + 10w^2 - 11) = 0$

$4(w^2 + 11)(w^2 - 1) = 0$

$4(w^2 + 11)(w + 1)(w - 1) = 0$

$w = -1 \text{ or } w = 1$

37. $4z^5 = 84z^3$

$4z^5 - 84z^3 = 0$

$4z^3(z^2 - 21) = 0$

$z = 0, z = \sqrt{21}, \text{ or } z = -\sqrt{21}$

38. $5b^3 + 15b^2 + 12b = -36$

$5b^3 + 15b^2 + 12b + 36 = 0$

$5b^2(b + 3) + 12(b + 3) = 0$

$(b + 3)(5b^2 + 12) = 0$

$b = -3$

39. $x^6 - 4x^4 - 9x^2 + 36 = 0$

$x^4(x^2 - 4) - 9(x^2 - 4) = 0$

$(x^2 - 4)(x^4 - 9) = 0$

$(x - 2)(x + 2)(x^2 - 3)(x^2 + 3) = 0$

$x = 2, x = -2, x = \sqrt{3}, \text{ or } x = -\sqrt{3}$

40. $48p^5 = 27p^3$

$48p^5 - 27p^3 = 0$

$3p^3(16p^2 - 9) = 0$

$(3p^3)(4p + 3)(4p - 3) = 0$

$p = 0, p = -\frac{3}{4}, \text{ or } p = \frac{3}{4}$

41. C;

$3x^4 - 27x^2 + 9x = x^3$

$3x^4 - x^3 - 27x^2 + 9x = 0$

$x(3x^3 - 27x - x^2 + 9) = 0$

$x(3x(x^2 - 9) - (x^2 - 9)) = 0$

$x(x^2 - 9)(3x - 1) = 0$

$x(x - 3)(x + 3)(3x - 1) = 0$

$x = 0, x = 3, x = -3, \text{ or } x = \frac{1}{3}$

42. $16x^3 - 44x^2 - 42x = 2x(8x^2 - 22x - 21)$

$= 2x(4x + 3)(2x - 7)$

43. $n^4 - 4n^2 - 60 = (n^2 + 6)(n^2 - 10)$

44. $-4b^4 - 500b = -4b(b^3 + 125)$
$= -4b(b + 5)(b^2 - 5b + 25)$

45. $36a^3 - 15a^2 + 84a - 35 = 36a^3 + 84a - 15a^2 - 35$
$= 12a(3a^2 + 7) - 5(3a^2 + 7)$
$= (3a^2 + 7)(12a - 5)$

46. $18c^4 + 57c^3 - 10c^2 = c^2(18c^2 + 57c - 10)$
$= c^2(6c - 1)(3c + 10)$

47. $2d^4 - 13d^2 - 45 = (2d^2 + 5)(d^2 - 9)$
$= (2d^2 + 5)(d + 3)(d - 3)$

48. $32x^5 - 108x^2 = 4x^2(8x^3 - 27)$
$= 4x^2(2x - 3)(4x^2 + 6x + 9)$

49. $8y^6 - 38y^4 - 10y^2 = 2y^2(4y^4 - 19y^2 - 5)$
$= 2y^2(4y^2 + 1)(y^2 - 5)$

50. $z^5 - 3z^4 - 16z + 48 = z^4(z - 3) - 16(z - 3)$
$= (z - 3)(z^4 - 16)$
$= (z - 3)(z^2 + 4)(z^2 - 4)$
$= (z - 3)(z^2 + 4)(z - 2)(z + 2)$

51. $A = \ell \cdot w \qquad \ell = 3x + 2, \qquad w = x + 4$
$48 = (3x + 2)(x + 4)$
$48 = 3x^2 + 14x + 8$
$0 = 3x^2 + 14x - 40$
$0 = (3x + 20)(x - 2)$

$x = -\frac{20}{3}$ or $x = 2$, but a side length cannot be negative so $x = 2$.

52. $V = \ell \cdot w \cdot h \qquad \ell = 2x, \qquad w = x - 1, \qquad h = x - 4$
$40 = (2x)(x - 1)(x - 4)$
$40 = (2x^2 - 2x)(x - 4)$
$40 = 2x^3 - 10x^2 + 8x$
$0 = 2x^3 - 10x^2 + 8x - 40$
$0 = 2x^2(x - 5) + 8(x - 5)$
$0 = (x - 5)(2x^2 + 8)$
The only real solution is $x = 5$.

53. $V = \frac{1}{3}\pi r^2 h \qquad r = 2x - 5, \qquad h = 3x$

$125\pi = \frac{1}{3}\pi(2x - 5)^2(3x)$
$375 = (4x^2 - 20x + 25)(3x)$
$375 = 12x^3 - 60x^2 + 75x$
$0 = 12x^3 - 60x^2 + 75x - 375$
$0 = 4x^3 - 20x^2 + 25x - 125$
$0 = 4x^2(x - 5) + 25(x - 5)$
$0 = (x - 5)(4x^2 + 25)$
The only real solution is $x = 5$.

54. $x^3y^6 - 27 = (xy^2)^3 - 3^3$
$= (xy^2 - 3)(x^2y^4 + 3xy^2 + 9)$

55. $7ac^2 + bc^2 - 7ad^2 - bd^2 = c^2(7a + b) - d^2(7a + b)$
$= (7a + b)(c^2 - d^2)$
$= (7a + b)(c - d)(c + d)$

56. $x^{2n} - 2x^n + 1 = (x^n - 1)(x^n - 1)$
$= (x^n - 1)^2$

57. $a^5b^2 - a^2b^4 + 2a^4b - 2ab^3 + a^3 - b^2$
$= a^2b^2(a^3 - b^2) + 2ab(a^3 - b^2) + (a^3 - b^2)$
$= (a^3 - b^2)(a^2b^2 + 2ab + 1)$
$= (a^3 - b^2)(ab + 1)(ab + 1)$
$= (a^3 - b^2)(ab + 1)^2$

Problem Solving

58. $V = \ell \cdot w \cdot h \quad \ell = 12x - 15, \quad w = 12x - 21, \quad h = x$
$945 = (12x - 15)(12x - 21)(x)$
$945 = (144x^2 - 432x + 315)x$
$0 = 144x^3 - 432x^2 + 315x - 945$
$0 = 16x^3 - 48x^2 + 35x - 105$
$0 = 16x^2(x - 3) + 35(x - 3)$
$0 = (x - 3)(16x^2 + 35)$
The only real solution is $x = 3$. The block is 3 meters high.

59. $V = \ell \cdot w \cdot h \quad \ell = x - 2, \quad w = 3x - 2, \quad h = 6x - 2$
$112 = (x - 2)(3x - 2)(6x - 2)$
$112 = (3x^2 - 8x + 4)(6x - 2)$
$112 = 18x^3 - 48x^2 + 24x - 6x^2 + 16x - 8$
$112 = 18x^3 - 54x^2 + 40x - 8$
$0 = 18x^3 - 54x^2 + 40x - 120$
$0 = 9x^3 - 27x^2 + 20x - 60$
$0 = 9x^2(x - 3) + 20(x - 3)$
$0 = (9x^2 + 20)(x - 3)$
$x = 3$ cm
$3x = 9$ cm
$6x = 18$ cm
The outer dimensions of the mold should be 3 cm × 9 cm × 18 cm.

60. a. $V = \ell \cdot w \cdot h$
Volume of platform 1 $= (8x)(6x)(x) = 48x^3$
Volume of platform 2 $= (6x)(4x)(x) = 24x^3$
Volume of platform 3 $= (4x)(2x)(x) = 8x^3$

b. $1250 = 48x^3 + 24x^3 + 8x^3$
$1250 = 80x^3$

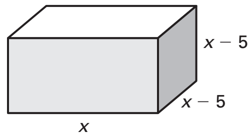
c. $1250 = 80x^3$
$15.625 = x^3$
$2.5 \text{ ft} = x$
Platform 1: 20 ft × 15 ft × 2.5 ft
Platform 2: 15 ft × 10 ft × 2.5 ft
Platform 3: 10 ft × 5 ft × 2.5 ft

61. $\ell = x$

$h = x - 5$

$w = x - 5$

$V = \ell \cdot w \cdot h$



$250 = (x)(x - 5)(x - 5)$

$250 = x(x^2 - 10x + 25)$

$250 = x^3 - 10x^2 + 25x$

$0 = x^3 - 10x^2 + 25x - 250$

$0 = x^2(x - 10) + 25(x - 10)$

$0 = (x^2 + 25)(x - 10)$

$x = 10$

$x - 5 = 5$

The dimensions of the prism should be 10 in. $\times$ 5 in. $\times$ 5 in.

62. Volume of steel = volume of outside − volume of inside

$6.42 = (x)(x - 2)(x + 8) - 89.58$

$6.42 = x(x^2 + 6x - 16) - 89.58$

$6.42 = x^3 + 6x^2 - 16x - 89.58$

$0 = x^3 + 6x^2 - 16x - 96$

$0 = x^2(x + 6) - 16(x + 6)$

$0 = (x + 6)(x^2 - 16)$

$0 = (x + 6)(x - 4)(x + 4)$

$x = 4$ feet

The outer dimensions of the tank should be 4 ft $\times$ 2 ft $\times$ 12 ft.

63. $V = \ell \cdot w \cdot h \quad \ell = x - 2, \quad w = 3 - 2x, \quad h = 3x + 4$

$\frac{7}{3} = (x - 2)(3 - 2x)(3x + 4)$

$\frac{7}{3} = (x - 2)(9x + 12 - 6x^2 - 8x)$

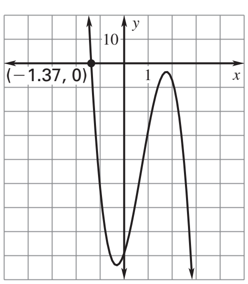
$\frac{7}{3} = 9x^2 + 12x - 6x^3 - 8x^2 - 18x - 24 + 12x^2 + 16x$

$\frac{7}{3} = -6x^3 + 13x^2 + 10x - 24$

$7 = -18x^3 + 39x^2 + 30x - 72$

$0 = -18x^3 + 39x^2 + 30x - 79$

From the graph, the only real real solution is −1.37. This result would yield a negative dimension of the platform. So, the volume cannot be $\frac{7}{3}$ cubic feet.



64. a.

y	$y^3 + y^2$
1	$1 + 1 = 2$
2	$8 + 4 = 12$
3	$27 + 9 = 36$
4	$64 + 16 = 80$
5	$125 + 25 = 150$
6	$216 + 36 = 252$
7	$343 + 49 = 392$
8	$512 + 64 = 576$
9	$729 + 81 = 810$
10	$1000 + 100 = 1100$

b. $x^3 + 2x^2 = 96$

$\frac{x^3}{8} + \frac{x^2}{4} = 12 \qquad \left(\frac{x}{2}\right)^3 + \left(\frac{x}{2}\right)^2 = 12$

$y = \frac{x}{2} \qquad 2 = \frac{x}{2} \qquad x = 4$

c. $3x^3 + 2x^2 = 512$

$\frac{27x^3}{8} + \frac{9x^2}{4} = 576 \qquad \left(\frac{3x}{2}\right)^3 + \left(\frac{3x}{2}\right)^2 = 576$

$y = \frac{3x}{2} \qquad 8 = \frac{3x}{2} \qquad x = \frac{16}{3}$

d. $ax^4 + bx^3 = c$ Original equation

$\frac{a^4x^4}{b^4} + \frac{a^3x^3}{b^3} = \frac{a^3c}{b^4}$ Multiply each side by $\frac{a^3}{b^4}$.

$\left(\frac{ax}{b}\right)^4 + \left(\frac{ax}{b}\right)^3 = \frac{a^3c}{b^4}$ Rewrite equation.

Then, create a table with values for $y^4 + y^3$.

65. a. The three solids form a cube with a smaller cube taken out of the corner. If the solids were connected, $V = a^3 - b^3$ because the side of the large cube = a and the side of the cube taken out = b.

b. I: $V = a \cdot a \cdot (a - b) = a^2(a - b)$

II: $V = a \cdot (b)(a - b) = ab(a - b)$

III: $V = b \cdot b(a - b) = b^2(a - b)$

c. $V = \text{I} + \text{II} + \text{II}$

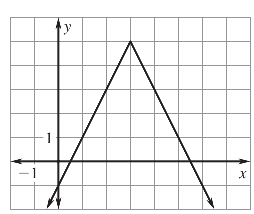
$= a^2(a - b) + ab(a - b) + b^2(a - b)$

$= (a - b)(a^2 + ab + b^2)$

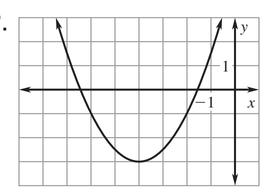
$= a^3 - b^3$

Mixed Review

66.



67.



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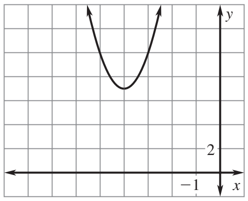

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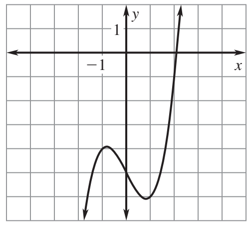

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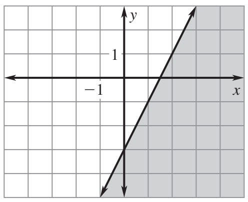

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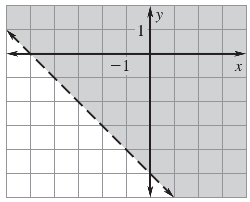

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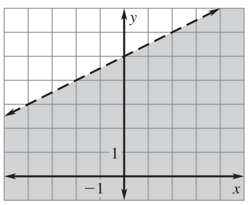
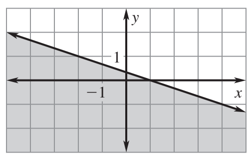
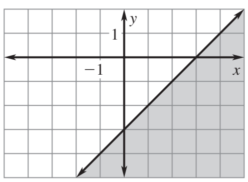


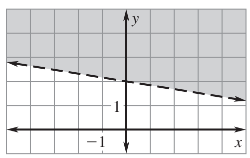
73.



74.



75.



76.

2	5	−3	4	−1	10
		10	14	36	70
	5	7	18	35	80

$f(2) = 80$

77.

−3	−3	0	1	−6	2	4
		9	−27	78	−216	642
	−3	9	−26	72	−214	646

$f(-3) = 646$

78.

−2	5	0	−4	12	0	20
		−10	20	−32	40	−80
	5	−10	16	−20	40	−60

$f(-2) = -60$

79.

4	−6	0	0	9	−15
		−24	−96	−384	−1500
	−6	−24	−96	−375	−1515

$f(4) = -1515$

Problem Solving Workshop 5.4 (p. 361)

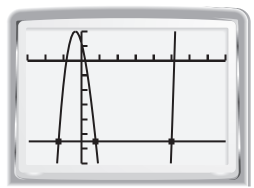
1. $y = x^3 + 4x^2 - 8x$

x	*y*
1	−3
2	8
3	39
4	96

$x = 4$

2. $y = x^3 - 9x^2 - 14x + 7$

$y = -33$



Intersections : $(-2.56, -33)$

$(1.56, -33)$

$(10, -33)$

$x = -2.56, 1.56, \text{ or } 10$

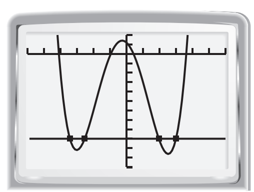
3. $y = 2x^3 - 11x^2 + 3x + 5$

x	*y*
1	−1
2	−17
3	−31
4	−31
5	−5
6	59

$x = 6$

4. $y = x^4 + x^3 - 15x^2 - 8x + 6$

$y = -45$



Intersections: $(-3.43, -45)$

$(-2.53, -45)$

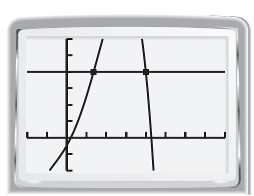
$(1.96, -45)$

$(3, -45)$

$x = -3.43, -2.53, 1.96, \text{ or } 3$

5. $y = -x^4 + 2x^3 + 6x^2 + 17x - 4$

$y = 32$



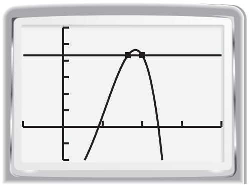
Intersections: $(1.45, 32)$

$(4, 32)$

$x = 1.45 \text{ or } 4$

6. $y = -3x^4 + 4x^3 + 8x^2 + 4x - 11$

$y = 13$



Intersections: $(1.64, 13)$

$(2, 13)$

$x = 1.64$ or 2

7. $y = 4x^4 - 16x^3 + 29x^2 - 95x$

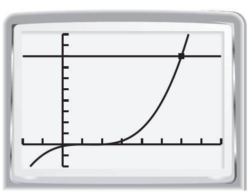
x	y
0	0
0.5	−42
1	−78
1.5	−111
2	−138
2.5	−150

$x = 2.5$

8. $y = (2x - 2)(x - 2)(x - 1)$

x	y
2	0
3	8
4	36
5	96
6	200

The volume is 200 cubic feet when $x = 6$.
Length $= 2x = 12$ ft
Width $= x = 6$ ft
Height $= x = 6$ ft



Intersection: $x = 6, y = 200$

Length $= 2x = 12$ ft

Width $= x = 6$ ft

Height $= x = 6$ ft

9. $x =$ height

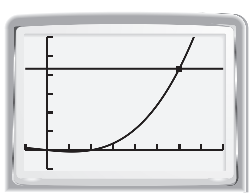
$x - 4 =$ width

$x + 6 =$ length

$V = \ell \cdot w \cdot h$

$y = (x + 6)(x - 4)(x)$

x	y
6	144
7	273
8	448
9	675
10	960
11	1309
12	1728



Intersection at: $x = 12$, $y = 1728$

$x = 12$

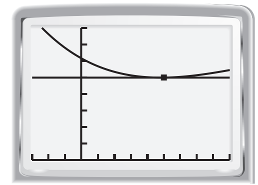
height $= x = 12$ ft

width $= x - 4 = 8$ ft

length $= x + 6 = 18$ ft

10. $y = 0.0000984x^4 - 0.00712x^3 + 0.162x^2 - 1.11x + 12.3$

x	y
1	11.35
2	10.67
3	10.24
4	10.02
5	9.97



$x = 5$ Intersection: $x = 5, y = 9.97$

In 1975, pineapple consumption was about 9.97 pounds per person.

Lesson 5.5

5.5 Guided Practice (pp. 362–365)

1.

$$\begin{array}{r} 2x^2 - 3x + 8 \\ x^2 + 2x - 1 \overline{\big)\, 2x^4 + x^3 + 0x^2 + x - 1} \\ \underline{2x^4 + 4x^3 - 2x^2} \\ -3x^3 + 2x^2 + x \\ \underline{-3x^3 - 6x^2 + 3x} \\ 8x^2 - 2x - 1 \\ \underline{8x^2 + 16x - 8} \\ -18x + 7 \end{array}$$

$$\frac{2x^4 + x^3 + x - 1}{x^2 + 2x - 1} = 2x^2 - 3x + 8 + \frac{-18x + 7}{x^2 + 2x - 1}$$

2.

$$\begin{array}{r} x^2 - 3x + 10 \\ x + 2 \overline{\big)\, x^3 - x^2 + 4x - 10} \\ \underline{x^3 + 2x^2} \\ -3x^2 + 4x \\ \underline{-3x^2 - 6x} \\ 10x - 10 \\ \underline{10x + 20} \\ -30 \end{array}$$

$$\frac{x^3 - x^2 + 4x - 10}{x + 2} = x^2 - 3x + 10 - \frac{30}{x + 2}$$

3.

$$\begin{array}{r|rrrr} -3 & 1 & 4 & -1 & -1 \\ & & -3 & -3 & 12 \\ \hline & 1 & 1 & -4 & 11 \end{array}$$

$$\frac{x^3 + 4x^2 - x - 1}{x + 3} = x^2 + x - 4 + \frac{11}{x + 3}$$

4.

$$\begin{array}{r|rrrr} 1 & 4 & 1 & -3 & 7 \\ & & 4 & 5 & 2 \\ \hline & 4 & 5 & 2 & 9 \end{array}$$

$$\frac{4x^3 + x^2 - 3x + 7}{x - 1} = 4x^2 + 5x + 2 + \frac{9}{x - 1}$$

5. $$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & 12 \\ & & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 - 6x^2 + 5x + 12 \\ &= (x-4)(x^2 - 2x - 3) \\ &= (x-4)(x-3)(x+1) \end{aligned}$$

6. $$\begin{array}{r|rrrr} 4 & 1 & -1 & -22 & 40 \\ & & 4 & 12 & -40 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 - x^2 - 22x + 40 \\ &= (x-4)(x^2 + 3x - 10) \\ &= (x-4)(x+5)(x-2) \end{aligned}$$

7. $$\begin{array}{r|rrrr} -2 & 1 & 2 & -9 & -18 \\ & & -2 & 0 & 18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 2x^2 - 9x - 18 \\ &= (x+2)(x^2 - 9) \\ &= (x+2)(x-3)(x+3) \end{aligned}$$

The other zeros are 3 and -3.

8. $$\begin{array}{r|rrrr} -2 & 1 & 8 & 5 & -14 \\ & & -2 & -12 & 14 \\ \hline & 1 & 6 & -7 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 8x^2 + 5x - 14 \\ &= (x+2)(x^2 + 6x - 7) \\ &= (x+2)(x+7)(x-1) \end{aligned}$$

The other zeros are -7 and 1.

9. $25 = -15x^3 + 40x$

$0 = -15x^3 + 40x - 25$

$$\begin{array}{r|rrrr} 1 & -15 & 0 & 40 & -25 \\ & & -15 & -15 & 25 \\ \hline & -15 & -15 & 25 & 0 \end{array}$$

$(x-1)(-15x^2 - 15x + 25) = 0$

$x \approx 0.9$ is the other positive solution. The company could still make the same profit producing about 900,000 shoes.

5.5 Exercises (pp. 366–368)

Skill Practice

1. If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

2. The red numbers represent the coefficients of the quotient and the blue number represents the remainder.

3. $$\begin{array}{r} x + 5 \\ x - 4\overline{)x^2 + x - 17} \\ \underline{x^2 - 4x} \\ 5x - 17 \\ \underline{5x - 20} \\ 3 \end{array}$$

$$\frac{x^2 + x - 17}{x - 4} = x + 5 + \frac{3}{x - 4}$$

4. $$\begin{array}{r} 3x + 4 \\ x - 5\overline{)3x^2 - 11x - 26} \\ \underline{3x^2 - 15x} \\ 4x - 26 \\ \underline{4x - 20} \\ -6 \end{array}$$

$$\frac{3x^2 - 11x - 26}{x - 5} = 3x + 4 - \frac{6}{x - 5}$$

5. $$\begin{array}{r} x^2 + 4x + 7 \\ x - 1\overline{)x^3 + 3x^2 + 3x + 2} \\ \underline{x^3 - x^2} \\ 4x^2 + 3x \\ \underline{4x^2 - 4x} \\ 7x + 2 \\ \underline{7x - 7} \\ 9 \end{array}$$

$$\frac{x^3 + 3x^2 + 3x + 2}{x - 1} = x^2 + 4x + 7 + \frac{9}{x - 1}$$

6. $$\begin{array}{r} 2x + 9 \\ 4x - 1\overline{)8x^2 + 34x - 1} \\ \underline{8x^2 - 2x} \\ 36x - 1 \\ \underline{36x - 9} \\ 8 \end{array}$$

$$\frac{8x^2 + 34x - 1}{4x - 1} = 2x + 9 + \frac{8}{4x - 1}$$

7. $$\begin{array}{r} 3x + 8 \\ x^2 + x\overline{)3x^3 + 11x^2 + 4x + 1} \\ \underline{3x^3 + 3x^2} \\ 8x^2 + 4x \\ \underline{8x^2 + 8x} \\ -4x + 1 \end{array}$$

$$\frac{3x^3 + 11x^2 + 4x + 1}{x^2 + x} = 3x + 8 + \frac{-4x + 1}{x^2 + x}$$

8.
$$\begin{array}{r} 7x + 11 \\ x^2 + 1\overline{)7x^3 + 11x^2 + 7x + 5} \\ \underline{7x^3 \qquad\quad + 7x} \\ 11x^2 + 0x + 5 \\ \underline{11x^2 + 0x + 11} \\ -6 \end{array}$$

$$\frac{7x^3 + 11x^2 + 7x + 5}{x^2 + 1} = 7x + 11 - \frac{6}{x^2 + 1}$$

9.
$$\begin{array}{r} 5x^2 - 12x + 37 \\ x^2 + 2x - 4\overline{)5x^4 - 2x^3 - 7x^2 + 0x - 39} \\ \underline{5x^4 + 10x^3 - 20x^2} \qquad\qquad \\ -12x^3 + 13x^2 + 0x \qquad \\ \underline{-12x^2 - 24x^2 - 48x} \qquad \\ 37x^2 + 48x - 39 \\ \underline{37x^2 + 74x - 148} \\ -26x + 109 \end{array}$$

$$\frac{5x^4 - 2x^3 - 7x^2 - 39}{x^2 + 2x - 4} = 5x^2 - 12x + 37 + \frac{-26x + 109}{x^2 + 2x - 4}$$

10.
$$\begin{array}{r} 4x^2 + 12x + 44 \\ x^2 - 3x - 2\overline{)4x^4 + 0x^3 + 0x^2 + 5x - 4} \\ \underline{4x^4 - 12x^3 - 8x^2} \qquad\qquad \\ 12x^3 + 8x^2 + 5x \qquad \\ \underline{12x^3 - 36x^2 - 24x} \qquad \\ 44x^2 + 29x - 4 \\ \underline{44x^2 - 132x - 88} \\ 161x + 84 \end{array}$$

$$\frac{4x^4 + 5x - 4}{x^2 - 3x - 2} = 4x^2 + 12x + 44 + \frac{161x + 84}{x^2 - 3x - 2}$$

11.
$$\begin{array}{r|rrr} 5 & 2 & -7 & 10 \\ & & 10 & 15 \\ \hline & 2 & 3 & 25 \end{array}$$

$$\frac{2x^2 - 7x + 10}{x - 5} = 2x + 3 + \frac{25}{x - 5}$$

12.
$$\begin{array}{r|rrr} 2 & 4 & -13 & -5 \\ & & 8 & -10 \\ \hline & 4 & -5 & -15 \end{array}$$

$$\frac{4x^2 + 13x - 5}{x - 2} = 4x - 5 - \frac{15}{x - 2}$$

13.
$$\begin{array}{r|rrr} -4 & 1 & 8 & 1 \\ & & -4 & -16 \\ \hline & 1 & 4 & -15 \end{array}$$

$$\frac{x^2 + 8x + 1}{x + 4} = x + 4 - \frac{15}{x + 4}$$

14.
$$\begin{array}{r|rrr} 3 & 1 & 0 & 9 \\ & & 3 & 9 \\ \hline & 1 & 3 & 18 \end{array}$$

$$\frac{x^2 + 9}{x - 3} = x + 3 + \frac{18}{x - 3}$$

15.
$$\begin{array}{r|rrrr} 4 & 1 & -5 & 0 & -2 \\ & & 4 & -4 & -16 \\ \hline & 1 & -1 & -4 & -18 \end{array}$$

$$\frac{x^3 - 5x^2 - 2}{x - 4} = x^2 - x - 4 - \frac{18}{x - 4}$$

16.
$$\begin{array}{r|rrrr} -3 & 1 & 0 & -4 & 6 \\ & & -3 & 9 & -15 \\ \hline & 1 & -3 & 5 & -9 \end{array}$$

$$\frac{x^3 - 4x + 6}{x + 3} = x^2 - 3x + 5 - \frac{9}{x + 3}$$

17.
$$\begin{array}{r|rrrrr} 6 & 1 & -5 & -8 & 13 & -12 \\ & & 6 & 6 & -12 & 6 \\ \hline & 1 & 1 & -2 & 1 & -6 \end{array}$$

$$\frac{x^4 - 5x^3 - 8x^2 + 13x - 12}{x - 6} = x^3 + x^2 - 2x + 1 - \frac{6}{x - 6}$$

18.
$$\begin{array}{r|rrrrr} -5 & 1 & 4 & 0 & 16 & -35 \\ & & -5 & 5 & -25 & 45 \\ \hline & 1 & -1 & 5 & -9 & 10 \end{array}$$

$$\frac{x^4 + 4x^3 + 16x - 35}{x + 5} = x^3 - x^2 + 5x - 9 + \frac{10}{x + 5}$$

19. The division was done correctly, however, there is an error in how the quotient is written.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -5 & 3 \\ & & 2 & 4 & -2 \\ \hline & 1 & 2 & -1 & 1 \end{array}$$

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 + 2x - 1 + \frac{1}{x - 2}$$

20. The error is that a zero was not used as a place holder for the missing x^2 term.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -5 & 3 \\ & & 2 & 4 & -2 \\ \hline & 1 & 2 & -1 & 1 \end{array}$$

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 + 2x - 1 + \frac{1}{x - 2}$$

21. $$\begin{array}{r|rrrr} 6 & 1 & -10 & 19 & 30 \\ & & 6 & -24 & -30 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 - 10x^2 - 19x + 30 \\ &= (x - 6)(x^2 - 4x - 5) \\ &= (x - 6)(x - 5)(x + 1) \end{aligned}$$

22. $$\begin{array}{r|rrrr} -4 & 1 & 6 & 5 & -12 \\ & & -4 & -8 & 12 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 6x^2 + 5x - 12 \\ &= (x + 4)(x^2 + 2x - 3) \\ &= (x + 4)(x + 3)(x - 1) \end{aligned}$$

23. $$\begin{array}{r|rrrr} 8 & 1 & -2 & -40 & -64 \\ & & 8 & 48 & 64 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 - 2x^2 - 40x - 64 \\ &= (x - 8)(x^2 + 6x + 8) \\ &= (x - 8)(x + 4)(x + 2) \end{aligned}$$

24. $$\begin{array}{r|rrrr} -10 & 1 & 18 & 95 & 150 \\ & & -10 & -80 & -150 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 18x^2 + 95x + 150 \\ &= (x + 10)(x^2 + 8x + 15) \\ &= (x + 10)(x + 3)(x + 5) \end{aligned}$$

25. $$\begin{array}{r|rrrr} -9 & 1 & 2 & -51 & 108 \\ & & -9 & 63 & -108 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 2x^2 - 51x + 108 \\ &= (x + 9)(x^2 - 7x + 12) \\ &= (x + 9)(x - 4)(x - 3) \end{aligned}$$

26. $$\begin{array}{r|rrrr} -2 & 1 & -9 & 8 & 60 \\ & & -2 & 22 & -60 \\ \hline & 1 & -11 & 30 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 - 9x^2 + 8x + 60 \\ &= (x + 2)(x^2 - 11x + 30) \\ &= (x + 2)(x - 5)(x - 6) \end{aligned}$$

27. $$\begin{array}{r|rrrr} 1 & 2 & -15 & 34 & -21 \\ & & 2 & -13 & 21 \\ \hline & 2 & -13 & 21 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 2x^3 - 15x^2 + 34x - 21 \\ &= (x - 1)(2x^2 - 13x + 21) \\ &= (x - 1)(2x - 7)(x - 3) \end{aligned}$$

28. $$\begin{array}{r|rrrr} 5 & 3 & -2 & -61 & -20 \\ & & 15 & 65 & 20 \\ \hline & 3 & 13 & 4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 3x^3 - 2x^2 - 61x - 20 \\ &= (x - 5)(3x^2 + 13x + 4) \\ &= (x - 5)(3x + 1)(x + 4) \end{aligned}$$

29. $$\begin{array}{r|rrrr} -3 & 1 & -2 & -21 & -18 \\ & & -3 & 15 & 18 \\ \hline & 1 & -5 & -6 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 - 2x^2 - 21x - 18 \\ &= (x + 3)(x^2 - 5x - 6) \\ &= (x + 3)(x - 6)(x + 1) \end{aligned}$$

The other zeros are 6 and -1.

30. $$\begin{array}{r|rrrr} 10 & 4 & -25 & -154 & 40 \\ & & 40 & 150 & -40 \\ \hline & 4 & 15 & -4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 4x^3 - 25x^2 - 154x + 40 \\ &= (x - 10)(4x^2 + 15x - 4) \\ &= (x - 10)(4x - 1)(x + 4) \end{aligned}$$

The other zeros are $\frac{1}{4}$ and -4.

31. $$\begin{array}{r|rrrr} 7 & 10 & -81 & 71 & 42 \\ & & 70 & -77 & -42 \\ \hline & 10 & -11 & -6 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 10x^3 - 81x^2 + 71x + 42 \\ &= (x - 7)(10x^2 - 11x - 6) \\ &= (x - 7)(5x + 2)(2x - 3) \end{aligned}$$

The other zeros are $-\frac{2}{5}$ and $\frac{3}{2}$.

32.

$$\begin{array}{r|rrrr} -4 & 3 & 34 & 72 & -64 \\ & & -12 & -88 & 64 \\ \hline & 3 & 22 & -16 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 3x^3 + 34x^2 + 72x - 64 \\ &= (x+4)(3x^2 + 22x - 16) \\ &= (x+4)(3x-2)(x+8) \end{aligned}$$

The other zeros are $\frac{2}{3}$ and -8.

33.

$$\begin{array}{r|rrrr} 9 & 2 & -10 & -71 & -9 \\ & & 18 & 72 & 9 \\ \hline & 2 & 8 & 1 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 2x^3 - 10x^2 - 71x - 9 \\ &= (x-9)(2x^2 + 8x + 1) \end{aligned}$$

$$x = \frac{-8 \pm \sqrt{64 - 4(2)}}{2(2)} = \frac{-8x \pm \sqrt{56}}{4}$$

$$= \frac{-4 \pm \sqrt{14}}{2}$$

The other zeros are about -0.13 and -3.87.

34.

$$\begin{array}{r|rrrr} -2 & 5 & -1 & -18 & 8 \\ & & -10 & 22 & -8 \\ \hline & 5 & -11 & 4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 5x^3 - x^2 - 18x + 8 \\ &= (x+2)(5x^2 - 11x + 4) \end{aligned}$$

$$x = \frac{11 \pm \sqrt{121 - 4(5)(4)}}{10} = \frac{11 \pm \sqrt{41}}{10}$$

The other zeros are about 0.46 and 1.74.

35. D;

$$\begin{array}{r|rrrr} -6 & 4 & 15 & -63 & -54 \\ & & -24 & 54 & 54 \\ \hline & 4 & -9 & -9 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 4x^3 + 15x^2 - 63x - 54 \\ &= (x+6)(4x^2 - 9x - 9) \\ &= (x+6)(4x+3)(x-3) \end{aligned}$$

The zeros are -6, $-\frac{3}{4}$, and 3.

36. $(x + 4)(x + 2) = x^2 + 6x + 8$

$$\ell = \frac{2x^3 + 17x^2 + 46x + 40}{x^2 + 6x + 8}$$

$$\begin{array}{r} 2x + 5 \\ x^2 + 6x + 8\overline{\smash{)}2x^3 + 17x^2 + 46x + 40} \\ \underline{2x^3 + 12x^2 + 16x\phantom{{}+40}} \\ 5x^2 + 30x + 40 \\ \underline{5x^2 + 30x + 40} \\ 0 \end{array}$$

The missing dimension is $2x + 5$.

37. $(x - 1)(x + 6) = x^2 + 5x - 6$

$$w = \frac{x^3 + 13x^2 + 34x - 48}{x^2 + 5x - 6}$$

$$\begin{array}{r} x + 8 \\ x^2 + 5x - 6\overline{\smash{)}x^3 + 13x^2 + 34x - 48} \\ \underline{x^3 + 5x^2 - 6x\phantom{{}-48}} \\ 8x^2 + 40x - 48 \\ \underline{8x^2 + 40x - 48} \\ 0 \end{array}$$

The missing dimension is $x + 8$.

38. a.

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -12 & 36 \\ & & 2 & -6 & -36 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 - 5x^2 - 12x + 36 \\ &= (x-2)(x^2 - 3x - 18) \\ &= (x-2)(x-6)(x+3) \end{aligned}$$

The other zeros are 6 and -3.

b. The factors are $(x - 2)$, $(x - 6)$, and $(x + 3)$.

c. The solutions are $x = 2$, $x = 6$, $x = -3$.

39. A;

$$\begin{array}{r|rrrr} 5 & 1 & -1 & k & -30 \\ & & 5 & 20 & 30 \\ \hline & 1 & 4 & k + 20 & 0 \end{array}$$

For $x - 5$ to be a factor, the remainder must be zero. In the last column 30 must be added to -30 to get zero. $k + 20$ must equal 6 in order to have a product of 30 to add to the -30. $k + 20 = 6$ so, $k = -14$.

40. a. $\frac{1}{2}$

b.

$$\begin{array}{c|cccc} \frac{1}{2} & 30 & 7 & -39 & 14 \\ & & 15 & 11 & -14 \\ \hline & 30 & 22 & -28 & 0 \end{array}$$

$f(x) = \left(x - \frac{1}{2}\right)\left(30x^2 + 22x - 28\right)$

c. $f(x) = \left(x - \frac{1}{2}\right)(2)\left(15x^2 + 11x - 14\right)$

$= (2x - 1)(3x - 2)(5x + 7)$

Problem Solving

41. $P = -x^3 + 4x^2 + x$

$4 = -x^3 + 4x^2 + x$

$0 = -x^3 + 4x^2 + x - 4$

$$\begin{array}{c|cccc} 4 & -1 & 4 & 1 & -4 \\ & & -4 & 0 & 4 \\ \hline & -1 & 0 & 1 & 0 \end{array}$$

$P = (x - 4)\left(-x^2 + 1\right)$

$= (x - 4)(1 - x)(1 + x)$

$x = 1$ is the only other positive solution. So, the company could still make the same profit by producing only 1 million T-shirts.

42. $P = -4x^3 + 12x^2 + 16x$

$48 = -4x^3 + 12x^2 + 16x$

$0 = -4x^3 + 12x^2 + 16x - 48$

$$\begin{array}{c|cccc} 3 & -4 & 12 & 16 & -48 \\ & & -12 & 0 & 48 \\ \hline & -4 & 0 & 16 & 0 \end{array}$$

$P = (x - 3)\left(-4x^2 + 16\right) = (x - 3)(4 - 2x)(4 + 2x)$

$x = 2$ is the only other positive solution. The company could produce 2 million MP3 players and still make the same profit.

43. Let a = average attendance per team.

$a = \frac{A}{T} = \frac{-1.95x^3 + 70.1x^2 - 188x + 2150}{14.8x + 725}$

$$\begin{array}{r} -0.132x^2 + 11.2x - 561.35 \\ 14.8x + 725 \overline{)-1.95x^3 + 70.1x^2 - 188x + 2150} \\ \underline{-1.95x^3 - 95.7x^2} \qquad\qquad\qquad \\ 165.8x^2 - 188x \qquad\quad \\ \underline{165.8x^2 + 8120x} \qquad\quad \\ -8308x + 2150 \\ \underline{-8308x - 406{,}978.75} \\ 409{,}128.75 \end{array}$$

A function is

$a = -0.132x^2 + 11.2x - 561 + \frac{409{,}129}{14.8x + 725}$

44. a. Total revenue

= (number of radios sold)(price per radio)

$= (x)\left(40 - 4x^2\right) = 40x - 4x^3$

An expression is $40x - 4x^3$.

b. $P = \left(40x - 4x^3\right) - 15x = 25x - 4x^3$

A function is $P = 25x - 4x^3$.

c. When $P = 24$:

$24 = 25x - 4x^3$

$0 = 4x^3 - 25x + 24$

You know that 1.5 is a solution, so $x - 1.5$ is a factor of $4x^3 - 25x + 24$.

$$\begin{array}{c|cccc} 1.5 & 4 & 0 & -25 & 24 \\ & & 6 & 9 & -24 \\ \hline & 4 & 6 & -16 & 0 \end{array}$$

So, $(x - 1.5)\left(4x^2 + 6x - 16\right) = 0$. Use the quadratic formula to find $x \approx 1.39$, the other positive solution. So, about 1.39 million radios also make a profit of \$24,000,000.

d. No. The solution $x \approx -2.88$ does not make sense because you cannot produce a negative number of radios.

45. Let P = the percent of visits to the national park that were overnight stays.

$P = \frac{S}{V} = \frac{-0.00722x^4 + 0.176x^3 - 1.40x^2 + 3.39x + 17.6}{3.10x + 256}$

$$\begin{array}{r} -0.00233x^3 + 0.249x^2 - 21x + 1735.287 \\ 3.10x + 256 \overline{)-0.00722x^4 + 0.176x^3 - 1.40x^2 + 3.39x + 17.6} \\ \underline{-0.00722x^4 - 0.596x^3} \qquad\qquad\qquad\qquad\qquad \\ 0.772x^3 - 1.40x^2 \qquad\qquad\qquad \\ \underline{0.772x^3 + 63.7x^2} \qquad\qquad\qquad \\ -65.1x^2 + 3.39x \qquad\quad \\ \underline{-65.1x^2 - 5376x} \qquad\quad \\ 5379.39x + 17.6 \\ \underline{5379.39x + 44{,}233.472} \\ -44215.872 \end{array}$$

So,

$P = -0.00233x^3 + 0.249x^2 - 21x + 1735 - \frac{444{,}216}{3.10x + 256}$.

The function for the percent of visits that were overnight stays is the overnight stays divided by the total visits.

46. $P = -6x^3 + 72x$

$96 = -6x^3 + 72x$

$0 = -6x^3 + 72x - 96$

$$\begin{array}{r|rrrr} 2 & -6 & 0 & 72 & -96 \\ & & -12 & -24 & 96 \\ \hline & -6 & -12 & 48 & 0 \end{array}$$

$P = (-6x^2 - 12x + 48)(x - 2)$

$= (-3x + 6)(2x + 8)(x - 2)$

$= -3(x - 2)(2)(x + 4)(x - 2)$

$= -6(x - 2)(x + 4)(x - 2)$

The zeros are at 2 and -4. Production levels cannot be negative so $x = 2$ is the only solution.

Mixed Review

47. $x - 4y < 5$ $\qquad$ $x - 4y < 5$

$1 - 4(4) < 5$ $\qquad$ $4 - 4(-1) < 5$

$1 - 16 < 5$ $\qquad$ $4 + 4 < 5$

$-15 < 5$ yes $\qquad$ $8 < 5$ no

(1, 4) is a solution.

48. $3x + 2y \ge 1$ $\qquad$ $3x + 2y \ge 1$

$3(-2) + 2(4) \ge 1$ $\qquad$ $3(1) + 2(-3) \ge 1$

$-6 + 8 \ge 1$ $\qquad$ $3 - 6 \ge 1$

$2 \ge 1$ yes $\qquad$ $-3 \ge 1$ no

$(-2, 4)$ is a solution.

49. $5x - 2y > 10$ $\qquad$ $5x - 2y > 10$

$5(4) - 2(6) > 10$ $\qquad$ $5(8) - 2(10) > 10$

$20 - 12 > 10$ $\qquad$ $40 - 20 > 10$

$8 > 10$ no $\qquad$ $20 > 10$ yes

(8, 10) is a solution.

50. $6x + 5y \le 15$ $\qquad$ $6x + 5y \le 15$

$6(-5) + 5(10) \le 15$ $\qquad$ $6(-1) + 5(4) \le 15$

$-30 + 50 \le 15$ $\qquad$ $-6 + 20 \le 15$

$20 \le 15$ no $\qquad$ $14 \le 15$ yes

$(-1, 4)$ is a solution.

51. $x^2 + 3x - 40 = 0$

$(x - 5)(x + 8) = 0$

$x - 5 = 0$ or $x + 8 = 0$

$x = 5$ or $x = -8$

52. $5x^2 + 13x + 6 = 0$

$(5x + 3)(x + 2) = 0$

$5x + 3 = 0$ or $x + 2 = 0$

$5x = -3$

$x = -\frac{3}{5}$ or $x = -2$

53. $x^2 + 7x + 2 = 0$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{41}}{2}$$

≈ -0.30 and -6.70

54. $4x^2 + 15x + 10 = 0$

$$x = \frac{-15 \pm \sqrt{(15)^2 - 4(4)(10)}}{2(4)} = \frac{-15 \pm \sqrt{65}}{8}$$

≈ -2.88 and -0.87

55. $2x^2 + 15x + 31 = 0$

$$x = \frac{-15 \pm \sqrt{(15)^2 - 4(2)(31)}}{2(2)} = \frac{-15 \pm \sqrt{-23}}{4} = \frac{-15 \pm i\sqrt{23}}{4}$$

The solutions are $\frac{-15 + i\sqrt{23}}{4}$ and $\frac{-15 - i\sqrt{23}}{4}$.

56. $x^2 + 2x + 10 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(10)}}{2(1)} = \frac{-2 \pm \sqrt{-36}}{2} = -1 \pm 3i$$

The solutions are $-1 + 3i$ and $-1 - 3i$.

57. $(x^2 - 4x + 15) + (-3x^2 + 6x - 12)$

$= x^2 - 4x + 15 - 3x^2 + 6x - 12$

$= -2x^2 + 2x + 3$

58. $(2x^2 - 5x + 8) - (5x^2 - 7x - 7)$

$= 2x^2 - 5x + 8 - 5x^2 + 7x + 7$

$= -3x^2 + 2x + 15$

59. $(3x - 4)(3x^3 + 2x^2 - 8)$

$= 3x(3x^3 + 2x^2 - 8) - 4(3x^3 + 2x^2 - 8)$

$= 9x^4 + 6x^3 - 24x - 12x^3 - 8x^2 + 32$

$= 9x^4 - 6x^3 - 8x^2 - 24x + 32$

60. $(3x - 5)^3 = (3x)^3 - 3(3x)^2(5) + 3(3x)(5)^2 - (5)^3$

$= 27x^3 - 135x^2 + 225x - 125$

Mixed Review of Problem Solving (p. 369)

1. a. $164{,}000{,}000{,}000 = 1.64 \times 10^{11}$ yards

b. $\frac{1.64 \times 10^{11}}{1.20 \times 10^2} = \frac{1.64}{1.20} \times \frac{10^{11}}{10^2}$

$= 1.367 \times 10^9$ football fields

2. a. $T(x) = l \cdot w \cdot h$

$= (4x)(x)(2x)$

$= 8x^3$

b. $C(x) = (4x - 2)(x - 2)(2x - 4)$

$= (4x^2 - 10x + 4)(2x - 4)$

$= 8x^3 - 36x^2 + 48x - 16$

c. $I(x) = T(x) - C(x)$

d. $I(x) = 8x^3 - (8x^3 - 36x^2 + 48x - 16)$

$= 36x^2 - 48x + 16$

$I(8) = 36(8)^2 - 48(8) + 16$

$= 2304 - 384 + 16$

$= 1936$

The volume of the insulation is 1936 cubic inches.

3. Cube: $V = x^3$ $\qquad \frac{A}{V} = \frac{6x^2}{x^3} = \frac{6}{x}$

$A = 6x^2$

Sphere: $V = \frac{4}{3}\pi\left(\frac{x}{2}\right)^3$ $\qquad \frac{A}{V} = \frac{\pi x^2}{\frac{\pi}{6}x^3} = \frac{6}{x}$

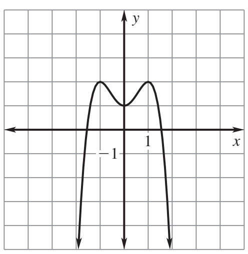
$= \frac{\pi}{6}x^3$

$A = 4\pi\left(\frac{x}{2}\right)^2$

$= \pi x^2$

The cells exchange at the same rate.

4. $f(x) = -x^4 + 2x^2 + 1$



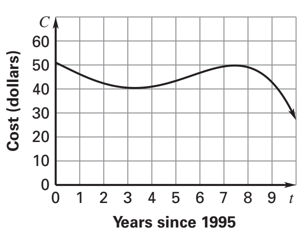
5. a. C is a degree 4 (quartic).

b.

***t*, year**	0	1	2	3	4
***C*, cost**	51	46.14	42.33	40.50	40.97

***t*, year**	5	6	7	8
***C*, cost**	43.38	46.73	49.38	49.05

c.



No, the model will not accurately predict cell phone bills beyond 2003. The model was not intended to be used past 2003. From the graph you can see that it drops sharply past 2003.

6. a. $R = x(100 - 10x) = 100x - 10x^3$

A function for the total revenue is $R(x) = 100x - 10x^3$.

b. $P = (100x - 10x^3) - 30x = 70x - 10x^3$

A function for the profit is $P(x) = 70x - 10x^3$.

c. When $P = 60$:

$60 = 70x - 10x^3$

$0 = 10x^3 - 70x + 60$

You know that 2 is a solution, so $x - 2$ is a factor of $10x^3 - 70x + 60$.

$$\begin{array}{r|rrrr} 2 & 10 & 0 & -70 & 60 \\ & & 20 & 40 & -60 \\ \hline & 10 & 20 & -30 & 0 \end{array}$$

So, $(x - 2)(10x^2 + 20x - 30) = 0$. Use factoring to find that $x = 1$ and $x = -3$ are the other solutions.

d. No. The solution $x = -3$ does not make sense because you cannot produce a negative number of cameras.

7. $V = \frac{1}{3}h(s^2)$

$48 = \frac{1}{3}(x)(3x - 6)^2$

$48 = \frac{x}{3}(9x^2 - 36x + 36)$

$48 = 3x^3 - 12x^2 + 12x$

$0 = 3x^3 - 12x^2 + 12x - 48$

$0 = 3x^2(x - 4) + 12(x - 4)$

$0 = (3x^2 + 12)(x - 4)$

$x = 4$

The height is 4 feet.

Lesson 5.6

5.6 Guided Practice (pp. 371–373)

1. Factors of the constant term: $\pm1, \pm3, \pm5, \pm15$

Factors of the leading coefficient: ±1

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}$

Simplified list: $\pm1, \pm3, \pm5, \pm15$

2. Factors of the constant term: $\pm1, \pm2, \pm3, \pm6$

Factors of the leading coefficient: $\pm1, \pm2$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{6}{2}$

Simplified list: $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

3. Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{9}{1}, \pm\frac{18}{1}$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

Because 1 is a zero of f, $f(x)$ can be writen as:

$f(x) = (x - 1)(x^2 - 3x - 18) = (x - 1)(x - 6)(x + 3)$

The zeros are -3, 1 and 6.

4. Possible rational zeros: $\pm 1, \pm 2, \pm 7, \pm 14$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 5 & 14 \\ & & 1 & -7 & -2 \\ \hline & 1 & -7 & -2 & -12 \end{array}$$

1 is not a zero.

Test $x = -1$:

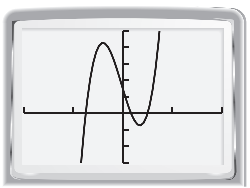
$$\begin{array}{r|rrrr} -1 & 1 & -8 & 5 & 14 \\ & & -1 & 9 & -14 \\ \hline & 1 & -9 & 14 & 0 \end{array}$$

Because -1 is a zero of f, $f(x)$ can be written as:

$f(x) = (x + 1)(x^2 - 9x + 14) = (x + 1)(x - 7)(x - 2)$

The zeros of f are -1, 2 and 7.

5. Possible rational zeros: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{6}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{1}{12}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{1}{24}, \pm\frac{1}{48}$



Reasonable values: $x = -\frac{3}{4}, x = \frac{3}{16}, x = \frac{1}{2}$

Check: $x = -\frac{3}{4}$:

$$\begin{array}{r|rrrr} -\frac{3}{4} & 48 & 4 & -20 & 3 \\ & & -36 & 24 & -3 \\ \hline & 48 & -32 & 4 & 0 \end{array}$$

$-\frac{3}{4}$ is a zero.

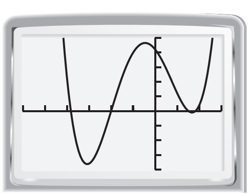
$f(x) = \left(x + \frac{3}{4}\right)(48x^2 - 32x + 4)$

$= \left(x + \frac{3}{4}\right)(4)(12x^2 - 8x + 1)$

$= (4x + 3)(6x - 1)(2x - 1)$

The real zeros of f are $-\frac{3}{4}, \frac{1}{6}$, and $\frac{1}{2}$.

6. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$



Reasonable zeros: $x = -\frac{7}{2}, x = -2, x = \frac{3}{2}, x = 2$

$$\begin{array}{r|rrrrr} -\frac{7}{2} & 2 & 5 & -18 & -19 & 42 \\ & & -7 & 7 & \frac{77}{2} & \frac{-273}{4} \\ \hline & 2 & -2 & -11 & \frac{39}{2} & \frac{-105}{4} \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 2 & 5 & -18 & -19 & 42 \\ & & -4 & -2 & 40 & -42 \\ \hline & 2 & 1 & -20 & 21 & 0 \end{array}$$

$f(x) = (x + 2)(2x^3 + x^2 - 20x + 21) = (x + 2) \cdot g(x)$

Possible rational zeros of $g(x)$: $\pm 1, \pm 3, \pm 7, \pm 21, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$

Reasonable zeros: $x = \frac{3}{2}, x = 2$

$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & 1 & -20 & 21 \\ & & 3 & 6 & -21 \\ \hline & 2 & 4 & -14 & 0 \end{array}$$

$f(x) = (x + 2) \cdot g(x)$

$= (x + 2)\left(x - \frac{3}{2}\right)(2x^2 + 4x - 14)$

$2x^2 + 4x - 14 = 0$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-14)}}{2(2)} = \frac{-4 \pm \sqrt{128}}{4}$$

$= -1 \pm 2\sqrt{2}$

The real zeros of f are: $-2, \frac{3}{2}, -1 + 2\sqrt{2}$, and $-1 - 2\sqrt{2}$.

7. $V = \frac{1}{3}(x^2)(x - 1)$

$6 = \frac{1}{3}x^2(x - 1)$

$18 = x^3 - x^2$

$0 = x^3 - x^2 - 18$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 0 & -18 \\ & & 2 & 2 & 4 \\ \hline & 1 & 1 & 2 & -14 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & 0 & -18 \\ & & 3 & 6 & 18 \\ \hline & 1 & 2 & 6 & 0 \end{array}$$

The other solutions, which satisfy $x^2 + 2x + 6$, are $x = -1 \pm i\sqrt{5}$ and can be discarded because they are imaginary. The only real solution is $x = 3$. The base is 3×3 feet. The height is $x - 1$ or 2 feet.

5.6 Exercises (pp. 374–377)

Skill Practice

1. If a polynomial function has integer coefficients, then every rational zero of the function has the form $\frac{p}{q}$ where p is a factor of the *constant term* and q is a factor of the *leading coefficient*.

2. To shorten the list, draw the graph to see approximately where the zeros are.

3. $f(x) = x^3 - 3x + 28$

Factors of constant term: $\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$

Factors of leading coefficient: ± 1

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}, \pm\frac{28}{1}$

Simplified list: $\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$

4. $g(x) = x^3 - 4x^2 + x - 10$

Factors of constant term: $\pm 1, \pm 2, \pm 5, \pm 10$

Factors of leading coefficient: ± 1

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}$

Simplified list: $\pm 1, \pm 2, \pm 5, \pm 10$

5. $f(x) = 2x^4 + 6x^3 - 7x + 9$

Factors of constant term: $\pm 1, \pm 3, \pm 9$

Factors of leading coefficient: $\pm 1, \pm 2$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{9}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

Simplified list: $\pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

6. $h(x) = 2x^3 + x^2 - x - 18$

Factors of constant term: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

Factors of leading coefficient: $\pm 1, \pm 2$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{9}{1}, \pm\frac{18}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{6}{2}, \pm\frac{9}{2}, \pm\frac{18}{2}$

Simplified list: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

7. $g(x) = 4x^5 + 3x^3 - 2x - 14$

Factors of the constant term: $\pm 1, \pm 2, \pm 7, \pm 14$

Factors of the leading coefficient: $\pm 1, \pm 2, \pm 4$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{7}{2}, \pm\frac{14}{2}, \pm\frac{1}{4}, \pm\frac{2}{4}, \pm\frac{7}{4}, \pm\frac{14}{4}$

Simplified list: $\pm 1, \pm 2, \pm 7, \pm 14, \pm\frac{1}{2}, \pm\frac{7}{2}, \pm\frac{1}{4}, \pm\frac{7}{4}$

8. $f(x) = 3x^4 + 5x^3 - 3x + 42$

Factors of the constant term: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$

Factors of the leading coefficient: $\pm 1, \pm 3$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}, \pm\frac{21}{1}, \pm\frac{42}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{6}{3}, \pm\frac{7}{3}, \pm\frac{14}{3}, \pm\frac{21}{3}, \pm\frac{42}{3}$

Simplified list: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{7}{3}, \pm\frac{14}{3}$

9. $h(x) = 8x^4 + 4x^3 - 10x + 15$

Factors of the constant term: $\pm 1, \pm 3, \pm 5, \pm 15$

Factors of the leading coefficient: $\pm 1, \pm 2, \pm 4, \pm 8$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, \pm\frac{15}{8}$

Simplified list: $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{7}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, \pm\frac{15}{8}$

10. $h(x) = 6x^3 - 3x^2 + 12$

Factors of the constant term: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Factors of the leading coefficient: $\pm 1, \pm 2, \pm 3, \pm 6$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{4}{2}, \pm\frac{6}{2}, \pm\frac{12}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{4}{3}, \pm\frac{6}{3}, \pm\frac{12}{3}, \pm\frac{1}{6}, \pm\frac{2}{6}, \pm\frac{3}{6}, \pm\frac{4}{6}, \pm\frac{6}{6}, \pm\frac{12}{6}$

Simplified list: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{6}$

11. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -12 & 35 & -24 \\ & & 1 & -11 & 24 \\ \hline & 1 & -11 & 24 & 0 \end{array}$$

$$f(x) = (x - 1)(x^2 - 11x + 24)$$

$$= (x - 1)(x - 3)(x - 8)$$

Real zeros are 1, 3, and 8.

12. Possible rational zeros: $\pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -5 & -22 & 56 \\ & & 1 & -4 & -26 \\ \hline & 1 & -4 & -26 & 30 \end{array}$$

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 1 & -5 & -22 & 56 \\ & & -1 & 6 & 16 \\ \hline & 1 & -6 & -16 & 72 \end{array}$$

Test $x = 2$:

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -22 & 56 \\ & & 2 & -6 & -56 \\ \hline & 1 & -3 & -28 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-2)(x^2-3x-28) \\ &= (x-2)(x-7)(x+4) \end{aligned}$$

Real zeros are 2, 7, and -4.

13. Possible rational zeros: $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -31 & -30 \\ & & 1 & 1 & -30 \\ \hline & 1 & 1 & -30 & -60 \end{array}$$

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -31 & -30 \\ & & -1 & 1 & 30 \\ \hline & 1 & -1 & -30 & 0 \end{array}$$

$$\begin{aligned} g(x) &= (x+1)(x^2-x-30) \\ &= (x+1)(x-6)(x+5) \end{aligned}$$

Real zeros are -1, 6, and -5.

14. Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm9, \pm12, \pm18, \pm24, \pm36, \pm72$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & 8 & -9 & -72 \\ & & 1 & 9 & 0 \\ \hline & 1 & 9 & 0 & -72 \end{array}$$

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 1 & 8 & -9 & -72 \\ & & -1 & -7 & 16 \\ \hline & 1 & 7 & -16 & -56 \end{array}$$

Test $x = 2$:

$$\begin{array}{r|rrrr} 2 & 1 & 8 & -9 & -72 \\ & & 2 & 20 & 22 \\ \hline & 1 & 10 & 11 & -50 \end{array}$$

Test $x = -2$:

$$\begin{array}{r|rrrr} -2 & 1 & 8 & -9 & -72 \\ & & -2 & -12 & 42 \\ \hline & 1 & 6 & -21 & -30 \end{array}$$

Test $x = 3$:

$$\begin{array}{r|rrrr} 3 & 1 & 8 & -9 & -72 \\ & & 3 & 33 & 72 \\ \hline & 1 & 11 & 24 & 0 \end{array}$$

$$\begin{aligned} h(x) &= (x-3)(x^2+11x+24) \\ &= (x-3)(x+8)(x+3) \end{aligned}$$

The real zeros are 3, -8, and -3.

15. Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Test $x = 1$:

$$\begin{array}{r|rrrrr} 1 & 1 & 7 & 26 & 44 & 24 \\ & & 1 & 8 & 34 & 78 \\ \hline & 1 & 8 & 34 & 78 & 102 \end{array}$$

Test $x = -1$:

$$\begin{array}{r|rrrrr} -1 & 1 & 7 & 26 & 44 & 24 \\ & & -1 & -6 & -20 & -24 \\ \hline & 1 & 6 & 20 & 24 & 0 \end{array}$$

$h(x) = (x+1)(x^3+6x^2+20x+24) = (x+1) \cdot g(x)$

Possible rational zeros of $g(x)$: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Test: $x = 2$:

$$\begin{array}{r|rrrr} 2 & 1 & 6 & 20 & 24 \\ & & 2 & 16 & 52 \\ \hline & 1 & 8 & 26 & 76 \end{array}$$

Test: $x = -2$:

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 20 & 24 \\ & & -2 & -8 & -24 \\ \hline & 1 & 4 & 12 & 0 \end{array}$$

$h(x) = (x + 1)(x + 2)(x^2 + 4x + 12)$

$0 = x^2 + 4x + 12$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(12)}}{2(1)}$$

$= -2 \pm 2i\sqrt{2}$, which are not real.

The only real solutions are -1 and -2.

16. Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Test $x = 1$:

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & -9 & 10 & -24 \\ & & 1 & -1 & -10 & 0 \\ \hline & 1 & -1 & -10 & 0 & -24 \end{array}$$

Test $x = -1$:

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & -9 & 10 & -24 \\ & & -1 & 3 & 6 & -16 \\ \hline & 1 & -3 & -6 & 16 & -40 \end{array}$$

Test $x = 2$:

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & -9 & 10 & -24 \\ & & 2 & 0 & -18 & -16 \\ \hline & 1 & 0 & -9 & -8 & -40 \end{array}$$

Test $x = -2$:

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -9 & 10 & -24 \\ & & -2 & 8 & 2 & -24 \\ \hline & 1 & -4 & -1 & 12 & -48 \end{array}$$

Test $x = 3$:

$$\begin{array}{r|rrrrr} 3 & 1 & -2 & -9 & 10 & -24 \\ & & 3 & 3 & -18 & -24 \\ \hline & 1 & 1 & -6 & -8 & -48 \end{array}$$

Test $x = -3$:

$$\begin{array}{r|rrrrr} -3 & 1 & -2 & -9 & 10 & -24 \\ & & -3 & 15 & -18 & 24 \\ \hline & 1 & -5 & 6 & -8 & 0 \end{array}$$

$f(x) = (x + 3)(x^3 - 5x^2 + 6x - 8) = (x + 3) \cdot g(x)$

Possible rational zeros of $g(x)$: $\pm1, \pm2, \pm4, \pm8$

Test $x = 4$:

$$\begin{array}{r|rrrr} 4 & 1 & -5 & 6 & -8 \\ & & 4 & -4 & 8 \\ \hline & 1 & -1 & 2 & 0 \end{array}$$

$f(x) = (x + 3)(x - 4)(x^2 - x + 2)$

$x^2 - x + 2 = 0$

$$x = \frac{1 \pm \sqrt{1 - 4(1)(2)}}{2}$$

$= \dfrac{1 \pm i\sqrt{7}}{2}$ which are not real.

The real solutions are -3 and 4.

17. Possible rational zeros: $\pm1, \pm2, \pm4, \pm8$

Test $x = 1$:

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -9 & -2 & 8 \\ & & 1 & 3 & -6 & -8 \\ \hline & 1 & 3 & -6 & -8 & 0 \end{array}$$

$f(x) = (x - 1)(x^3 + 3x^2 - 6x - 8) = (x - 1) \cdot g(x)$

Possible rational zeros of $g(x)$: $\pm1, \pm2, \pm4, \pm8$

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

$f(x) = (x - 1)(x + 1)(x^2 + 2x - 8)$

$= (x - 1)(x + 1)(x + 4)(x - 2)$

The real zeros are 1, -1, -4, and 2.

18. Possible rational zeros: $\pm1, \pm5, \pm25$

Test $x = 1$:

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -16 & -40 & -25 \\ & & 1 & 1 & -15 & -55 \\ \hline & 1 & 1 & -15 & -55 & -80 \end{array}$$

Test $x = -1$:

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -16 & -40 & -25 \\ & & -1 & 1 & 15 & 25 \\ \hline & 1 & -1 & -15 & -25 & 0 \end{array}$$

$g(x) = (x + 1)(x^3 - x^2 - 15x - 25) = (x + 1) \cdot f(x)$

Possible zeros of $f(x)$: $\pm1, \pm5, \pm25$

Test $x = 5$:

$$\begin{array}{r|rrrr} 5 & 1 & -1 & -15 & -25 \\ & & 5 & 20 & 25 \\ \hline & 1 & 4 & 5 & 0 \end{array}$$

$g(x) = (x + 1)(x - 5)(x^2 + 4x + 5)$

$x^2 + 4x + 5 = 0$

$x = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$

$= -2 \pm i$, which are not real.

The only real zeros are -1 and 5.

19. Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{2}, \pm \frac{1}{4}$

Reasonable rational zeros: $x = 1$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -20 & 16 \\ & & 4 & 4 & -16 \\ \hline & 4 & 4 & -16 & 0 \end{array}$$

$f(x) = (x - 1)(4x^2 + 4x - 16)$

$= (x - 1)(4)(x^2 + x - 4)$

$0 = x^2 + x - 4$

$x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{17}}{2}$

Real zeros: $1, \dfrac{-1 + \sqrt{17}}{2}, \dfrac{-1 - \sqrt{17}}{2}$

20. Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{5}{4}, \pm \frac{15}{4}$

Reasonable zeros: $x = -1, x = \frac{3}{2}, x = \frac{5}{2}$

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 4 & -12 & -1 & 15 \\ & & -4 & 16 & -15 \\ \hline & 4 & -16 & 15 & 0 \end{array}$$

$f(x) = (x + 1)(4x^2 - 16x + 15)$

$= (x + 1)(2x - 3)(2x - 5)$

The real zeros are $-1, \frac{3}{2}$, and $\frac{5}{2}$.

21. Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$

Reasonable zeros: $x = -3, x = -\frac{5}{3}, x = \frac{1}{2}$

Test $x = -3$:

$$\begin{array}{r|rrrr} -3 & 6 & 25 & 16 & -15 \\ & & -18 & -21 & 15 \\ \hline & 6 & 7 & -5 & 0 \end{array}$$

$f(x) = (x + 3)(6x^2 + 7x - 5)$

$= (x + 3)(2x - 1)(3x + 5)$

Real zeros are $-3, \frac{1}{2}$, and $-\frac{5}{3}$.

22. Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{16}{3}$

Reasonable zeros: $x = \frac{2}{3}, x = 2, x = 4$

Test $x = \frac{2}{3}$:

$$\begin{array}{r|rrrr} \frac{2}{3} & -3 & 20 & -36 & 16 \\ & & -2 & 12 & -16 \\ \hline & -3 & 18 & -24 & 0 \end{array}$$

$f(x) = \left(x - \frac{2}{3}\right)(-3x^2 + 18x - 24)$

$= \left(x - \frac{2}{3}\right)(3)(-x^2 + 6x - 8)$

$= (3x - 2)(-x + 2)(x - 4)$

Real zeros are $\frac{2}{3}$, 2, and 4.

23. C; Possible zeros: $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

24. Possible zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

Reasonable zeros: $x = -2, x = -1$, and $x = 2$

Test $x = -2$:

$$\begin{array}{r|rrrr} -2 & 2 & 2 & -8 & -8 \\ & & -4 & 4 & 8 \\ \hline & 2 & -2 & -4 & 0 \end{array}$$

$f(x) = (x + 2)(2x^2 - 2x - 4)$

$= (x + 2)(2x + 2)(x - 2)$

The real zeros are $-2, -1$, and 2.

25. Possible rational zeros: $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

Reasonable zeros: $x = -1, x = \frac{3}{2}, x = 3$

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 2 & -7 & 0 & 9 \\ & & -2 & 9 & -9 \\ \hline & 2 & -9 & 9 & 0 \end{array}$$

$g(x) = (x + 1)(2x^2 - 9x + 9) = (x + 1)(2x - 3)(x - 3)$

The real zeros are $-1, \frac{3}{2}$, and 3.

26. Possible rational zeros: $\pm1, \pm3, \pm5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

Reasonable zeros: $x = -\frac{5}{2}, x = 1$, and $x = 3$

Test $x = -\frac{5}{2}$:

$$\begin{array}{r|rrrr} -\frac{5}{2} & 2 & -3 & -14 & 15 \\ & & -5 & 20 & -15 \\ \hline & 2 & -8 & 6 & 0 \end{array}$$

$h(x) = \left(x + \frac{5}{2}\right)(2x^2 - 8x + 6)$

$= \left(x + \frac{5}{2}\right)(2)(x^2 - 4x + 3)$

$= (2x + 5)(x - 3)(x - 1)$

The real zeros are $-\frac{5}{2}$, 3, and 1.

27. Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Reasonable zeros: $x = -4, x = -\frac{1}{3}$, and $x = 3$

Test $x = -4$:

$$\begin{array}{r|rrrr} -4 & 3 & 4 & -35 & -12 \\ & & -12 & 32 & 12 \\ \hline & 3 & -8 & -3 & 0 \end{array}$$

$f(x) = (x + 4)(3x^2 - 8x - 3) = (x + 4)(3x + 1)(x - 3)$

The real zeros are $-4, -\frac{1}{3}$, and 3.

28. Possible real zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Reasonable zeros: $x = -6, x = -1, x = \frac{2}{3}$

Test $x = -6$:

$$\begin{array}{r|rrrr} -6 & 3 & 19 & 4 & -12 \\ & & -18 & -6 & 12 \\ \hline & 3 & 1 & -2 & 0 \end{array}$$

$f(x) = (x + 6)(3x^2 + x - 2)$

$= (x + 6)(3x - 2)(x + 1)$

The real zeros are $-6, -1$, and $\frac{2}{3}$.

29. Possible real zeros: $\pm1, \pm2, \pm7, \pm14, \pm\frac{1}{2}, \pm\frac{7}{2}$

Reasonable zeros: $x = -\frac{7}{2}, x = -1$, and $x = 2$

Test $x = -\frac{7}{2}$:

$$\begin{array}{r|rrrr} -\frac{7}{2} & 2 & 5 & -11 & -14 \\ & & -7 & 7 & 14 \\ \hline & 2 & -2 & -4 & 0 \end{array}$$

$g(x) = \left(x + \frac{7}{2}\right)(2x^2 - 2x - 4)$

$= \left(x + \frac{7}{2}\right)(2)(x^2 - x - 2)$

$= (2x + 7)(x - 2)(x + 1)$

The real zeros are $-\frac{7}{2}$, 2, and -1.

30. Possible real solutions: $\pm1, \pm2, \pm4, \pm\frac{1}{2}$

Resonable zeros: $x = -4$ and $x = \frac{1}{2}$

Test $x = -4$:

$$\begin{array}{r|rrrrr} -4 & 2 & 9 & 5 & 3 & -4 \\ & & -8 & -4 & -4 & 4 \\ \hline & 2 & 1 & 1 & -1 & 0 \end{array}$$

$g(x) = (x + 4)(2x^3 + x^2 + x - 1) = (x + 4) \cdot f(x)$

Possible real solutions of $f(x)$: $\pm1, \pm\frac{1}{2}$

Test $x = \frac{1}{2}$:

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 1 & 1 & -1 \\ & & 1 & 1 & 1 \\ \hline & 2 & 2 & 2 & 0 \end{array}$$

$g(x) = (x + 4)\left(x - \frac{1}{2}\right)(2x^2 + 2x + 2)$

$= (x + 4)\left(x - \frac{1}{2}\right)(2)(x^2 + x + 1)$

$= (x + 4)(2x - 1)(x^2 + x + 1)$

$x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$= \frac{-1 \pm i\sqrt{3}}{2}$, which are imaginary numbers

The only real solutions are -4 and $\frac{1}{2}$.

31. Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$

Reasonable zeros: $x = -2$ and $x = 2$

Test $x = -2$:

-2	2	-1	-7	4	-4
		-4	10	-6	4
	2	-5	3	-2	0

$h(x) = (x + 2)(2x^3 - 5x^2 + 3x - 2) = (x + 2) \cdot f(x)$

Possible rational zeros of f: $\pm 1, \pm 2, \pm\frac{1}{2}$

Test $x = 2$:

2	2	-5	3	-2
		4	-2	2
	2	-1	1	0

$h(x) = (x + 2)(x - 2)(2x^2 - x + 1)$

$2x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{1 \pm i\sqrt{7}}{4}, \text{ which are not real.}$$

The real solutions are $x = -2$ and $x = 2$.

32. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Reasonable zeros: $x = -3$, and $x = 4$

Test $x = -3$:

-3	3	-6	-32	35	-12
		-9	45	-39	12
	3	-15	13	-4	0

$h(x) = (x + 3)(3x^3 - 15x^2 + 13x - 4) = (x + 3) \cdot g(x)$

Possible rational zeros of g: $\pm 1, \pm 2, \pm 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Test $x = 4$:

4	3	-15	13	-4
		12	-12	4
	3	-3	1	0

$h(x) = (x + 3)(x - 4)(3x^2 - 3x + 1)$

$3x^2 - 3x + 1 = 0$

$$x = \frac{3 \pm \sqrt{3^2 - 4(3)(1)}}{2(3)} = \frac{3 \pm i\sqrt{3}}{6}, \text{ which are not real.}$$

The only real solutions are -3 and 4.

33. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$.

Reasonable zeros: $x = -2, x = 1, x = \frac{5}{2}$, and $x = 3$

Test $x = -2$:

-2	2	-9	0	37	-30
		-4	26	-52	30
	2	-13	26	-15	0

$f(x) = (x + 2)(2x^3 - 13x^2 + 26x - 15) = (x + 2) \cdot g(x)$

Possible rational zeros of g: $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

Test $x = 1$:

1	2	-13	26	-15
		2	-11	15
	2	-11	15	0

$f(x) = (x + 2)(x - 1)(2x^2 - 11x + 15)$

$\quad = (x + 2)(x - 1)(2x - 5)(x - 3)$

The real zeros are $-2, 1, \frac{5}{2}$, and 3.

34. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Reasonable zeros: $x = -2, x = -1, x = 1, x = 2$, and $x = 3$

Test $x = -2$:

-2	1	-3	-5	15	4	-12
		-2	10	-10	-10	12
	1	-5	5	5	-6	0

$f(x) = (x + 2)(x^4 - 5x^3 + 5x^2 + 5x - 6)$

$\quad = (x + 2) \cdot g(x)$

Possible rational zeros of g: $\pm 1, \pm 2, \pm 3, \pm 6$

Test $x = -1$:

-1	1	-5	5	5	-6
		-1	6	-11	6
	1	-6	11	-6	0

$f(x) = (x + 2)(x + 1)(x^3 - 6x^2 + 11x - 6)$

$\quad = (x + 2)(x + 1) \cdot h(x)$

Possible rational zeros of h: $\pm 1, \pm 2, \pm 3, \pm 6$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$f(x) = (x + 2)(x + 1)(x - 1)(x^2 - 5x + 6)$

$= (x + 2)(x + 1)(x - 1)(x - 2)(x - 3)$

The real zeros of f are $-2, -1, 1, 2$, and 3.

35. Possible rational zeros: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$

Reasonable zeros: $x = -3, x = \frac{1}{2}$, and $x = 1$

Test $x = -3$:

$$\begin{array}{r|rrrrrr} -3 & 2 & 5 & -3 & -2 & -5 & 3 \\ & & -6 & 3 & 0 & 6 & -3 \\ \hline & 2 & -1 & 0 & -2 & 1 & 0 \end{array}$$

$h(x) = (x + 3)(2x^4 - x^3 - 2x + 1) = (x + 3) \cdot f(x)$

Possible rational zeros of f: $\pm 1, \pm\frac{1}{2}$

Test $x = \frac{1}{2}$:

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -1 & 0 & -2 & 1 \\ & & 1 & 0 & 0 & -1 \\ \hline & 2 & 0 & 0 & -2 & 0 \end{array}$$

$h(x) = (x + 3)\left(x - \frac{1}{2}\right)(2x^3 - 2)$

$= (x + 3)\left(x - \frac{1}{2}\right)(2)(x^3 - 1)$

$= (x + 3)(2x - 1)(x - 1)(x^2 + x + 1)$

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{-1 \pm i\sqrt{3}}{2}$, which are not real.

The only real solutions of h are $-3, \frac{1}{2}$, and 1.

36. The error is that only the positive factors were listed as possible rational zeros.

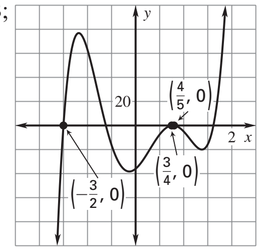
Possible zeros: $\pm 1, \pm 2, \pm 7, \pm 14$

37. The error is that the factors of the coefficient were divided by the factors of the constant term. The possible zeros are: $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

38. *Sample answer:* $f(x) = 4x^2 + 3$

Possible zeros: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

39. B;



40. sr factors. Each factor of a_0 (there are s) is divided by each factor of a_n (there are r), then at most there are sr possible rational zeros.

41. $f(x) = x^3 - 2x^2 - x + 2$

Possible rational zeros: $\pm 1, \pm 2$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -1 & 2 \\ & & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$f(x) = (x - 1)(x^2 - x - 2)$

$= (x - 1)(x - 2)(x + 1)$

Real zeros are -1, 1, and 2, which matches graph B.

42. $g(x) = x^3 - 3x^2 + 2$

Possible rational zeros: $\pm 1, \pm 2$

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 0 & 2 \\ & & 1 & -2 & -2 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

$g(x) = (x - 1)(x^2 - 2x - 2)$

$x^2 - 2x - 2 = 0$

$x = \dfrac{2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$

$= 1 \pm \sqrt{3}$

Real zeros: 1, $1 + \sqrt{3}$, and $1 - \sqrt{3}$, which matches graph C.

43. $h(x) = x^3 + x^2 - x + 2$

Possible rational zeros: $\pm 1, \pm 2$

Test $x = -2$:

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -1 & 2 \\ & & -2 & 2 & -2 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$h(x) = (x + 2)(x^2 - x + 1)$

$x^2 - x + 1 = 0$

$x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{1 \pm i\sqrt{3}}{2}$, which are imaginary numbers.

The only real zero is -2, which matches graph A.

44. It is not possible for a cubic function to have more than 3 real zeros. At most a cubic function can have 3 real zeros. The number of possible zeros is determined by degree. The cubic function will either have 1 real zero or 3 real zeros, and never no zeros because the graphs of the functions cross the x-axis at least once.

Problem Solving

45. $V = \ell^2 h \qquad h = \ell + 4$

$63 = \ell^2(\ell + 4)$

$63 = \ell^3 + 4\ell^2$

$0 = \ell^3 + 4\ell^2 - 63$

Possible zeros: 1, 3, 7, 9, 21, 63

Test $\ell = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 0 & -63 \\ & & 1 & 5 & 5 \\ \hline & 1 & 5 & 5 & -58 \end{array}$$

Test $\ell = 3$:

$$\begin{array}{r|rrrr} 3 & 1 & 4 & 0 & -63 \\ & & 3 & 21 & 63 \\ \hline & 1 & 7 & 21 & 0 \end{array}$$

$v = (\ell - 3)(\ell^2 + 7\ell + 21)$

$\ell = 3$ is the only real zero.

Dimensions: 3 in. $\times$ 3 in. $\times$ 7 in.

46. $V = \ell \cdot w \cdot d \qquad w = d + 5 \qquad \ell = d + 35$

$2000 = (d + 35)(d + 5)(d)$

$2000 = (d^2 + 40d + 175)d$

$0 = d^3 + 40d^2 + 175d - 2000$

Reasonable zero: $d = 5$

Test $d = 5$:

$$\begin{array}{r|rrrr} 5 & 1 & 40 & 175 & -2000 \\ & & 5 & 225 & 2000 \\ \hline & 1 & 45 & 400 & 0 \end{array}$$

$V = (d - 5)(d^2 + 45d + 400)$

The only positive real value for d is 5.

Dimensions: Depth $= d = 5$ feet

Width $= d + 5 = 10$ feet

Length $= d + 35 = 40$ feet

47. $V = x(x - 1)(x - 2)$

$24 = x(x^2 - 3x + 2)$

$0 = x^3 - 3x^2 + 2x - 24$

Possible rational solutions: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

48. $V = \frac{1}{3}(x^2(2x - 5))$

$3 = \frac{1}{3}(2x^3 - 5x^2)$

$9 = 2x^3 - 5x^2$

$0 = 2x^3 - 5x^2 - 9$

Possible rational solutions: $\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

49. a. $20{,}300 = -10t^3 + 140t^2 - 20t + 18{,}150$

$0 = -10t^3 + 140t^2 - 20t - 2150$

b. Possible solutions: $\pm1, \pm2, \pm5$, and ±10

c.

$$\begin{array}{r|rrrr} 1 & -10 & 140 & -20 & -2150 \\ & & -10 & 130 & 110 \\ \hline & -10 & 130 & 110 & -2040 \end{array}$$

$$\begin{array}{r|rrrr} 2 & -10 & 140 & -20 & -2150 \\ & & -20 & 240 & 440 \\ \hline & -10 & 120 & 220 & -1710 \end{array}$$

$$\begin{array}{r|rrrr} 5 & -10 & 140 & -20 & -2150 \\ & & -50 & 450 & 2150 \\ \hline & -10 & 90 & 430 & 0 \end{array}$$

$t = 5$ is an actual solution.

$t = 5$ corresponds to 1999.

50. a. $56{,}300 = 12t^4 - 264t^3 + 2028t^2 - 3924t + 43{,}916$

$0 = 12t^4 - 264t^3 + 2028t^2 - 3924t - 12{,}384$

b. Possible solutions: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm9$

c. Test $t = 4$:

$$\begin{array}{r|rrrrr} 4 & 12 & -264 & 2028 & -3924 & -12{,}384 \\ & & 48 & -864 & 4656 & 2928 \\ \hline & 12 & -216 & 1164 & 732 & -9456 \end{array}$$

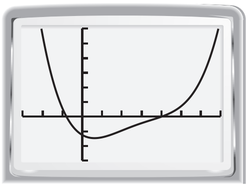
Test $t = 6$:

$$\begin{array}{r|rrrrr} 6 & 12 & -264 & 2028 & -3924 & -12{,}384 \\ & & 72 & -1152 & 5256 & 7992 \\ \hline & 12 & -192 & 876 & 1332 & -4392 \end{array}$$

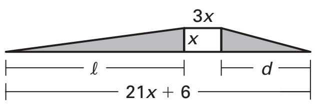
Test $t = 8$:

$$\begin{array}{r|rrrrr} 8 & 12 & -264 & 2028 & -3924 & -12{,}384 \\ & & 96 & -1344 & 5472 & 12{,}384 \\ \hline & 12 & -168 & 684 & 1548 & 0 \end{array}$$

$t = 8$ is a solution.

d. 

The only reasonable solution is 8 because the other solutions are either negative or imaginary. The year 1998 corresponds to the solution.

51. 

Not drawn to scale

Ramp length $= (21x + 6) - 3x = 18x + 6$

$d = \frac{1}{3}(18x + 6) = 6x + 2 \qquad \ell = \frac{2}{3}(18x + 6) = 12x + 4$

$V = 3x\left[\frac{1}{2}(x)(6x + 2) + \frac{1}{2}(x)(12x + 4)\right]$

$150 = 3x[3x^2 + x + 6x^2 + 2x]$

$50 = x[9x^2 + 3x]$

$0 = 9x^3 + 3x^2 - 50$

The possible rational solutions: $\pm1, \pm2, \pm5, \pm10, \pm25, \pm50, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{25}{3}, \pm\frac{50}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{5}{9}, \pm\frac{10}{9}, \pm\frac{25}{9}, \pm\frac{50}{9}$

Test $x = \frac{5}{3}$:

$$\begin{array}{r|rrrr} \frac{5}{3} & 9 & 3 & 0 & -50 \\ & & 15 & 30 & 50 \\ \hline & 9 & 18 & 30 & 0 \end{array}$$

$(9x^2 + 18x + 30)\left(x - \frac{5}{3}\right) = 0$

$x = \frac{5}{3}$ is the only positive, real solution.

right ramp: $\ell = 12$ ft $\qquad$ left ramp: $\ell = 24$ ft

$h = \frac{5}{3}$ ft $\qquad h = \frac{5}{3}$ ft

$w = 5$ ft $\qquad w = 5$ ft

Mixed Review

52. $4x - 6 = 18$

$4x = 24$

$x = 6$

53. $3y + 7 = -14$

$3y = -21$

$y = -7$

54. $|2p + 5| = 15$

$2p + 5 = 15$ or $2p + 5 = -15$

$2p = 10$ or $2p = -20$

$p = 5$ or $p = -10$

55. $49z^2 - 14z + 1 = 0$

$(7z - 1)^2 = 0$

$7z - 1 = 0$

$z = \frac{1}{7}$

56. $8x^2 - 30x + 7 = 0$

$(4x - 1)(2x - 7) = 0$

$4x - 1 = 0$ or $2x - 7 = 0$

$4x = 1$ or $2x = 7$

$x = \frac{1}{4}$ or $x = \frac{7}{2}$

57. $-3(q + 2)^2 = -18$

$(q + 2)^2 = 6$

$q + 2 = \pm\sqrt{6}$

$q = \pm\sqrt{6} - 2$

58.
$$\begin{array}{ccc} A & & B \\ \begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix} & X = & \begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix} \end{array}$$

$$A^{-1} = \frac{1}{-1 - (-10)}\begin{bmatrix} -1 & -5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{9} & -\frac{5}{9} \\ \frac{2}{9} & \frac{1}{9} \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -\frac{1}{9} & -\frac{5}{9} \\ \frac{2}{9} & \frac{1}{9} \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$$

59.
$$\begin{array}{ccc} A & & B \\ \begin{bmatrix} -2 & 1 \\ 4 & 0 \end{bmatrix} & X = & \begin{bmatrix} 6 & 0 \\ -1 & 10 \end{bmatrix} \end{array}$$

$$A^{-1} = \frac{1}{0 - 4}\begin{bmatrix} 0 & -1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} \\ 1 & \frac{1}{2} \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 0 & \frac{1}{4} \\ 1 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 6 & 0 \\ -1 & 10 \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & \frac{5}{2} \\ \frac{11}{2} & 5 \end{bmatrix}$$

60.
$$\begin{array}{ccc} A & & B \\ \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} & X = & \begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix} \end{array}$$

$$A^{-1} = \frac{1}{10 - 12}\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} -1 & \frac{3}{2} \\ 2 & -\frac{5}{2} \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -1 & \frac{3}{2} \\ 2 & -\frac{5}{2} \end{bmatrix}\begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -7 & -\frac{7}{2} \\ -6 & 12 & \frac{13}{2} \end{bmatrix}$$

61. $$\overset{A}{\begin{bmatrix} 2 & -8 \\ 3 & -7 \end{bmatrix}} X = \overset{B}{\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & -3 \end{bmatrix}}$$

$$A^{-1} = \frac{1}{-14 - (-24)} \begin{bmatrix} -7 & 8 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{7}{10} & \frac{4}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix}$$

$$x = A^{-1}B = \begin{bmatrix} -\frac{7}{10} & \frac{4}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{31}{10} & \frac{-14}{5} & -\frac{19}{5} \\ \frac{9}{10} & -\frac{6}{5} & -\frac{6}{5} \end{bmatrix}$$

62. Discriminant $= b^2 - 4ac = (-4)^2 - 4(1)(11) = -28$

There are two imaginary solutions.

63. Discriminant $= b^2 - 4ac = (-14)^2 - 4(1)(49) = 0$

There is one real solution.

64. Discriminant $= b^2 - 4ac$

$= (-8)^2 - 4(3)(-5) = 64 + 60 = 124$

There are two real solutions.

65. Discriminant $= b^2 - 4ac = (-5)^2 - 4(-2)(-3)$

$= 25 - 24 = 1$

There are two real solutions.

66. Discriminant $= b^2 - 4ac = (18)^2 - 4(81)(1)$

$= 324 - 324 = 0$

There is one real solution.

67. Discriminant $= b^2 - 4ac = (0)^2 - 4(7)(5) = -140$

There are two imaginary solutions.

Quiz 5.4–5.6 (p. 377)

1. $2x^3 - 54 = 2(x^3 - 27) = 2(x^3 - 3^3)$

$= 2(x - 3)(x^2 + 3x + 9)$

2. $x^3 - 3x^2 + 2x - 6 = x^2(x - 3) + 2(x - 3)$

$= (x - 3)(x^2 + 2)$

3. $x^3 + x^2 + x + 1 = x^2(x + 1) + (x + 1)$

$= (x + 1)(x^2 + 1)$

4. $6x^5 - 150x = 6x(x^4 - 25) = 6x(x^2 - 5)(x^2 + 5)$

5. $3x^4 - 24x^2 + 48 = 3(x^4 - 8x^2 + 16)$

$= 3(x^2 - 4)(x^2 - 4)$

$= 3(x - 2)(x + 2)(x - 2)(x + 2)$

$= 3(x - 2)^2(x + 2)^2$

6. $2x^3 - 3x^2 - 12x + 18 = x^2(2x - 3) - 6(2x - 3)$

$= (2x - 3)(x^2 - 6)$

7.
$$\begin{array}{r} x^2 - 4x + 14 \\ x^2 + 5x - 2\overline{)x^4 + x^3 - 8x^2 + 5x + 5} \\ \underline{x^4 + 5x^3 - 2x^2} \\ -4x^3 - 6x^2 + 5x \\ \underline{-4x^3 - 20x^2 + 8x} \\ 14x^2 - 3x + 5 \\ \underline{14x^2 + 70x - 28} \\ -73x + 33 \end{array}$$

$$\frac{x^4 + x^3 - 8x^2 + 5x + 5}{x^2 + 5x - 2} = x^2 - 4x + 14 + \frac{-73x + 33}{x^2 + 5x - 2}$$

8.
$$\begin{array}{r|rrrr} -7 & 4 & 27 & 3 & 64 \\ & & -28 & 7 & -70 \\ \hline & 4 & -1 & 10 & -6 \end{array}$$

$$\frac{4x^3 + 27x^2 + 3x + 64}{x + 7} = 4x^2 - x + 10 - \frac{6}{x + 7}$$

9. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10,$

$\pm 15, \pm 30, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

Reasonable zero: $x = -\frac{1}{2}$:

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -19 & 50 & 30 \\ & & -1 & 10 & -30 \\ \hline & 2 & -20 & 60 & 0 \end{array}$$

$f(x) = \left(x + \frac{1}{2}\right)(2x^2 - 20x + 60)$

$= \left(x + \frac{1}{2}\right)(2)(x^2 - 10x + 30)$

$= (2x + 1)(x^2 - 10x + 30)$

$x^2 - 10x + 30 = 0$

$x = \dfrac{10 \pm \sqrt{(-10)^2 - 4(1)(30)}}{2}$

The zeros are $-\frac{1}{2}$, $5 + i\sqrt{5}$, and $5 - i\sqrt{5}$. The only real zero is $-\frac{1}{2}$.

10. Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14,$
$\pm 28, \pm 56$

Reasonable zero: $x = 8$

$$\begin{array}{r|rrrr} 8 & 1 & -4 & -25 & -56 \\ & & 8 & 32 & 56 \\ \hline & 1 & 4 & 7 & 0 \end{array}$$

$f(x) = (x - 8)(x^2 + 4x + 7)$

The zeros are 8, $-2 + i\sqrt{3}$, and $-2 - i\sqrt{3}$ so $x = 8$ is the only real zero.

11. Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Reasonable zeros: $x = -6, x = -1, x = 1$, and $x = 2$

$$\begin{array}{r|rrrrr} -6 & 1 & 4 & -13 & -4 & 12 \\ & & -6 & 12 & 6 & -12 \\ \hline & 1 & -2 & -1 & 2 & 0 \end{array}$$

$f(x) = (x + 6)(x^3 - 2x^2 - x + 2) = (x + 6) \cdot g(x)$

Possible rational zeros of g: $\pm1, \pm2$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -1 & 2 \\ & & -1 & 3 & -2 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

$f(x) = (x + 6)(x + 1)(x^2 - 3x + 2)$

$= (x + 6)(x + 1)(x - 2)(x - 1)$

The real zeros are $-6, -1, 2$, and 1.

12. Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20,$

$\pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{4}, \pm\frac{5}{4}$

Reasonable zeros: $x = -\frac{5}{2}$ and $x = \frac{1}{2}$

$$\begin{array}{r|rrrrr} -\frac{5}{2} & 4 & 0 & -5 & 42 & -20 \\ & & -10 & 25 & -50 & 20 \\ \hline & 4 & -10 & 20 & -8 & 0 \end{array}$$

$f(x) = \left(x + \frac{5}{2}\right)(4x^3 - 10x^2 + 20x - 8)$

$= \left(x + \frac{5}{2}\right)(2)(2x^3 - 5x^2 + 10x - 4)$

$= (2x + 5) \cdot g(x)$

Possible zeros of g: $\pm1, 2, \pm4, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -5 & 10 & -4 \\ & & 1 & -2 & 4 \\ \hline & 2 & -4 & 8 & 0 \end{array}$$

$f(x) = (2x + 5)\left(x - \frac{1}{2}\right)(2x^2 - 4x + 8)$

$= (2x + 5)\left(x - \frac{1}{2}\right)(2)(x^2 - 2x + 4)$

$= (2x + 5)(2x - 1)(x^2 - 2x + 4)$

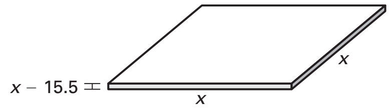
$x^2 - 2x + 4 = 0$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$

$= 1 \pm i\sqrt{3}$, which are imaginary numbers.

The real zeros are $-\frac{5}{2}$ and $\frac{1}{2}$.

13.



$V = x \cdot x \cdot (x - 15.5)$

$128 = x^3 - 15.5x^2$

$0 = x^3 - 15.5x^2 - 128$

Possible zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm32, \pm64, \pm128$

Reasonable zeros: 16, 32, 64, 128 (positive and > 15.5)

$$\begin{array}{r|rrrr} 16 & 1 & -15.5 & 0 & -128 \\ & & 16 & 8 & 128 \\ \hline & 1 & 0.5 & 8 & 0 \end{array}$$

$v = (x - 16)(x^2 + 0.5x + 8)$

The zeros are $x = 16$ and $\frac{-0.5 \pm i\sqrt{31.75}}{2}$. But $x = 16$ is the only solution.

Dimensions: 16 ft $\times$ 16 ft $\times \frac{1}{2}$ ft

Spreadsheet Activity 5.6 (p. 378)

1.

	A	B
1	x	$f(x)$
2	0	10
3	1	0
4	2	6
5	3	64
6	4	210
7	5	480

By the location principal, 1 is a zero.

$f(x) = (x - 1)(6x^2 - 4x - 10)$

$= (x - 1)(3x - 5)(2x + 2)$

The zeros are 1, $\frac{5}{3}$, and -1.

2.

	A	B
1	x	$f(x)$
2	0	-28
3	1	-390
4	2	-1026
5	3	-1300
6	4	0
7	5	4662

	C	D
1	x	$g(x)$
2	-5	-1728
3	-4	-765
4	-3	-242
5	-2	-35
6	-1	0
7	0	7

By the location principal, 4 is a zero.

$f(x) = (x - 4)(24x^3 + 58x^2 + 41x + 7) = (x - 4) \cdot g(x)$

By the location principal, -1 is a zero.

$f(x) = (x - 4)(x + 1)(24x^2 + 34x + 7)$

$= (x - 4)(x + 1) \cdot h(x)$

	E	F
1	x	$h(x)$
2	−3	121
3	−2	35
4	−1	−3
5	0	7
6	1	65
7	2	171

By the location principal, there is a zero between −2 and −1, and between −1 and 0.

$-\frac{7}{4}$ and $-\frac{7}{6}$ fall between −2 and −1 while $-\frac{7}{8}, -\frac{7}{12}, -\frac{7}{24}, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{8}, -\frac{1}{12}, -\frac{1}{24}$ fall between −1 and 0.

Using synthetic division, $x = -\frac{7}{6}$ and $-\frac{1}{4}$ are zeros of $f(x)$.

$f(x) = (x - 4)(x + 1)\left(x + \frac{7}{6}\right)\left(x + \frac{1}{4}\right)$

The real zeros are 4, −1, $-\frac{7}{6}$, and $-\frac{1}{4}$.

3.

	A	B
1	x	$f(x)$
2	−5	0
3	−4	874
4	−3	1102
5	−2	900
6	−1	484
7	0	70
8	1	−126

By the location principal there is a zero at $x = -5$ and a zero between 0 and 1.

$f(x) = 36x^3 + 109x^2 - 341x + 70$

$= (x + 5)(36x^2 - 71x + 14)$

$= (x + 5)(9x - 2)(4x - 7)$

The real zeros are −5, $\frac{2}{9}$, and $\frac{7}{4}$.

4.

	A	B
1	x	$f(x)$
2	−1	0
3	0	−132
4	1	−560
5	2	−990
6	3	−840
7	4	4968

By the location principal there is a zero between $x = 3$ and $x = 4$ along with a zero at $x = -1$.

$f(x) = 12x^4 + 25x^3 - 160x^2 - 305x - 132$

$= (x + 1)(12x^3 + 13x^2 - 173x - 132)$

$x = \frac{11}{3}$ is the only possible rational zero that falls between 3 and 4, synthetic division confirms it is a factor.

$f(x) = (x + 1)\left(x - \frac{11}{3}\right)(12x^2 + 57x + 36)$

$= (x + 1)(3x - 11)(4x^2 + 19x + 12)$

$= (x + 1)(3x - 11)(4x + 3)(x + 4)$

The real zeros are −1, $\frac{11}{3}$, $-\frac{3}{4}$, and −4.

Lesson 5.7

5.7 Guided Practice (pp. 379–383)

1. 4 solutions because it is a polynomial of degree 4.

2. 3 zeros because it is a polynomial of degree 3.

3. Possible rational zeros: ±1, ±3, ±9

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 15 & 9 \\ & & -1 & -6 & -9 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

$f(x) = (x + 1)(x^2 + 6x + 9) = (x + 1)(x + 3)^2$

The zeros are −3, −3, and −1.

4. Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrrrr} -2 & 1 & -2 & 0 & 8 & -13 & 6 \\ & & -2 & 8 & -16 & 16 & -6 \\ \hline & 1 & -4 & 8 & -8 & 3 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 8 & -8 & 3 \\ & & 1 & -3 & 5 & -3 \\ \hline & 1 & -3 & 5 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 5 & -3 \\ & & 1 & -2 & 3 \\ \hline & 1 & -2 & 3 & 0 \end{array}$$

$f(x) = x^5 - 2x^4 + 8x^2 - 13x + 6$
$= (x + 2)(x - 1)^2(x^2 - 2x + 3)$
$= (x + 2)(x - 1)^2[x - (1 + i\sqrt{2})][x - (1 - i\sqrt{2})]$

The zeros are $-2, 1, 1, 1 + i\sqrt{2}$, and $1 - i\sqrt{2}$.

5. $f(x) = (x + 1)(x - 2)(x - 4)$
$= (x^2 - x - 2)(x - 4)$
$= x^3 - x^2 - 2x - 4x^2 + 4x + 8$
$= x^3 - 5x^2 + 2x + 8$

6. $f(x) = (x - 4)[x - (1 + \sqrt{5})][x - (1 - \sqrt{5})]$
$= (x - 4)[(x - 1) + \sqrt{5}][(x - 1) - \sqrt{5}]$
$= (x - 4)[(x - 1)^2 - 5]$
$= (x - 4)[(x^2 - 2x + 1) - 5]$
$= (x - 4)(x^2 - 2x - 4)$
$= x^3 - 2x^2 - 4x - 4x^2 + 8x + 16$
$= x^3 - 6x^2 + 4x + 16$

7. $f(x) = (x - 2)(x - 2i)(x + 2i)[x - (4 - \sqrt{6})][x - (4 + \sqrt{6})]$
$= (x - 2)[x^2 - (2i)^2][(x - 4) - \sqrt{6}][(x - 4) + \sqrt{6}]$
$= (x - 2)(x^2 + 4)[(x - 4)^2 - 6]$
$= (x^3 + 4x - 2x^2 - 8)[(x^2 - 8x + 16) - 6]$
$= (x^3 - 2x^2 + 4x - 8)(x^2 - 8x + 10)$
$= x^5 - 2x^4 + 4x^3 - 8x^2 - 8x^4 + 16x^3 - 32x^2 + 64x + 10x^3 - 20x^2 + 40x - 80$
$= x^5 - 10x^4 + 30x^3 - 60x^2 + 104x - 80$

8. $f(x) = (x - 3)[x - (3 - i)][x - (3 + i)]$
$= (x - 3)[(x - 3) + i][(x - 3) - i]$
$= (x - 3)[(x - 3)^2 - i^2]$
$= (x - 3)[(x^2 - 6x + 9) + 1]$
$= (x - 3)(x^2 - 6x + 10)$
$= x^3 - 6x^2 + 10x - 3x^2 + 18x - 30$
$= x^3 - 9x^2 + 28x - 30$

9. $f(x) = x^3 + 2x - 11$

The coefficients in $f(x)$ have 1 sign change so f has 1 positive real zero.

$f(-x) = (-x)^3 + 2(-x) - 11 = -x^3 - 2x - 11$

The coefficients of $f(-x)$ have 0 sign changes so f has 0 negative real zeros.

The possible number of zeros for f are 1 positive real zero and 2 imaginary zeros.

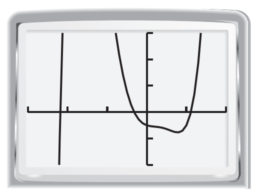
10. $g(x) = 2x^4 - 8x^3 + 6x^2 - 3x + 1$

The coefficients of $g(x)$ have 4 sign changes so g has 4, 2, or zero positive real zeros.

$g(-x) = 2(-x)^4 - 8(-x)^3 + 6(-x)^2 - 3(-x) + 1$
$= 2x^4 + 8x^3 + 6x^2 + 3x + 1$

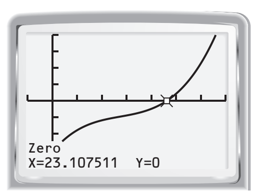
The coefficients of $g(-x)$ have zero sign changes so g has zero negative real zeros.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
4	0	0	4
2	0	2	4
0	0	4	4

11. 

Zeros $\approx x = -2.18$, $x = -0.32$, and $x = 1.11$

12. $0 = 0.00547x^3 - 0.225x^2 + 3.62x - 31.0$



The tachometer reading is about 2310 RPM.

5.7 Exercises (pp. 383–386)

Skill Practice

1. For the equation $(x - 1)^2(x + 2) = 0$ a *repeated* solution is 1 because the factor $x - 1$ appears twice.

2. Complex conjugates include imaginary numbers in the form $a + bi$ and $a - bi$ and irrational conjugates include irrational numbers in the form $a + \sqrt{b}$ and $a - \sqrt{b}$.

3. 4 solutions **4.** 3 solutions **5.** 6 solutions

6. 4 solutions **7.** 7 solutions **8.** 12 solutions

9. D; 6 solutions

10. Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

$$f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$$
$$= (x + 1)(x^3 - 7x^2 + 14x - 8)$$
$$= (x + 1)(x - 1)(x^2 - 6x + 8)$$
$$= (x + 1)(x - 1)(x - 4)(x - 2)$$

Zeros are: -1, 1, 2, and 4.

11. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$$f(x) = x^4 + 5x^3 - 7x^2 - 29x + 30$$
$$= (x + 5)(x^3 - 7x + 6)$$
$$= (x + 5)(x + 3)(x^2 - 3x + 2)$$
$$= (x + 5)(x + 3)(x - 1)(x - 2)$$

The zeros are -5, -3, 1 and 2.

12. Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$f(x) = x^4 - 9x^2 - 4x + 12$$
$$= (x + 2)(x^3 - 2x^2 - 5x + 6)$$
$$= (x + 2)(x + 2)(x^2 - 4x + 3)$$
$$= (x + 2)^2(x - 3)(x - 1)$$

The zeros are -2, 1, and 3.

13. Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$f(x) = x^3 + 5x^2 - 4x - 20$$
$$= x^2(x + 5) - 4(x + 5)$$
$$= (x + 5)(x^2 - 4)$$
$$= (x + 5)(x - 2)(x + 2)$$

The zero are -5, -2, and 2.

14. Possible rational solutions: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$f(x) = x^4 + 15x^2 - 16$$
$$= (x + 1)(x^3 - x^2 + 16x - 16)$$
$$= (x + 1)(x - 1)(x^2 + 16)$$
$$= (x + 1)(x - 1)(x - 4i)(x + 4i)$$

The zeros are -1, 1, $-4i$, and $4i$.

15. Possible rational solutions: $\pm 1, \pm 2, \pm 4, \pm 8$

$$f(x) = x^4 + x^3 + 2x^2 + 4x - 8$$
$$= (x + 2)(x^3 - x^2 + 4x - 4)$$
$$= (x + 2)(x - 1)(x^2 + 4)$$
$$= (x + 2)(x - 1)(x - 2i)(x + 2i)$$

The zeros are -2, 1, $2i$, and $-2i$.

16. Possible rational solutions: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$f(x) = x^4 + 4x^3 + 7x^2 + 16x + 12$$
$$= (x + 1)(x^3 + 3x^2 + 4x + 12)$$
$$= (x + 1)(x + 3)(x^2 + 4)$$
$$= (x + 1)(x + 3)(x - 2i)(x + 2i)$$

The zeros are $-1, -3$, $2i$, and $-2i$.

17. Possible rational solutions: $\pm 1, \pm 2$

$$g(x) = x^4 - 2x^3 - x^2 - 2x - 2$$
$$= (x^2 + 1)(x^2 - 2x - 2)$$
$$= (x - i)(x + i)[x - (1 + \sqrt{3})][x - (1 - \sqrt{3})]$$

The zeros are i, $-i$, $1 + \sqrt{3}$, and $1 - \sqrt{3}$

18. Possible rational solutions: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

$$g(x) = 4x^4 + 4x^3 - 11x^2 - 12x - 3$$
$$= \left(x + \frac{1}{2}\right)(4x^3 + 2x^2 - 12x - 6)$$
$$= \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(4x^2 - 12)$$
$$= \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x - \sqrt{3})(x + \sqrt{3})$$

The zeros are $-\frac{1}{2}$, $-\frac{1}{2}$, $-\sqrt{3}$, and $\sqrt{3}$.

19. Possible rational solutions: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$h(x) = 2x^4 + 13x^3 + 19x^2 - 10x - 24$$
$$= (x + 4)(2x^3 + 5x^2 - x - 6)$$
$$= (x + 4)(x + 2)(2x^2 + x - 3)$$
$$= (x + 4)(x + 2)(2x + 3)(x - 1)$$

The zeros are -4, -2, $-\frac{3}{2}$, and 1.

20. $f(x) = (x - 1)(x - 2)(x - 3)$
$$= (x^2 - 3x + 2)(x - 3)$$
$$= x^3 - 3x^2 + 2x - 3x^2 + 9x - 6$$
$$= x^3 - 6x^2 + 11x - 6$$

21. $f(x) = (x + 2)(x - 1)(x - 3)$
$$= (x^2 + x - 2)(x - 3)$$
$$= x^3 + x^2 - 2x - 3x^2 - 3x + 6$$
$$= x^3 - 2x^2 - 5x + 6$$

22. $f(x) = (x + 5)(x + 1)(x - 2)$
$$= (x^2 + 6x + 5)(x - 2)$$
$$= x^3 + 6x^2 + 5x - 2x^2 - 12x - 10$$
$$= x^3 + 4x^2 - 7x - 10$$

23. $f(x) = (x + 3)(x - 1)(x - 6)$
$$= (x^2 + 2x - 3)(x - 6)$$
$$= x^3 + 2x^2 - 3x - 6x^2 - 12x + 18$$
$$= x^3 - 4x^2 - 15x + 18$$

24. $f(x) = (x - 2)(x - i)(x + i)$
$$= (x - 2)[(x^2 - (i^2)]$$
$$= (x - 2)(x^2 + 1)$$
$$= x^3 - 2x^2 + x - 2$$

25. $f(x) = (x - 3i)(x + 3i)[x - (2 - i)][x - (2 + i)]$
$= [x^2 - (3i)^2][(x - 2) + i][(x - 2) - i]$
$= (x^2 + 9)[(x - 2)^2 - (i)^2]$
$= (x^2 + 9)[x^2 - 4x + 4 + 1]$
$= (x^2 + 9)(x^2 - 4x + 5)$
$= x^4 - 4x^3 + 5x^2 + 9x^2 - 36x + 45$
$= x^4 - 4x^3 + 14x^2 - 36x + 45$

26. $f(x) = (x + 1)(x - 2)(x + 3i)(x - 3i)$
$= (x^2 - x - 2)[x^2 - (3i)^2]$
$= (x^2 - x - 2)(x^2 + 9)$
$= x^4 - x^3 - 2x^2 + 9x^2 - 9x - 18$
$= x^4 - x^3 + 7x^2 - 9x - 18$

27. $f(x) = (x - 5)(x - 5)[x - (4 + i)][x - (4 - i)]$
$= (x^2 - 10x + 25)[(x - 4) - i][(x - 4) + i]$
$= (x^2 - 10x + 25)[(x - 4)^2 - i^2]$
$= (x^2 - 10x + 25)[x^2 - 8x + 16 - (-1)]$
$= (x^2 - 10x + 25)(x^2 - 8x + 17)$
$= x^4 - 10x^3 + 25x^2 - 8x^3 + 80x^2 - 200x + 17x^2$
$- 170x + 425$
$= x^4 - 18x^3 + 122x^2 - 370x + 425$

28. $f(x) = (x - 4)(x - \sqrt{5})(x + \sqrt{5})$
$= (x - 4)[x^2 - (\sqrt{5})^2]$
$= (x - 4)(x^2 - 5)$
$= x^3 - 4x^2 - 5x + 20$

29. $f(x) = (x + 4)(x - 1)[x - (2 - \sqrt{6})][x - (2 + \sqrt{6})]$
$= (x^2 + 3x - 4)[(x - 2) + \sqrt{6}][(x - 2) - \sqrt{6}]$
$= (x^2 + 3x - 4)[(x - 2)^2 - (\sqrt{6})^2]$
$= (x^2 + 3x - 4)(x^2 - 4x + 4 - 6)$
$= (x^2 + 3x - 4)(x^2 - 4x - 2)$
$= x^4 + 3x^3 - 4x^2 - 4x^3 - 12x^2 + 16x - 2x^2 - 6x + 8$
$= x^4 - x^3 - 18x^2 + 10x + 8$

30. $f(x) = (x + 2)(x - 2)(x + 1)(x - 3)(x - \sqrt{11})(x + \sqrt{11})$
$= (x^2 - 4)(x^2 - 2x - 3)[x^2 - (\sqrt{11})^2]$
$= (x^4 - 2x^3 - 3x^2 - 4x^2 + 8x + 12)(x^2 - 11)$
$= (x^4 - 2x^3 - 7x^2 + 8x + 12)(x^2 - 11)$
$= x^6 - 2x^5 - 7x^4 + 8x^3 + 12x^2 - 11x^4 + 22x^3$
$+ 77x^2 - 88x - 132$
$= x^6 - 2x^5 - 18x^4 + 30x^3 + 89x^2 - 88x - 132$

31. $f(x) = (x - 3)[x - (4 + 2i)][x - (4 - 2i)][x - (1 + \sqrt{7})]$
$[x - (1 - \sqrt{7})]$
$= (x - 3)[(x - 4) - 2i][(x - 4) + 2i][(x - 1) - \sqrt{7}]$
$[(x - 1) + \sqrt{7}]$
$= (x - 3)[(x - 4)^2 - (2i)^2][(x - 1)^2 - (\sqrt{7})^2]$
$= (x - 3)[(x^2 - 8x + 16) + 4][(x^2 - 2x + 1) - 7]$
$= (x - 3)(x^2 - 8x + 20)(x^2 - 2x - 6)$
$= (x^3 - 8x^2 + 20x - 3x^2 + 24x - 60)(x^2 - 2x - 6)$
$= (x^3 - 11x^2 + 44x - 60)(x^2 - 2x - 6)$
$= x^5 - 11x^4 + 44x^3 - 60x^2 - 2x^4 + 22x^3 - 88x^2$
$+ 120x - 6x^3 + 66x^2 - 264x + 360$
$= x^5 - 13x^4 + 60x^3 - 82x^2 - 144x + 360$

32. The error is that the conjugate of $1 + i$ was not included as a factor.

$f(x) = (x - 2)[x - (1 + i)][x - (1 - i)]$
$= (x - 2)[(x - 1) - i][(x - 1) + i]$
$= (x - 2)[(x - 1)^2 - i^2]$
$= (x - 2)[(x^2 - 2x + 1) + 1]$
$= (x - 2)(x^2 - 2x + 2)$
$= x^3 - 2x^2 + 2x - 2x^2 + 4x - 4$
$= x^3 - 4x^2 + 6x - 4$

33. *Sample answer:*

$f(x) = (x - 1)(x - 1)(x - 2)(x - i)(x + i)$
$= (x^2 - 2x + 1)(x - 2)[x^2 - (i^2)]$
$= (x^3 - 2x^2 + x - 2x^2 + 4x - 2)(x^2 + 1)$
$= (x^3 - 4x^2 + 5x - 2)(x^2 + 1)$
$= x^5 - 4x^4 + 5x^3 - 2x^2 + x^3 - 4x^2 + 5x - 2$
$= x^5 - 4x^4 + 6x^3 - 6x^2 + 5x - 2$

34. $f(x) = x^4 - x^2 - 6$

The coefficients of $f(x)$ have 1 sign change, so f can only have 1 positive zero.

$f(-x) = (-x)^4 - (-x)^2 - 6 = x^4 - x^2 - 6$

The coefficients of $f(-x)$ have 1 sign change, so f can only have 1 negative zero. Possible numbers of zeros:

1 positive real zero, 1 negative real zero, and 2 imaginary zeros.

35. $g(x) = -x^3 + 5x^2 + 12$

The coefficients of $f(x)$ have 1 sign change, so g can only have 1 positive zero.

$g(-x) = -(-x)^3 + 5(-x)^2 + 12$
$= x^3 + 5x^2 + 12$

There are no sign changes in the coefficients of $g(-x)$ so there are no negative zeros. Possible numbers of zeros:

1 positive real zero, 0 negative real zeros, and 2 imaginary zeros.

36. $g(x) = x^3 - 4x^2 + 8x + 7$

The coefficients of $f(x)$ have 2 sign changes, so f can have 2 or no positive zeros.

$$g(-x) = (-x)^3 - 4(-x)^2 + 8(-x) + 7$$
$$= -x^3 - 4x^2 - 8x + 7$$

The coefficients of $g(-x)$ have 1 sign change, so g has 1 negative zero. Possible numbers of zeros:

2 positive real zeros, 1 real negative zero, and 0 imaginary zeros.

0 positive real zeros, 1 real negative zero, and 2 imaginary zeros.

37. $h(x) = x^5 - 2x^3 - x^2 + 6x + 5$

The coefficients of $h(x)$ have 2 sign changes, so h has either 2 or 0 positive real zeros.

$$h(-x) = (-x)^5 - 2(-x)^3 - (-x)^2 + 6(-x) + 5$$
$$= -x^5 + 2x^3 - x^2 - 6x + 5$$

The coefficients of $h(-x)$ have 3 sign changes so h has 3 or 1 real positive zeros. Possible numbers of zeros:

2 positive real zeros, 3 negative real zeros, and 0 imaginary zeros.

0 positive real zeros, 3 negative real zeros, and 2 imaginary zeros.

2 positive real zeros, 1 negative real zero, and 2 imaginary zeros.

0 positive real zeros, 1 negative real zero, and 4 imaginary zeros.

38. $h(x) = x^5 - 3x^3 + 8x - 10$

The coefficients of $h(x)$ have 3 sign changes, so h has 3 or 1 positive real zeros.

$$h(-x) = (-x)^5 - 3(-x)^3 + 8(-x) - 10$$
$$= -x^5 + 3x^3 - 8x - 10$$

The coefficients of $h(-x)$ have 2 sign changes, so h has 2 or 0 negative real zeros. Possible numbers of zeros:

3 positive real zeros, 2 negative real zeros, 0 imaginary zeros.

1 positive real zero, 2 negative real zeros, 2 imaginary zeros.

3 positive real zeros, 0 negative real zeros, 2 imaginary zeros.

1 positive real zero, 0 negative real zero, 4 imaginary zeros.

39. $f(x) = x^5 + 7x^4 - 4x^3 - 3x^2 + 9x - 15$

The coefficients of $f(x)$ have 3 sign changes, so f has 3 or 1 positive real zeros.

$$f(-x) = (-x)^5 + 7(-x)^4 - 4(-x)^3 - 3(-x)^2 + 9(-x) - 15$$
$$= -x^5 + 7x^4 + 4x^3 - 3x^2 - 9x - 15$$

The coefficients of $f(-x)$ have 2 sign changes, so f has 2 or 0 negative real zeros. Possible numbers of zeros:

3 positive real zeros, 2 negative real zeros, and 0 imaginary zeros.

1 positive real zero, 2 negative real zeros, and 2 imaginary zeros.

3 positive real zeros, 0 negative real zeros, and 2 imaginary zeros.

1 positive real zero, 0 negative real zero, and 4 imaginary zeros.

40. $g(x) = x^6 + x^5 - 3x^4 + x^3 + 5x^2 + 9x - 18$

The coefficients of $g(x)$ have 3 sign changes so g has 3 or 1 positive real zeros.

$$g(-x) = (-x)^6 + (-x)^5 - 3(-x)^4 + (-x)^3 + 5(-x)^2 + 9(-x) - 18$$
$$= x^6 - x^5 - 3x^4 - x^3 + 5x^2 - 9x - 18$$

The coefficients of $g(-x)$ have 3 sign changes so g has 3 or 1 negative real zeros. Possible numbers of zeros:

3 positive real zeros, 3 negative real zeros, and 0 imaginary zeros.

3 positive real zeros, 1 negative real zero, and 2 imaginary zeros.

1 positive real zero, 3 negative real zeros, and 2 imaginary zeros.

1 positive real zero, 1 negative real zero, and 4 imaginary zeros.

41. $f(x) = x^7 + 4x^4 - 10x + 25$

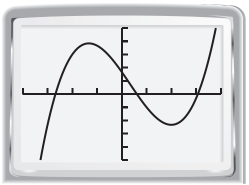
The coefficients of $f(x)$ have 2 sign changes, so f has 2 or 0 positive real zeros.

$$f(-x) = (-x)^7 + 4(-x)^4 - 10(-x) + 25$$
$$= -x^7 + 4x^4 + 10x + 25$$

The coefficients of $f(-x)$ have 1 sign change, so f has 1 negative real zero. Possible numbers of zeros:

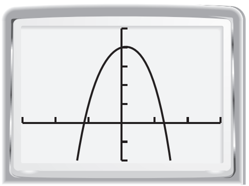
2 positive real zeros, 1 negative real zero, and 4 imaginary zeros.

0 positive real zeros, 1 negative real zero, and 6 imaginary zeros.

42. 

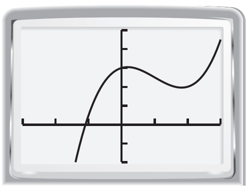
$f(x) = x^3 - x^2 - 8x + 5$

zeros: $x \approx -2.68$,
$x \approx 0.61$,
$x \approx 3.07$

43. 

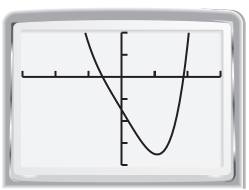
$f(x) = -x^4 - 4x^2 + x + 8$

zeros: $x \approx -1.14$,
$x \approx 1.28$

44. 

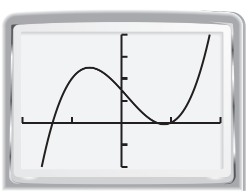
$g(x) = x^3 - 3x^2 + x + 6$

zeros: $x \approx -1.09$

45. 

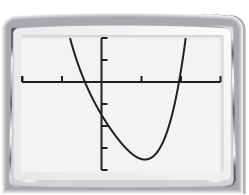
$h(x) = x^4 - 5x - 3$

zeros: $x \approx -0.58$,
$x \approx 1.88$

46. 

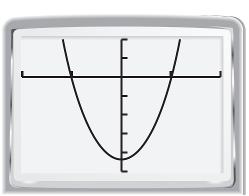
$h(x) = 3x^3 - x^2 - 5x + 3$

zeros: $x \approx -1.39$,
$x \approx 0.72$,
$x = 1$

47. 

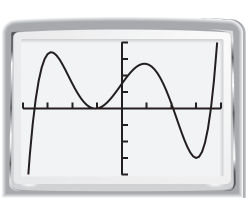
$g(x) = x^4 - x^3 + 2x^2 - 6x - 3$

zeros: $x \approx -0.42$,
$x \approx 1.95$

48. 

$f(x) = 2x^6 + x^4 + 31x^2 - 35$

zeros: $x \approx -1.01$
$x \approx 1.01$

49. 

$g(x) = x^5 - 16x^3 - 3x^2 + 42x + 30$

zeros: $x \approx -3.53$,
$x \approx -1.14$
$x = -1$,
$x \approx 2.07$
$x \approx 3.60$

50. Imaginary numbers come in pairs so if a function has one remaining zero and no imaginary zeros yet, then the remaining zero must be real as the imaginary zero will not have a pair.

51. Possible numbers of zeros:

3 positive real zeros, 0 negative real zeros, 0 imaginary zeros.

1 positive real zero, 2 negative real zeros, 0 imaginary zeros.

1 positive real zero, 0 negative real zeros, 2 imaginary zeros.

0 positive real zeros, 3 negative real zeros, 0 imaginary zeros.

0 positive real zeros, 1 negative real zero, 2 imaginary zeros.

2 positive real zeros, 1 negative real zero, 0 imaginary zeros.

52. C; $f(x) = x^5 - 4x^3 + 6x^2 + 12x - 6$

$f(x)$ has 3 sign changes: 3 or 1 positive real zeros

$f(-x) = -x^5 + 4x^3 + 6x^2 - 12x - 6$

$f(-x)$ has 2 sign changes: 2 or 0 negative real zeros

53. The graph crosses the x-axis 3 times, twice across the negative x-axis and once across the positive x-axis. So, the function has 2 negative real zeros, 1 positive real zero, and 0 imaginary zeros.

54. The graph crosses the negative x-axis once and the positive x-axis once. Since it is a quartic function there must be 4 zeros so the function has 1 negative real zero, 1 positive real zero, and 2 imaginary zeros.

55. The function has 5 zeros. Since it only crosses the x-axis once its zeros are 4 imaginary zeros and 1 negative real zero.

56.
$$\begin{aligned} f(x) &= x^3 - 2x^2 + 2x + 5i \\ f(2-i) &= (2-i)^3 - 2(2-i)^2 + 2(2-i) + 5i \\ &= 2 - 11i - 2(3-4i) + 4 - 2i + 5i \\ &= 2 - 11i - 6 + 8i + 4 - 2i + 5i = 0 \\ f(2+i) &= (2+i)^3 - 2(2+i)^2 + 2(2+i) + 5i \\ &= 2 + 11i - 2(3+4i) + 4 + 2i + 5i \\ &= 2 + 11i - 6 - 8i + 4 + 2i + 5i \\ &= 10i \neq 0 \end{aligned}$$

57.
$$\begin{aligned} f(x) &= x^3 + 2x^2 + 2i - 2 \\ f(-1+i) &= (-1+i)^3 + 2(-1+i)^2 + 2i - 2 \\ &= 2 + 2i + 2(-2i) + 2i - 2 \\ &= 2 + 2i - 4i + 2i - 2 = 0 \\ f(-1-i) &= (-1-i)^3 + 2(-1-i)^2 + 2i - 2 \\ &= 2 - 2i + 2(2i) + 2i - 2 \\ &= 2 - 2i + 4i + 2i - 2 = 4i \neq 0 \end{aligned}$$

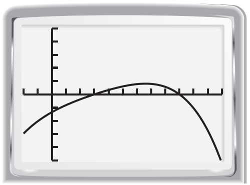
58. Because not all of the coefficients are real, the functions do not contradict the theorem.

Problem Solving

59. $R = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650)$

$0 = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650) - 1.5$

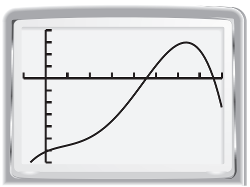
The zeros occur when $t = 3$ and $t = 9$, so after 3 years and 9 years the revenue will be \$15 million.



60. $N = -0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 2.5$

$0 = -0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 122.5$

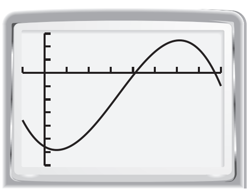
The zero is at about 9, so 9 years after 1990 (1999) there were 120 inland lakes in Michigan infested with zebra mussels.



61. $S = -0.015x^3 + 0.6x^2 - 2.4x + 19$

$0 = -0.015x^3 + 0.6x^2 - 2.4x - 56$

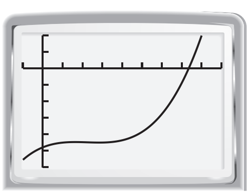
There are two positive zeros of this function, $x \approx 16.4$ and $x \approx 30.9$, but $x \approx 16.4$ is the most likely amount.



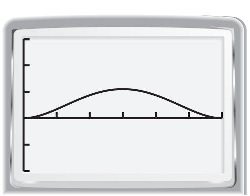
62. $P = 0.0035t^3 - 0.235t^2 + 4.87t + 243$

$0 = 0.0035t^3 - 0.235t^2 + 4.87t - 479$

When $t = 73.5$ years (1963), the population will reach 722 thousand.



63. The zeros are at about 0 and 60. The answers make sense because if books are set at the very end of the bookshelf there will be nearly no warping.



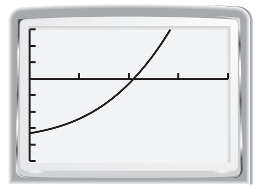
64. a.

	Year 1	Year 2	Year 3	Year 4
Value of 1st deposit	1000	$1000g$	$1000g^2$	$1000g^3$
Value of 2nd deposit		1000	$1000g$	$1000g^2$
Value of 3rd deposit			1000	$1000g$
Value of 4th deposit				1000

b. $v = 1000g^3 + 1000g^2 + 1000g + 1000$

c. $4300 = 1000g^3 + 1000g^2 + 1000g + 1000$

$0 = 1000g^3 + 1000g^2 + 1000g - 3300$



The growth factor can be found by substituting 4300 for v and then solving for g; $g = 1.05$. The annual interest rate is $g - 1 = 5\%$ interest.

65. V = volume of base + volume of pyramid

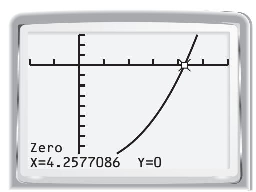
$$1000 = (2x + 6)^2(x) + \frac{1}{3}[(2x)^2(x)]$$

$$1000 = (4x^2 + 24x + 36)(x) + \frac{1}{3}[4x^2 \cdot x]$$

$$1000 = 4x^3 + 24x^2 + 36x + \frac{4}{3}x^3$$

$$0 = \frac{16}{3}x^3 + 24x^2 + 36x - 1000$$

$x \approx 4.26$ feet



Mixed Review

66. $$\left|\begin{array}{ccc} 5 & -9 & 2 \\ 4 & 2 & 3 \\ 0 & 1 & -1 \end{array}\right| \begin{array}{cc} 5 & -9 \\ 4 & 2 \\ 0 & 1 \end{array}$$

$= (-10 + 0 + 8) - (0 + 15 + 36) = -2 - 51 = -53$

67. $$\left|\begin{array}{ccc} 3 & 12 & -1 \\ 5 & 9 & 0 \\ -6 & 4 & -2 \end{array}\right| \begin{array}{cc} 3 & 12 \\ 5 & 9 \\ -6 & 4 \end{array}$$

$= (-54 + 0 - 20) - (54 + 0 - 120) = -74 + 66$
$= -8$

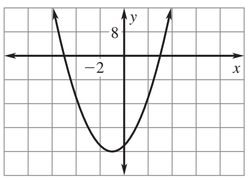
68. $\left|\begin{array}{ccc|cc} 15 & 4 & -9 & 15 & 4 \\ 10 & 0 & 2 & 10 & 0 \\ -8 & 2 & -7 & -8 & 2 \end{array}\right.$

$= (0 - 64 - 180) - (0 + 60 - 280) = -244 + 220$
$= -24$

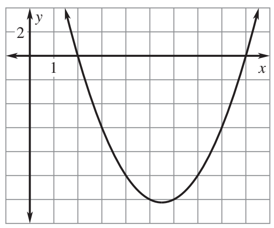
69. $\left|\begin{array}{ccc|cc} -2 & 1 & -3 & -2 & 1 \\ 7 & 0 & 2 & 7 & 0 \\ -6 & 2 & -4 & -6 & 2 \end{array}\right.$

$= (0 - 12 - 42) - (0 - 8 - 28) = -54 + 36 = -18$

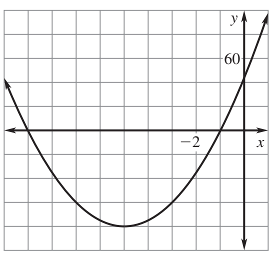
70. $y = 2(x + 5)(x - 3)$



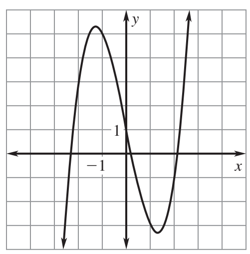
71. $y = (x - 2)(x - 9)$



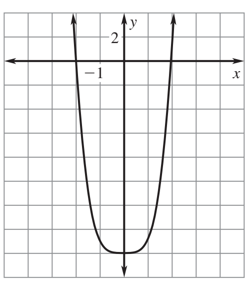
72. $y = 5(x + 1)(x + 9)$



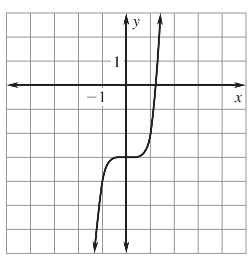
73. $y = x^3 - 5x + 1$



74. $y = x^4 - 16$



75. $y = x^5 - 3$



76. $y = a(x + 2)(x - 4)$
$-4 = a(2 + 2)(2 - 4)$
$-4 = a(4)(-2)$
$-4 = -8a$
$a = \frac{1}{2}$ so, $y = \frac{1}{2}(x + 2)(x - 4)$

77. $y = a(x + 5)(x + 1)$
$6 = a(-2 + 5)(-2 + 1)$
$6 = a(3)(-1)$
$6 = -3a$
$-2 = a$ so, $y = -2(x + 5)(x + 1)$

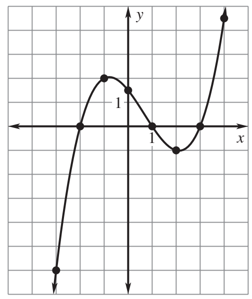
78. $y = a(x - 2)(x - 7)$
$-2 = a(4 - 2)(4 - 7)$
$-2 = a(2)(-3)$
$-2 = -6a$
$\frac{1}{3} = a$ so, $y = \frac{1}{3}(x - 2)(x - 7)$

Lesson 5.8

5.8 Guided Practice (p. 389)

1.

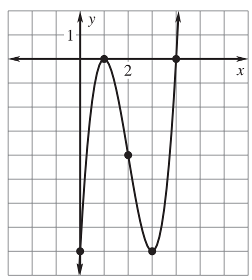
x	-3	-1	0	2	4
y	-6	2	1.5	-1	4.5



The x-intercepts are -2, 1, and 3.
Local maximum at $(-0.79, 2.05)$
Local minimum at $(2.12, -1.02)$

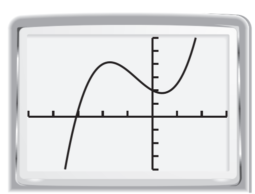
2.

x	0	2	3	5
y	-8	-4	-8	32

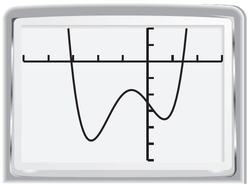


The x-intercepts are 1 and 4.
Local maximum at $(1, 0)$
Local minimum at $(3, -8)$

3.



The x-intercept is $x \approx -3.07$.
Local maximum $(-1.72, 4.13)$
Local minimum $(0.39, 1.80)$

4. 

The x-intercepts are $x \approx -3.07$ and $x \approx 1.42$.

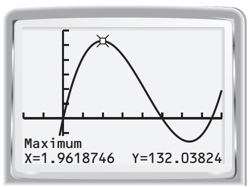
Local maximum at $(-0.65, -3.47)$

Local minimums at $(-2.28, -9.61)$, $(0.68, -7.03)$

5. $V = (15 - 2x)(10 - 2x) \cdot x$

$= (150 - 50x + 4x^2)x$

$= 4x^3 - 50x^2 + 150x$



You should make the cuts about 2 inches long.

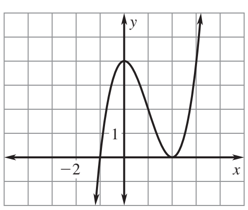
The maximum volume is 132 cubic inches. The dimensions of the box are 2 in. $\times$ 6 in. $\times$ 11 in.

5.8 Exercises (pp. 390–392)

Skill Practice

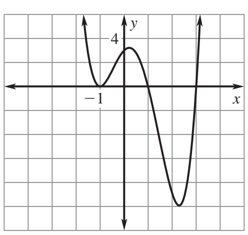
1. A local maximum or local minimum of a polynomial function occurs at a *turning* point of the function's graph.

2. *Sample answer:* A local maximum of a function is a point that is higher than all the nearby points. The local maximum is a description of the behavior of a function at a specific area of the graph and does not necessarily reflect the behavior of the function over a large interval, so it cannot be classified as the maximum value of the function.

3. 

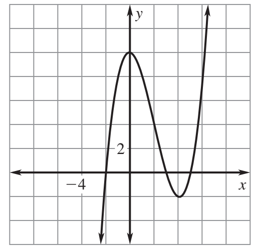
$f(x) = (x - 2)^2(x + 1)$

x	−2	0	1	3
y	−16	4	2	4

4. 

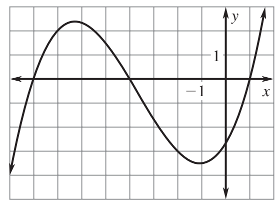
$f(x) = (x + 1)^2(x - 1)(x - 3)$

x	−2	0	2	4
y	15	3	−9	75

5. 

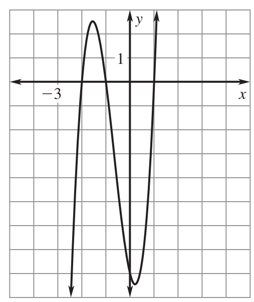
$g(x) = \frac{1}{3}(x - 5)(x + 2)(x - 3)$

x	−3	−1	0	1	2	4	6
y	−16	8	10	8	4	−2	8

6. 

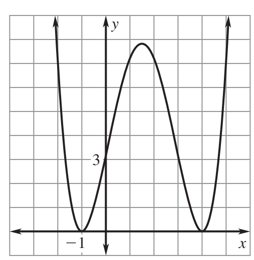
$h(x) = \frac{1}{12}(x + 4)(x + 8)(x - 1)$

x	−9	−7	−5	−3	−1	0	2
y	−4.2	2	1.5	−1.7	−3.5	−2.7	5

7. 

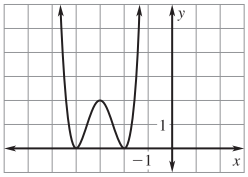
$h(x) = 4(x + 1)(x + 2)(x - 1)$

x	−3	0	2
y	−32	−8	48

8. 

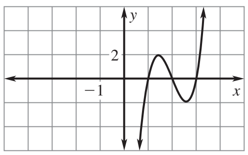
$f(x) = 0.2(x - 4)^2(x + 1)^2$

x	−2	0	1	2	3	5
y	7.2	3.2	7.2	7.2	3.2	7.2

9. 

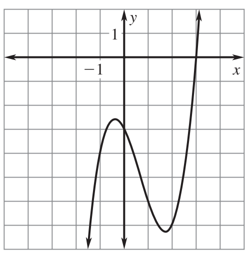
$f(x) = 2(x + 2)^2(x + 4)^2$

x	−5	−3	−1
y	18	2	18

10. 

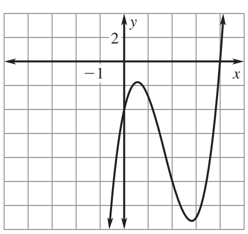
$h(x) = 5(x - 1)(x - 2)(x - 3)$

x	0	1.5	2.5	4
y	−30	1.9	−1.9	30

11. 

$g(x) = (x - 3)\left(x^2 + x + 1\right)$

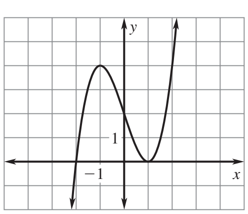
x	−1	1	2	4
y	−4	−6	−7	21

12. 

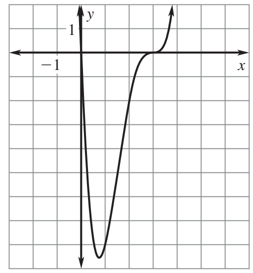
$h(x) = (x - 4)\left(2x^2 - 2x + 1\right)$

x	0	1	2	3	5
y	−4	−3	−10	−13	41

13. The zeros of f are -2 and 1, not -1 and 2; the y-intercept is 2, not -2.



14. The function f, is a quartic equation, thus its end behavior as $x \to +\infty$ and as $x \to -\infty$ will be the same.



15. Turning Pts.: $(-0.5, 0.5)$ local maximum
$(0.9, -1.5)$ local minimum

Zeros: $x \approx -0.75, x \approx 0, x \approx 1.4$

The function must be at least a degree 3 function.

16. Turning Pts.: $(-0.5, -2.5)$ local maximum
$(1.5, -5.75)$ local minimum

Zero: $x \approx 2.8$

The function must be at least a degree 3 function.

17. Turning Pts.: $(1, 0)$ local maximum
$(2, -2)$ local minimum
$(3, 0)$ local maximum

Zeros: $x = 1, x = 3$

The function must be at least a degree 4 function.

18. Turning Pts.: $(-1.5, -5)$ local minimum
$(-0.25, -2)$ local maximum
$(0.5, -2.5)$ local minimum

Zeros: $x = -2, x = 1$

The function must be at least a degree 4 function.

19. Turning Pts.: $(-2.2, -38)$ local minimum
$(-1.1, 0.8)$ local maximum
$(0.3, -40)$ local minimum
$(1.9, 8)$ local maximum
$(2.75, -10.5)$ local minimum

Zeros: $x \approx -1.4, x \approx -1, x \approx 1.5, x \approx 3$

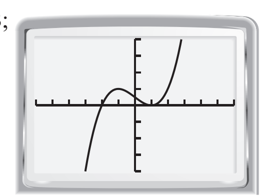
The function must be at least a degree 6 function.

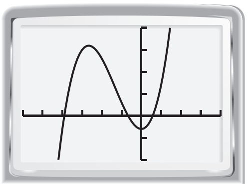
20. Turning Pts.: $(-1.6, -2)$ local minimum
$(-0.5, -0.5)$ local maximum
$(0.5, -2)$ local minimum

Zeros: $x \approx -2, x \approx 1$

The function must be at least a degree 4 function.

21. B;



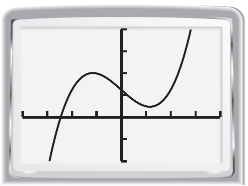
22. 

$f(x) = 2x^3 + 8x^2 - 3$

x-intercepts: $x \approx -3.90, x \approx -0.67, x \approx 0.57$

Local maximum: $(-2.67, 15.96)$

Local minimum: $(0, -3)$

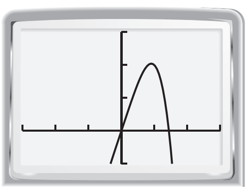
23. 

$g(x) = 0.5x^3 - 2x + 2.5$

x-intercept: $x = -2.46$

Local maximum: $(-1.15, 4.04)$

Local minimum: $(1.15, 0.96)$

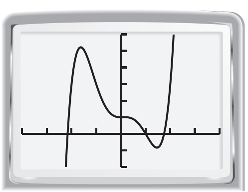
24. 

$h(x) = -x^4 + 3x$

x-intercepts: $x = 0, x \approx 1.44$

Local maximum: $(0.91, 2.04)$

Local minimum: none

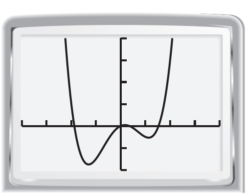
25. 

$f(x) = x^5 - 4x^3 + x^2 + 2$

x-intercepts: $x \approx -2.16, x = 1, x \approx 1.75$

Local maximums: $(-1.63, 10.47), (0.17, 2.01)$

Local minimums: $(0, 2), (1.46, -1.68)$

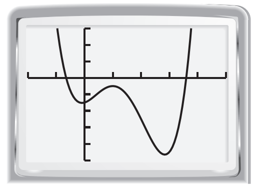
26. 

$g(x) = x^4 - 3x^2 + x$

x-intercepts: $x \approx -1.88, x = 0,$
$x \approx 0.35, x \approx 1.53$

Local maximum: $(0.17, 0.08)$

Local minimums: $(-1.3, -3.5)$, $(1.13, -1.07)$

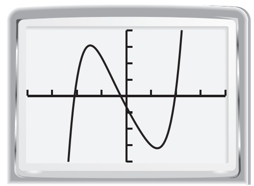
27. 

$h(x) = x^4 - 5x^3 + 2x^2 + x - 3$

x-intercepts: $x \approx -0.77, x \approx 4.54$

Local maximum: $(0.47, -2.6)$

Local minimums: $(-0.16, -3.09), (3.44, -39.40)$

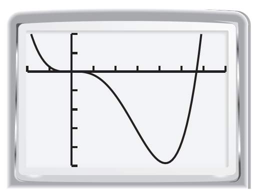
28. 

$h(x) = x^5 + 2x^2 - 17x - 4$

x-intercepts: $x \approx -2.10, x \approx -0.23,$
$x \approx 1.97$

Local maximum: $(-1.46, 18.45)$

Local minimum: $(1.25, -19.07)$

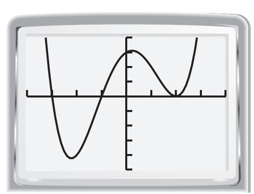
29. 

$g(x) = 0.7x^4 - 8x^3 + 5x$

x-intercepts: $x \approx -0.77, x = 0, x \approx 11.37$

Local maximum: $(0.47, 1.55)$

Local minimums: $(-0.45, -1.49), (8.55, 1216.68)$

30. D; 

Local maximum: $(0.22, 12.44)$

Local minimums: $(-2.22, -16.95), (2, 0)$

31. Quadratic functions only have 1 turning point; 1 maximum or 1 minimum. Since the turning point of a quadratic function is either the highest or lowest point on the curve, the term local is not needed. Cubic functions however, have ends stretch in different directions, so when there is a maximum or minimum it is never the absolute minimum or maximum. The term local is used to describe these locally described points.

32. A cubic function sometimes has a turning point. For example; the function $f(x) = x^3 + x - 1$ doesn't have a turning point but the function $f(x) = \frac{1}{4}x^3 - 3x$ has two turning points.

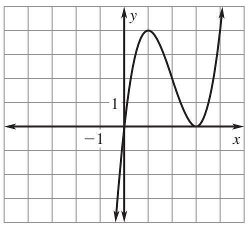
33. Cubic: $f(x) = (x + 2)(x)(x - 4) = x^3 - 2x^2 - 8x$

Quartic: $f(x) = (x + 2)(x^2)(x - 4) = x^4 - 2x^3 - 8x^2$

Fifth-degree: $f(x) = (x + 2)^2(x^2)(x - 4)$

$= x^5 - 12x^3 - 16x^2$

34.

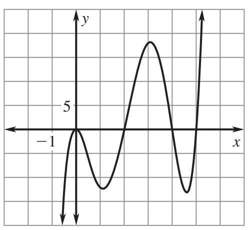


$f(x) = x(x - 3)^2$

Domain: All real numbers

Range: All real numbers

35.

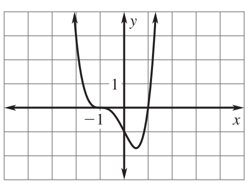


$f(x) = x^2(x - 2)(x - 4)(x - 5)$

Domain: All real numbers

Range: All real numbers

36.

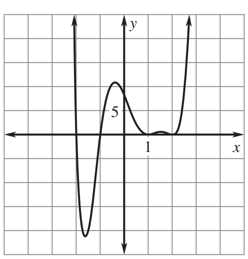


$f(x) = (x + 1)^3(x - 1)$

Domain: All real numbers

Range: $y \geq -1.6875$

37.



$f(x) = (x + 2)(x + 1)(x - 1)^2(x - 2)^2$

Domain: All real numbers

Range: $y \geq -21.28453$

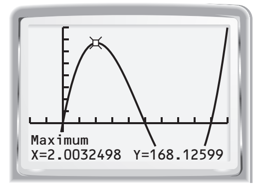
38. The domain and range of odd-degree polynomial functions are all real numbers. The domain of even-degree polynomial functions is all real numbers and the range either is greater than the minimum or less than the maximum value.

Problem Solving

39. $V = (10 - 2x)(18 - 2x)(x)$

$= (180 - 56x + 4x^2)(x)$

$= 4x^3 - 56x^2 + 180x$

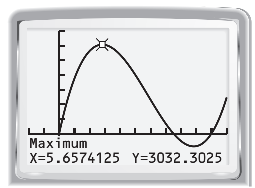


The maximum volume is about 168 cubic inches and the dimensions are 2 in. by 6 in. by 14 in.

40. $V = (30 - 2x)(40 - 2x)(x)$

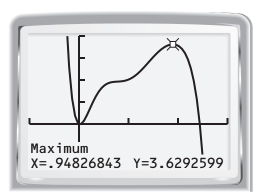
$= (1200 - 140x + 4x^2)(x)$

$= 4x^3 - 140x^2 + 1200x$



The maximum volume is about 3032 cm^3 and the dimensions are 5.7 cm by 18.6 cm by 28.6 cm.

41.



At about 0.95 second into the stroke the swimmer is going the fastest.

42. a. $600 = \pi r^2 + \pi r\ell$

$0 = \pi r^2 + \pi r\ell - 600$

$-\pi r\ell = \pi r^2 - 600$

$\ell = \frac{\pi r^2 - 600}{-\pi r}$

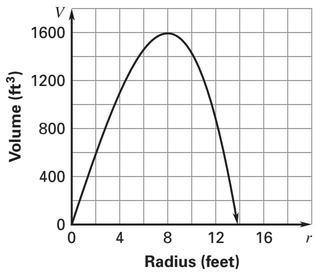
$= \frac{600 - \pi r^2}{\pi r}$

b. $V = \frac{1}{2}\pi r^2\left(\frac{600 - \pi r^2}{\pi r}\right)$

$= \frac{1}{2}r(600 - \pi r^2)$

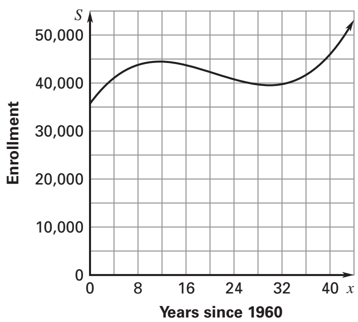
$= 300r - \frac{1}{2}\pi r^3$

c.



The maximum volume is about 1600 cubic feet, it occurs when $r = 8$ ft and $\ell = 15.9$ ft.

43. a.

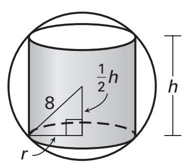


b. The turning points are: (11.66, 44,970.9) and (29.8, 40,078.2)

The number of students enrolled increased steadily before 1971 and decreased steadily between 1971 and 1990. After 1990 the number of students enrolled increased steadily again.

c. range: $36,300 \le S \le 47,978$

44. Write r as a function of h:



$$r^2 + \left(\frac{1}{2}h\right)^2 = 8^2$$

$$r^2 = 64 - \frac{h^2}{4}$$

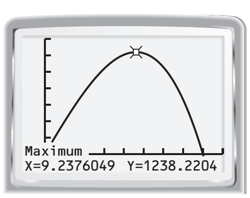
$$r = \sqrt{64 - \frac{h^2}{4}}$$

Volume of a cylinder $= \pi r^2 h$

$$= \pi\left(\sqrt{64 - \frac{h^2}{4}}\right)^2 \cdot h$$

$$= \pi\left(64 - \frac{h^2}{4}\right) \cdot h$$

$$= 64\pi h - \frac{\pi h^3}{4}$$



From the graph, you can see that $h \approx 9.2$ maximizes the volume. The maximum volume is about 1238 cubic units.

Mixed Review

45. $y = ax$
$5 = a \cdot 1$
$a = 5$
$y = 5x$

46. $y = ax$
$8 = a(-2)$
$-4 = a$
$y = -4x$

47. $y = ax$
$-5 = a \cdot 3$
$-\frac{5}{3} = a$
$y = -\frac{5}{3}x$

48. $y = ax$
$6 = a \cdot 4$
$\frac{3}{2} = a$
$y = \frac{3}{2}x$

49. $y = ax$
$-2 = a \cdot 5$
$-\frac{2}{5} = a$
$y = -\frac{2}{5}x$

50. $y = ax$
$-4 = a(-12)$
$\frac{1}{3} = a$
$y = \frac{1}{3}x$

51. $y = a(x - 5)^2 + 4$
$12 = a(3 - 5)^2 + 4$
$8 = 4a$
$a = 2$
$y = 2(x - 5)^2 + 4$

52. $y = a(x + 4)^2 - 6$
$3 = a(2 + 4)^2 - 6$
$9 = 36a$
$a = \frac{1}{4}$
$y = \frac{1}{4}(x + 4)^2 - 6$

53. $y = ax^2 + bx + c$

Use $(-3, 12)$: $12 = a(-3)^2 + b(-3) + c$
$12 = 9a - 3b + c$

Use $(-2, 3)$: $3 = a(-2)^2 + b(-2) + c$
$3 = 4a - 2b + c$

Use $(1, 0)$: $0 = a(1)^2 + b(1) + c$
$0 = a + b + c$

$9a - 3b + c = 12$
$4a - 2b + c = 3$
$a + b + c = 0 \rightarrow a = -b - c$
$9(-b - c) - 3b + c = 12$
$-9b - 9c - 3b + c = 12$
$-12b - 8c = 12$
$4(-b - c) - 2b + c = 3$
$-4b - 4c - 2b + c = 3$
$-6b - 3c = 3$

$-12b - 8c = 12$
$-6b - 3c = 3 \quad \xrightarrow{\times -2}$

$-12b - 8c = 12$
$\underline{12b + 6c = -6}$
$-2c = 6$
$c = -3$

$-6b - 3(3) = 3 \rightarrow b = 1$
$a = -1 - (-3) = 2$

A quadratic function is $y = 2x^2 + x - 3$.

54. $y = ax^2 + bx + c$

Use $(-2, 19)$: $19 = a(-2)^2 + b(-2) + c$

$19 = 4a - 2b + c$

Use $(2, -5)$: $-5 = a(2)^2 + b(2) + c$

$-5 = 4a + 2b + c$

Use $(4, -11)$: $-11 = a(4)^2 + b(4) + c$

$-11 = 16a + 4b + c$

$4a - 2b + c = 19 \rightarrow c = 19 - 4a + 2b$

$4a + 2b + c = -5$

$16a + 4b + c = -11$

$4a + 2b + (19 - 4a + 2b) = -5$

$4a + 2b + 19 - 4a + 2b = -5$

$4b = -24$

$b = -6$

$16a + 4b + (19 - 4a + 2b) = -11$

$16a + 4b + 19 - 4a + 2b = -11$

$12a + 6b = -30$

$12a + 6(-6) = -30$

$a = \frac{1}{2}$

$c = 19 - 4\left(\frac{1}{2}\right) + 2(-6) = 5$

A quadratic function is $y = \frac{1}{2}x^2 - 6x + 5$.

55. $(3^2x^3)^5 = (3^2)^5(x^3)^5$ Power of a product property

$= 59{,}049x^{15}$ Power of a power property

56. $(x^2y^4)^{-1} = \frac{1}{x^2y^4}$ Negative exponent property

57. $(xy^3)(x^{-2}y)^{-3} = (xy^3)(x^6y^{-3})$

Power of a product property

$= x^7y^0$ Product of powers property

$= x^7$ Zero exponent property

58. $-4x^{-3}y^0 = -4x^{-3}$ Zero exponent property

$= -\frac{4}{x^3}$ Negative exponent property

59. $\frac{x^6}{x^{-2}} = x^{6-(-2)}$ Quotient of powers property

$= x^8$

60. $\frac{3x^2y}{12xy^{-1}} = \frac{3}{12}x^{2-1}y^{1-(-1)}$ Quotient of powers property

$= \frac{1}{4}xy^2$

61. $\frac{8xy}{7x^4} \cdot \frac{7x^5y}{4y^2} = \left(\frac{8}{7}x^{1-4}y\right) \cdot \left(\frac{7}{4}x^5y^{1-2}\right)$

Quotient of powers property

$= \left(\frac{8}{7}x^{-3}y\right)\left(\frac{7}{4}x^5y^{-1}\right)$

$= 2x^2y^0$ Product of powers property

$= 2x^2$ Zero exponent property

62. $\left(\frac{5x^3y^7}{25x^2y^4}\right)^3 = \left(\frac{5}{25}x^{3-2}y^{7-4}\right)^3$ Quotient of powers property

$= \left(\frac{1}{5}xy^3\right)^3$

$= \left(\frac{1}{5}\right)^3x^3(y^3)^3$ Power of a product property

$= \frac{1}{125}x^3y^9$ Power of a power property

Lesson 5.9

5.9 Guided Practice (pp. 394–396)

1. $f(x) = a(x + 4)(x - 2)(x - 5)$

$10 = a(0 + 4)(0 - 2)(0 - 5)$

$10 = 40a$

$\frac{1}{4} = a$

$f(x) = \frac{1}{4}(x + 4)(x - 2)(x - 5)$

2. $f(x) = a(x + 1)(x - 2)(x - 3)$

$-12 = a(0 + 1)(0 - 2)(0 - 3)$

$-12 = 6a$

$-2 = a$

$f(x) = -2(x + 1)(x - 2)(x - 3)$

3.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
1	5	12	22	35	51	70
4	7	10	13	16	19	
3	3	3	3	3		

Each second order difference is 3, so the second-order differences are constant.

4.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	
6	15	22	21	6	-29	
9	7	-1	-15	-35		1st order differences
-2	-8	-14	-20			2nd order differences
-6	-6	-6				3rd order differences

Cubic function:

$f(x) = ax^3 + bx^2 + cx + d$

$a(1)^3 + b(1)^2 + c(1) + d = 6$

$a + b + c + d = 6$

$a(2)^3 + b(2)^2 + c(2) + d = 15$

$8a + 4b + 2c + d = 15$

$a(3)^3 + b(3)^2 + c(3) + d = 22$

$27a + 9b + 3c + d = 22$

$a(4)^3 + b(4)^2 + c(4) + d = 21$

$64a + 16b + 4c + d = 21$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 6 \\ 15 \\ 22 \\ 21 \end{bmatrix}$$

$$A \qquad X = B$$

$X = A^{-1}B$

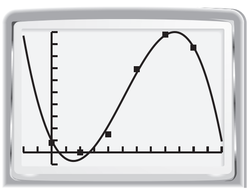
Using a graphing calculator, the solution is $a = -1$, $b = 5$, $c = 1$, and $d = 1$. So, a polynomial model is $f(x) = -x^3 + 5x^2 + x + 1$.

5. Regression model:

$y = 2.7130x^3 - 25.468x^2 + 71.82x - 45.7$

6. Regression model:

$y = -0.5868x^3 + 8.985x^2 - 23.37x + 9.6$



5.9 Exercises (pp. 397–399)

Skill Practice

1. When the x-values in a data set are equally spaced, the differences of consecutive y-values are called *finite differences*.

2. First-order differences are differences between consecutive y-values. Second-order differences are differences between consecutive first-order differences.

3. $f(x) = a(x + 1)(x - 2)(x - 3)$
   $3 = a(0 + 1)(0 - 2)(0 - 3)$
   $3 = 6a$
   $\frac{1}{2} = a$
   $f(x) = \frac{1}{2}(x + 1)(x - 2)(x - 3)$

4. $f(x) = a(x + 4)(x + 1)(x - 1)$
   $\frac{4}{3} = a(0 + 4)(0 + 1)(0 - 1)$
   $\frac{4}{3} = -4a$
   $-\frac{1}{3} = a$
   $f(x) = -\frac{1}{3}(x + 4)(x + 1)(x - 1)$

5. $f(x) = a(x + 3)(x - 1)(x - 4)$
   $2 = a(0 + 3)(0 - 1)(0 - 4)$
   $2 = 12a$
   $\frac{1}{6} = a$
   $f(x) = \frac{1}{6}(x + 3)(x - 1)(x - 4)$

6. $f(x) = a(x + 3)(x)(x - 4)$
   $10 = a(-1 + 3)(-1)(-1 - 4)$
   $10 = 10a$
   $a = 1$
   $f(x) = x(x + 3)(x - 4)$

7. $f(x) = a(x + 2)(x + 1)(x - 2)$
   $-8 = a(0 + 2)(0 + 1)(0 - 2)$
   $-8 = -4a$
   $2 = a$
   $f(x) = 2(x + 2)(x + 1)(x - 2)$

8. $f(x) = a(x + 3)(x - 1)(x - 4)$
   $2 = a(3 + 3)(3 - 1)(3 - 4)$
   $2 = -12a$
   $-\frac{1}{6} = a$
   $f(x) = -\frac{1}{6}(x + 3)(x - 1)(x - 4)$

9. $f(x) = a(x + 5)(x)(x - 6)$
   $-12 = a(1 + 5)(1)(1 - 6)$
   $-12 = -30a$
   $\frac{2}{5} = a$
   $f(x) = \frac{2}{5}x(x + 5)(x - 6)$

10. B; $f(x) = a(x + 3)(x + 1)(x - 3)$
   $3 = a(0 + 3)(0 + 1)(0 - 3)$
   $3 = -9a$
   $-\frac{1}{3} = a$
   $f(x) = -\frac{1}{3}(x + 3)(x + 1)(x - 3)$

11. The student replaced x with 3 instead of 1 and replaced y with 1 instead of 3.
   $3 = a(1 + 1)(1 - 2)(1 - 5)$
   $3 = 8a$
   $\frac{3}{8} = a$

12.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
−5	30	125	310	615	1070	1705

35 95 185 305 455 635

60 90 120 150 180

30 30 30 30

Each third-order difference is 30, so the third-order differences are nonzero and constant.

13.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
3	2	−3	−12	−25	−42	−63

−1 −5 −9 −13 −17 −21

−4 −4 −4 −4 −4

Each second-order difference is -4, so the second-order differences are nonzero and constant.

14.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
0	6	56	210	552	1190	2256

6 50 154 342 638 1066

44 104 188 296 428

60 84 108 132

24 24 24

Each fourth-order difference is 24, so the fourth-order differences are nonzero and constant.

15.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
−3	0	11	30	57	92	135

3 11 19 27 35 43

8 8 8 8 8

Each second-order difference is 8, so the second-order differences are nonzero and constant.

16.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
−3	−9	−11	−3	21	67	141

−6 −2 8 24 46 74

4 10 16 22 28

6 6 6 6

Each third-order difference is 6, so the third-order differences are nonzero and constant.

17.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
0	54	462	2004	6180	15,450	33,474

54 408 1542 4176 9270 18,024

354 1134 2634 5094 8754

780 1500 2460 3660

720 960 1200

240 240

Each fifth-order difference is 240, so the fifth-order differences are nonzero and constant.

18.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
0	−3	−8	−15	−24	−35

−3 −5 −7 −9 −11 1st order differences

−2 −2 −2 −2 2nd order differences

Quadratic function:

$f(n) = an^2 + bn + c$

$a(1)^2 + b(1) + c = 0 \rightarrow a + b + c = 0$

$a(2)^2 + b(2) + c = -3 \rightarrow 4a + 2b + c = -3$

$a(3)^2 + b(3) + c = -8 \rightarrow 9a + 3b + c = -8$

$a + b + c = 0 \rightarrow c = -a - b$

$4a + 2b + c = -3$

$9a + 3b + c = -8$

$4a + 2b + (-a - b) = -3$

$3a + b = -3$

$9a + 3b + (-a - b) = -8$

$8a + 2b = -8$

$3a + b = -3 \rightarrow b = -3 - 3a$

$8a + 2b = -8$

$8a + 2(-3 - 3a) = -8$

$8a - 6 - 6a = -8 \rightarrow a = -1$

$b = -3 - 3(-1) = 0$

$c = -(-1) - 0 = 1$

The quadratic function is $f(77) = -x^2 + 1$.

19.

	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
	11	14	9	−4	−25	−54
1st order differences	3	−5	−13	−21	−29	
2nd order differences	−8	−8	−8	−8		

Quadratic function: $f(n) = an^2 + bn + c$.

$a(1)^2 + b(1) + c = 11 \rightarrow a + b + c = 11$

$a(2)^2 + b(2) + c = 14 \rightarrow 4a + 2b + c = 14$

$a(3)^3 + b(3) + c = 9 \rightarrow 9a + 3b + c = 9$

$a + b + c = 11 \rightarrow c = 11 - a - b$

$4a + 2b + c = 14$

$9a + 3b + c = 9$

$4a + 2b + (11 - a - b) = 14$

$3a + b = 3$

$9a + 3b + (11 - a - b) = 9$

$8a + 2b = -2$

$3a + b = 3 \rightarrow b = 3 - 3a$

$8a + 2b = -2$

$8a + 2(3 - 3a) = -2$

$8a + 6 - 6a = -2 \rightarrow a = -4$

$b = 3 - 3(-4) = 15$

$c = 11 - (-4) - 15 = 0$

The quadratic function is $f(n) = -4x^2 + 15x$.

20.

	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
	−12	−14	−10	6	40	98
1st order differences	−2	4	16	34	58	
2nd order differences	6	12	18	24		
3rd order differences	6	6	6			

Cubic function: $f(x) = ax^3 + bx^2 + cx + d$

$a(1)^3 + b(1)^2 + c(1) + d = -12$

$a + b + c + d = -12$

$a(2)^3 + b(2)^2 + c(2) + d = -14$

$8a + 4b + 2c + d = -14$

$a(3)^3 + b(3)^2 + c(3) + d = -10$

$27a + 9b + 3c + d = -10$

$a(4)^3 + a(4)^2 + a(4) + d = 6$

$64a + 16b + 4c + d = 6$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -12 \\ -14 \\ -10 \\ 6 \end{bmatrix}$$

$$A \qquad X = B$$

$$X = A^{-1}B$$

Using a graphing calculator, the solution is $a = 1$, $b = -3$, $c = 0$, and $d = -10$. So, a polynomial function is $f(x) = x^3 - 3x^2 - 10$.

21.

	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
	5	14	27	41	53	60
1st order differences	9	13	14	12	7	
2nd order differences	4	1	−2	−5		
3rd order differences	−3	−3	−3			

Cubic function: $f(x) = ax^3 + bx^2 + cx + d$

$a(1)^3 + b(1)^2 + c(1) + d = 5$

$a + b + c + d = 5$

$a(2)^3 + b(2)^2 + c(2) + d = 14$

$8a + 4b + 2c + d = 14$

$a(3)^3 + b(3)^2 + c(3) + d = 27$

$27a + 9b + 3c + d = 27$

$a(4)^3 + b(4)^2 + c(4) + d = 41$

$64a + 16b + 4c + d = 41$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \\ 27 \\ 41 \end{bmatrix}$$

$$A \qquad X = B$$

$$X = A^{-1}B$$

Using a graphing calculator, the solution is $a = -\frac{1}{2}$, $b = 5$, $c = -\frac{5}{2}$, and $d = 3$. So, a polynomial model is

$f(x) = -\frac{1}{2}x^3 + 5x^2 - \frac{5}{2}x + 3$.

22. *Sample answers:*

$f(x) = a(x + 1)(x + 3)(x)$

$6 = a(2 + 1)(2 + 3)(2)$

$6 = 30a$

$\frac{1}{5} = a$

$f(x) = \frac{1}{5}(x)(x + 1)(x + 3)$

$$f(x) = a(x+1)(x+3)(x-1)$$
$$6 = a(2+1)(2+3)(2-1)$$
$$6 = 15a$$
$$\frac{2}{5} = a$$
$$f(x) = \frac{2}{5}(x+1)(x+3)(x-1)$$

23. For a function of degree 4, it takes 5 points to determine a quartic function.

For a function of degree 5, it takes 6 points to determine a quintic function.

To determine a quartic function $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, a system of 5 equations in 5 variables

$$\begin{bmatrix} x_1^{\ 4} & x_1^{\ 3} & x_1^{\ 2} & x_1 & 1 \\ x_2^{\ 4} & x_2^{\ 3} & x_2^{\ 2} & x_2 & 1 \\ x_3^{\ 4} & x_3^{\ 3} & x_3^{\ 2} & x_3 & 1 \\ x_4^{\ 4} & x_4^{\ 3} & x_4^{\ 2} & x_4 & 1 \\ x_5^{\ 4} & x_5^{\ 3} & x_5^{\ 2} & x_5 & 1 \end{bmatrix} \begin{bmatrix} a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \\ f(x_4) \\ f(x_5) \end{bmatrix}$$

should be solved to find a_0, a_1, a_2, a_3, and a_4, where each $(x_i, f(x_i))$, $i = 1, 2, 3, 4, 5$ is a point.

To determine a quintic function $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, a system of 6 equations in 6 variables (a_i, $i = 0, 1, 2, \ldots, 5$) should be solved.

$$\begin{bmatrix} x_1^{\ 5} & x_1^{\ 4} & x_1^{\ 3} & x_1^{\ 2} & x_1 & 1 \\ x_2^{\ 5} & x_2^{\ 4} & x_2^{\ 3} & x_2^{\ 2} & x_2 & 1 \\ x_3^{\ 5} & x_3^{\ 4} & x_3^{\ 3} & x_3^{\ 2} & x_3 & 1 \\ x_4^{\ 5} & x_4^{\ 4} & x_4^{\ 3} & x_4^{\ 2} & x_4 & 1 \\ x_5^{\ 5} & x_5^{\ 4} & x_5^{\ 3} & x_5^{\ 2} & x_5 & 1 \\ x_6^{\ 5} & x_6^{\ 4} & x_6^{\ 3} & x_6^{\ 2} & x_6 & 1 \end{bmatrix} \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \\ f(x_4) \\ f(x_5) \\ f(x_6) \end{bmatrix}$$

24. $f(k) = ak^3 + bk^2 + ck + d$

$$\begin{aligned} f(k+1) &= a(k+1)^3 + b(k+1)^2 + c(k+1) + d \\ &= ak^3 + (3a+b)k^2 + (3a+2b+c)k \\ &\quad + a + b + c + d \\ f(k+2) &= a(k+2)^3 + b(k+2)^2 + c(k+2) + d \\ &= ak^3 + (6a+b)k^2 + (12a+4b+c)k \\ &\quad + 8a + 4b + 2c + d \\ f(k+3) &= a(k+3)^3 + b(k+3)^2 + c(k+3) + d \\ &= ak^3 + (9a+b)k^2 + (27a+6b+c)k \\ &\quad + 27a + 9b + 3c + d \\ f(k+4) &= a(k+4)^3 + b(k+4)^2 + c(k+4) + d \\ &= ak^3 + (12a+b)k^2 + (48a+8b)k \\ &\quad + 64a + 16b + 4c + d \\ f(k+5) &= a(k+5)^3 + b(k+5)^2 + c(k+4) + d \\ &= ak^3 + (15a+b)k^2 + (75a+10b)k \\ &\quad + 125a + 25b + 5c + d \end{aligned}$$

First Differences

$f(k) \rightarrow 3ak^2 + (3a + 2b)k + a + b + c$

$f(k+1) \rightarrow 3ak^2 + (9a + 2b)k + 7a + 3b + c$

$f(k+2) \rightarrow 3ak^2 + (15a + 2b)k + 19a + 5b + c$

$f(k+3) \rightarrow 3ak + (21a + 2b)k + 37a + 7b + c$

$f(k+4) \rightarrow 3ak^2 + (27a + 2b)k + 61a + 9b + c$

$f(k+5)$

Second Differences	*Third Differences*
$\Rightarrow$ $bak + 6a + 2b$	$\Rightarrow$ $6a$
$\Rightarrow$ $bak + 12a + 2b$	$\Rightarrow$ $6a$
$\Rightarrow$ $bak + 18a + 2b$	$\Rightarrow$ $6a$
$\Rightarrow$ $bak + 24a + 2b$	

25.

$d(3)$	$d(4)$	$d(5)$	$d(6)$	$d(7)$	$d(8)$	
0	2	5	9	14	20	
	2	3	4	5	6	1st order differences
		1	1	1	1	2nd order differences

Quadratic function: $d(n) = an^2 + bn + c$

$a(3)^2 + b(3) + c = 0 \rightarrow 9a + 3b + c = 0$

$a(4)^2 + b(4) + c = 2 \rightarrow 16a + 4b + c = 2$

$a(5)^2 + b(5) + c = 5 \rightarrow 25a + 5b + c = 5$

$9a + 3b + c = 0 \rightarrow c = -9a - 3b$

$16a + 4b + c = 2$

$25a + 5b + c = 5$

$16a + 4b + (-9a - 3b) = 2$

$7a + b = 2 \rightarrow b = 2 - 7a$

$25a + 5b + (-9a - 3b) = 5$

$16a + 2b = 5$

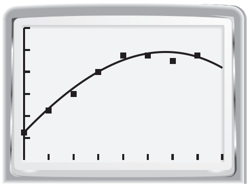
$16a + 2(2 - 7a) = 5 \rightarrow a = \frac{1}{2}$

$b = 2 - 7\left(\frac{1}{2}\right) = -\frac{3}{2}$

$c = -9\left(\frac{1}{2}\right) - 3\left(-\frac{3}{2}\right) = 0$

The number of diagonals of a polygon with n sides can be modeled by $d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$.

26.



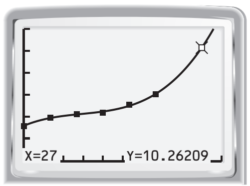
A polynomial model is

$p(t) = -0.013t^3 - 0.30t^2 + 4.7t + 131.$

27. a. A polynomial model is

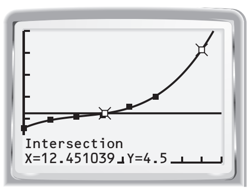
$m(t) = 0.00082t^3 - 0.0215t^2 + 0.249t + 3.17.$

b.



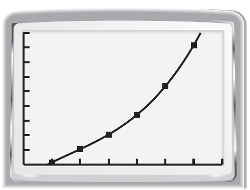
In 2010, the average U.S. movie ticket price will be about \$10.26.

c.



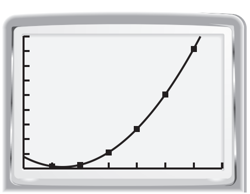
The average U.S. movie ticket price was \$4.50 in 1996.

28. *Yard work:*



A polynomial function for the profit for the yard work is $p(t) = 8.333t^3 - 40t^2 + 241.7t - 180.$

Pet care:



A polynomial function for the profit for the pet care is $p(t) = 75t^2 - 205t + 160$. The yard work business will achieve the greatest long-term profit. The cubic model for the yard work will increase more rapidly than the quadratic model for the pet care.

29.

$R(1)$	$R(2)$	$R(3)$	$R(4)$	$R(5)$	$R(6)$	$R(7)$
2	4	8	16	30	52	84

2 4 8 14 22 32

2 4 6 8 10

2 2 2 2

Each third-order difference is 2, so the third-order differences are constant.

30.

$p(1)$	$p(2)$	$p(3)$	$p(4)$	$p(5)$	$p(6)$
2	4	8	15	26	42

2 4 7 11 16 1st order differences

2 3 4 5 2nd order differences

1 1 1 3rd order differences

Cubic function: $p(x) = ax^3 + bx^2 + cx + d$

$a(1)^3 + b(1)^2 + c(1) + d = 2 \rightarrow$

$a + b + c + d = 2$

$a(2)^3 + b(2)^2 + c(2) + d = 4 \rightarrow$

$8a + 4b + 2c + d = 4$

$a(3)^3 + b(3)^2 + c(3) + d = 8 \rightarrow$

$27a + 9b + 3c + d = 8$

$a(4)^3 + b(4)^2 + c(4) + d = 15 \rightarrow$

$64a + 16b + 4c + d = 15$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 2 \\ 4 \\ 8 \\ 15 \end{bmatrix}}_{B}$$

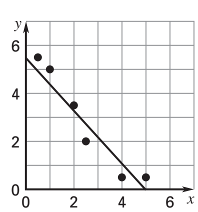
$$X = A^{-1}B$$

Using a graphing calculator, the solution is $a = \frac{1}{6}$, $b = 0$, $c = \frac{5}{6}$, and $d = 1$. So, a polynomial model for the maximum number of pieces is $p(c) = \frac{1}{6}c^3 + \frac{5}{6}c + 1.$

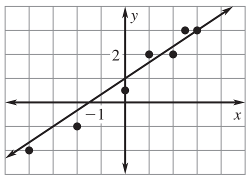
When $c = 8$, $p(c) = \frac{1}{6}(8)^3 + \frac{5}{6}(8) + 1 = 93$ pieces.

Mixed Review

31.



Sample answer: $y = -1.1x + 5.5$

32. 

Sample answer: $y = \frac{2}{3}x + 1$

33. $x^2 - 19x + 48 = (x - 16)(x - 3)$

34. $18x^2 + 30x - 12 = 6(3x^2 + 5x - 2) = 6(3x - 1)(x + 2)$

35. $64x^2 - 144x + 81 = (8x - 9)^2$

36. $18x^3 + 33x^2 - 30x = 3x(6x^2 + 11x - 10)$
$= 3x(3x - 2)(2x + 5)$

37. $64x^3 + 27 = (4x + 3)(16x^2 - 12x + 9)$

38. $3x^5 - 66x^3 - 225x = 3x(x^4 - 22x^2 - 75)$
$= 3x(x^2 - 25)(x^2 + 3)$
$= 3x(x - 5)(x + 5)(x^2 + 3)$

39. $5x^2 = 10$
$x^2 = 2$
$x = \pm\sqrt{2}$
$x = -\sqrt{2}$ or $x = \sqrt{2}$

40. $24x^2 = 6$
$x^2 = \frac{1}{4}$
$x = \pm\sqrt{\frac{1}{4}}$
$x = -\frac{1}{2}$ or $x = \frac{1}{2}$

41. $9x^2 + 2 = 6$
$9x^2 = 4$
$x^2 = \frac{4}{9}$
$x = \pm\sqrt{\frac{4}{9}}$
$x = -\frac{2}{3}$ or $x = \frac{2}{3}$

42. $7x^2 - 4 = 8$
$7x^2 = 12$
$x^2 = \frac{12}{7}$
$x = \pm\sqrt{\frac{12}{7}}$
$x \approx -1.31$ or $x \approx 1.31$

43. $-x^2 + 16 = 5x^2 - 12$
$-x^2 = 5x^2 - 28$
$-6x^2 = -28$
$x^2 = \frac{14}{3}$
$x = \pm\sqrt{\frac{14}{3}}$
$x \approx -2.16$ or $x \approx 2.16$

44. $4x^2 + 3 = -4x^2 + 15$
$4x^2 = -4x^2 + 12$
$8x^2 = 12$
$x^2 = \frac{12}{8}$
$x = \pm\sqrt{\frac{3}{2}}$
$x \approx -1.2$ or $x \approx 1.2$

Quiz 5.7–5.9 (p. 399)

1. $f(x) = x^3 - 4x^2 - 11x + 30$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$f(x) = (x - 2)(x^2 - 2x - 15) = (x - 2)(x - 5)(x + 3)$

The zeros are -3, 2, and 5.

2. $f(x) = 2x^4 - 2x^3 - 49x^2 + 9x + 180$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm9, \pm10, \pm12, \pm15, \pm18, \pm20, \pm30, \pm36, \pm45, \pm60, \pm90, \pm180$

$$\begin{array}{r|rrrrr} -4 & 2 & -2 & -49 & 9 & 180 \\ & & -8 & 40 & 36 & -180 \\ \hline & 2 & -10 & -9 & 45 & 0 \end{array}$$

$f(x) = (x + 4)(2x^3 - 10x^2 - 9x + 45)$

$$\begin{array}{r|rrrr} 5 & 2 & -10 & -9 & 45 \\ & & 10 & 0 & -45 \\ \hline & 2 & 0 & -9 & 0 \end{array}$$

$f(x) = (x + 4)(x - 5)(2x^2 - 9)$
$0 = (x + 4)(x - 5)(2x^2 - 9)$
$x + 4 = 0$ or $x - 5 = 0$ or $2x^2 - 9 = 0$
$x = -4$ or $x = 5$ or $x = -\frac{3\sqrt{2}}{2}$ or $x = \frac{3\sqrt{2}}{2}$

The zeros are -4, $-\frac{3\sqrt{2}}{2}$, $\frac{3\sqrt{3}}{2}$, and 5.

3. $f(x) = (x + 4)(x + 1)(x - 2)$
$= (x^2 + 5x + 4)(x - 2)$
$= x^3 + 5x^2 + 4x - 2x^2 - 10x - 8$
$= x^3 + 3x^2 - 6x - 8$

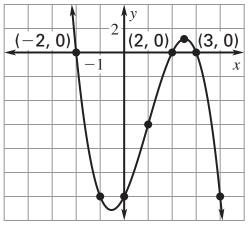
4. $f(x) = (x - 4)[x - (1 + i)][x - (1 - i)]$
$= (x - 4)[(x - 1) - i][(x - 1) + i]$
$= (x - 4)[(x - 1)^2 - i^2]$
$= (x - 4)(x^2 - 2x + 1 + 1)$
$= (x - 4)(x^2 - 2x + 2)$
$= x^3 - 2x^2 + 2x - 4x^2 + 8x - 8$
$= x^3 - 6x^2 + 10x - 8$

5. $f(x) = (x + 3)(x - 5)[x - (7 + \sqrt{2})][x - (7 - \sqrt{2})]$
$= (x + 3)(x - 5)[(x - 7) - \sqrt{2}][(x - 7) + \sqrt{2}]$
$= (x + 3)(x - 5)[(x - 7)^2 - (\sqrt{2})^2]$
$= (x + 3)(x - 5)(x^2 - 14x + 49 - 2)$
$= (x^2 - 2x - 15)(x^2 - 14x + 47)$
$= x^4 - 14x^3 + 47x^2 - 2x^3 + 28x^2 - 94x - 15x^2 + 210x - 705$
$= x^4 - 16x^3 + 60x^2 + 116x - 705$

6. $f(x) = (x - 1)(x - 2i)(x + 2i)[x - (3 - \sqrt{6})] \cdot [x - (3 + \sqrt{6})]$
$= (x - 1)[x^2 - (2i)^2][(x - 3) + \sqrt{6}][(x - 3) - \sqrt{6}]$
$= (x - 1)(x^2 + 4)[(x - 3)^2 - (\sqrt{6})^2]$
$= (x^3 + 4x - x^2 - 4)(x^2 - 6x + 9 - 6)$
$= (x^3 - x^2 + 4x - 4)(x^2 - 6x + 3)$
$= x^5 - x^4 + 4x^3 - 4x^2 - 6x^4 + 6x^3 - 24x^2 + 24x + 3x^3 - 3x^2 + 12x - 12$
$= x^5 - 7x^4 + 13x^3 - 31x^2 + 36x - 12$

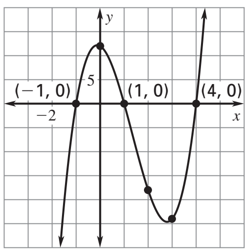
7. $f(x) = -(x-3)(x-2)(x+2)$

x	-2.5	-1	0	1	2.5	4
y	12.4	-12	-12	-6	1.1	-12



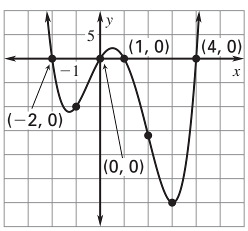
8. $f(x) = 3(x-1)(x+1)(x-4)$

x	-1.5	0	2	3	4.5
y	-20.6	12	-18	-24	28.9



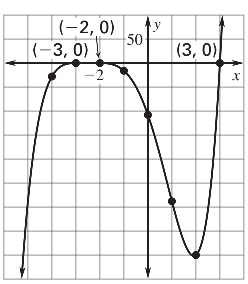
9. $f(x) = x(x-4)(x-1)(x+2)$

x	-2.5	-1	2	3	4.5
y	28.4	-10	-16	-30	51.2



10. $f(x) = (x-3)(x+2)^2(x+3)^2$

x	-4	-1	0	1	2	3.25
y	-28	-16	-108	-288	-400	269



11. $f(x) = a(x+5)(x+2)(x-2)$

Use (1, 9): $9 = a(1+5)(1+2)(1-2)$

$$-\frac{1}{2} = a$$

$f(x) = -\frac{1}{2}(x+5)(x+2)(x-2)$

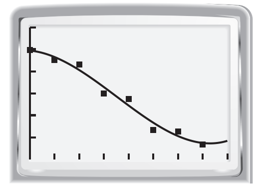
12. $f(x) = a(x+1)(x-2)(x-4)$

Use (0, 16): $16 = a(0+1)(0-2)(0-4)$

$$2 = a$$

$f(x) = 2(x+1)(x-2)(x-4)$

13.



A polynomial model is

$D(t) = 0.99x^3 - 10.5t^2 - 5t + 847.$

Mixed Review of Problem Solving (p. 400)

1. a. Volume (cubic in.) = Length (in.) • Width (in.) • Height (in.)

$180 = (x+5) \cdot (x+1) \cdot x$

$180 = (x^2 + 6x + 5)(x)$

$0 = x^3 + 6x^2 + 5x - 180$

b. Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm9, \pm10, \pm12, \pm15, \pm18, \pm20, \pm30, \pm36, \pm45, \pm60, \pm90$ ±180

c.

$$\begin{array}{r|rrrr} 4 & 1 & 6 & 5 & -180 \\ & & 4 & 40 & 180 \\ \hline & 1 & 10 & 45 & 0 \end{array}$$

$0 = (x-4)(x^2 + 10x + 45)$

$x - 4 = 0$ or $x^2 + 10x + 45 = 0$

$x = 4$ or $x = \dfrac{-10 \pm \sqrt{10^2 - 4(1)(45)}}{2} = \dfrac{-10 \pm \sqrt{-80}}{2}$

The solutions are 4, $-5 + 2i\sqrt{5}$, $-5 - 2i\sqrt{5}$.
The only real solution is $x = 4$.

d. The dimensions are 9 in. long by 5 in. wide by 4 in. tall.

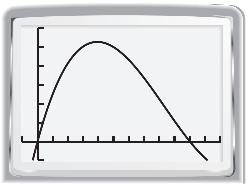
2. a. Volume (cubic inches) = Length (inches) • Width (inches) • Height (inches)

$V = (30 - 2x) \cdot (20 - 2x) \cdot x$

$= (600 - 100x + 4x^2)x$

$= 4x^3 - 100x^2 + 600x$

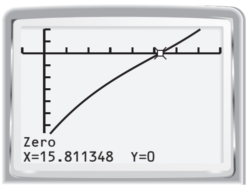
b.



c. The maximum occurs at (3.92, 1056.3), so the dimensions are about 4 inches by 12 inches by 22 inches.

d. The maximum volume is about 1056 cubic inches.

3. $$R = 0.629t^3 - 27.8t^2 + 744t + 19{,}600$$
$$26{,}900 = 0.629t^3 - 27.8t^2 + 744t + 19{,}600$$
$$0 = 0.629t^3 - 27.8t^2 + 744t - 7300$$



In 1995, there were 26,900,000 retirees receiving social security benefits.

4. $f(x) = (x + 2)(x - 1)[x - (4 - i)][x - (4 + i)]$
$= (x^2 + x - 2)[(x - 4) + i][(x - 4) - i]$
$= (x^2 + x - 2)[(x - 4)^2 - i^2]$
$= (x^2 + x - 2)(x^2 - 8x + 16 + 1)$
$= (x^2 + x - 2)(x^2 - 8x + 17)$
$= x^4 - 8x^3 + 17x^2 + x^3 - 8x^2 + 17x - 2x^2 + 16x - 34$
$= x^4 - 7x^3 + 7x^2 + 33x - 34$

5. *Sample answer:* $f(x) = 2x^2 + 5x + 15$

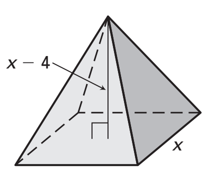
Possible rational zeros: $\frac{\pm 3}{\pm 1}, \frac{\pm 5}{\pm 1}, \frac{\pm 3}{\pm 2}, \frac{\pm 5}{\pm 2}$

$0 = 2x^2 + 5x + 15$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(15)}}{2(2)} = \frac{-5 \pm \sqrt{-95}}{4}$$

The solutions are $\frac{-5 + i\sqrt{95}}{4}$ and $\frac{-5 - i\sqrt{95}}{4}$.

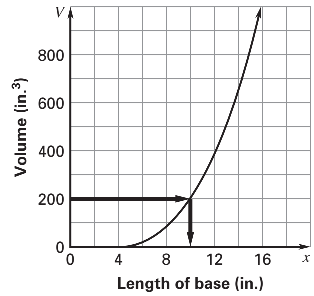
6. a.



b. Volume (cubic inches) $= \frac{1}{3}$ · Area of base (square inches) · Height (inches)

$$V = \frac{1}{3} \quad (x^2) \quad \cdot \quad (x - 4)$$
$$= \frac{1}{3}(x^2)(x - 4)$$
$$= \frac{1}{3}x^3 - \frac{4}{3}x^2$$

c.



d. $200 = \frac{1}{3}x^3 - \frac{4}{3}x^2$
$600 = x^3 - 4x^2$
$0 = x^3 - 4x^2 - 600$

10	1	−4	0	−600
		10	60	600
	1	6	60	0

$0 = (x - 10)(x^2 - 6x + 60)$
$x - 10 = 0$ or $x^2 - 6x + 60 = 0$
$$x = 10 \text{ or } x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(60)}}{2(1)} = 3 \pm i\sqrt{51}$$

The only reasonable solution is $x = 10$. This checks with the answer from part (c). So, the square pyramid has a side length of 10 inches and a height of $10 - 4 = 6$ inches.

7.

$p(1)$	$p(2)$	$p(3)$	$p(4)$	$p(5)$	
4	2	6	22	56	
−2	4	16	34		1st order differences
6	12	18			2nd order differences
6	6				3rd order differences

Cubic function: $p(t) = at^3 + bt^2 + ct + d$

Use (1, 4): $a(1)^3 + b(1)^2 + c(1) + d = 4$
$a + b + c + d = 4$

Use (2, 2): $a(2)^3 + b(2)^2 + c(2) + d = 2$
$8a + 4b + 2c + d = 2$

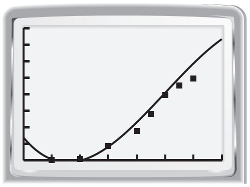
Use (3, 6): $a(3)^3 + b(3)^2 + c(3) + d = 6$
$27a + 9b + 3c + d = 6$

Use (4, 22): $a(4)^3 + b(4)^2 + c(4) + d = 22$
$64a + 16b + 4c + d = 22$

$$\overset{A}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix}} \overset{X}{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}} = \overset{B}{\begin{bmatrix} 4 \\ 2 \\ 6 \\ 22 \end{bmatrix}}$$
$$X = A^{-1}B$$

Using a graphing calculator, the solution is $a = 1$, $b = -3$, $c = 0$, and $d = 6$. So, the profit can be modeled by $p(t) = t^3 - 3t^2 + 6$.

When $t = 7$, $p(t) = (7)^3 - 3(7)^2 + 6 = \$202$.

8. 

A polynomial model is
$W(\ell) = -3.24\ell^2 + 0.0542\ell^2 - 1.71\ell + 13.9$.

Chapter 5 Review (pp. 402–406)

1. At each of its turning points, the graph of a polynomial function has a *local maximum* or a *local minimum*.

2. A solution of a polynomial equation is repeated if a factor is raised to a power greater then 1 or the factor is repeated.

3. A number is in scientific notation if it is in the form $c \times 10^n$ where $1 \le c < 10$ and n is an integer.

4. The function f has $4 - 1 = 3$ turning points.

5. $2^2 \cdot 2^5 = 2^{2+5}$ Product of powers property
$= 2^7$ Simplify and evaluate power.
$= 128$

6. $(3^2)^{-3}(3^3) = 3^{-6}3^3$ Power of a power property
$= 3^{-6+3}$ Product of powers property
$= 3^{-3}$ Simplify exponent.
$= \dfrac{1}{3^3}$ Negative exponent property
$= \dfrac{1}{27}$ Simplify and evaluate power.

7. $(x^{-2}y^5)^2 = (x^{-2})^2(y^5)^2$ Power of a product property
$= x^{-4}y^{10}$ Power of a power property
$= \dfrac{y^{10}}{x^4}$ Negative exponent property

8. $(3x^4y^{-2})^{-3} = (3)^{-3}(x^4)^{-3}(y^{-2})^{-3}$
Power of a product property
$= 3^{-3}x^{-12}y^6$ Power of a power property
$= \dfrac{y^6}{3^3x^{12}}$ Negative exponent property
$= \dfrac{y^6}{27x^{12}}$ Simplify and evaluate power.

9. $\left(\dfrac{3}{4}\right)^{-2} = \dfrac{3^{-2}}{4^{-2}}$ Power of a quotient property
$= \dfrac{4^2}{3^2}$ Negative exponent property
$= \dfrac{16}{9}$ Simplify and evaluate powers.

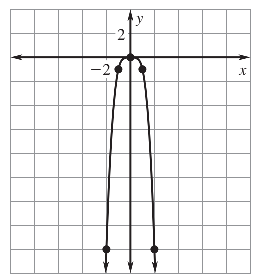
10. $\dfrac{8 \times 10^7}{2 \times 10^3} = \dfrac{8}{2} \times \dfrac{10^7}{10^3}$
$= 4 \times 10^{7-3}$ Quotient of powers property
$= 4 \times 10^4$ Simplify and evaluate power.

11. $\left(\dfrac{x^2}{y^{-2}}\right)^{-4} = \dfrac{(x^2)^{-4}}{(y^{-2})^{-4}}$ Power of quotient property
$= \dfrac{x^{-8}}{y^8}$ Power of a power property
$= \dfrac{1}{x^8y^8}$ Negative exponent property

12. $\dfrac{2x^{-6}y^5}{16x^3y^{-2}} = \dfrac{1}{8}x^{-6-3}y^{5-(-2)}$
Quotient of powers property
$= \dfrac{1}{8}x^{-9}y^7$ Simplify exponents.
$= \dfrac{y^7}{8x^9}$ Negative exponent property

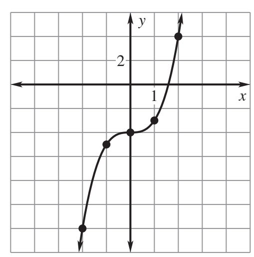
13. $f(x) = -x^4$

x	−2	−1	0	1	2
***f*(*x*)**	−16	−1	0	−1	−16



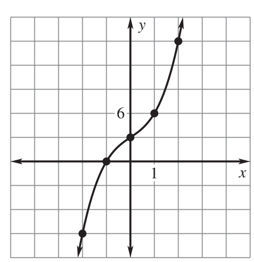
14. $f(x) = x^3 - 4$

x	−2	−1	0	1	2
***f*(*x*)**	−12	−5	−4	−3	4



15. $f(x) = x^3 + 2x + 3$

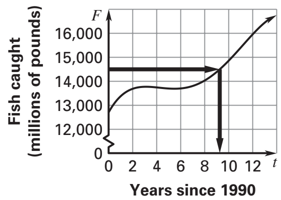
x	−2	−1	0	1	2
***f*(*x*)**	−9	0	3	6	15



16. $F = -0.907t^4 + 28.0t^3 - 258t^2 + 902t + 12{,}700$

t	0	2	4	6	8
$f(t)$	12,700	13,681	13,740	13697	14,025

t	10	12
$f(t)$	14,850	15,948



In 1999, the fish caught for human consumption was greater than 14.5 billion pounds.

17. $(5x^3 - x + 3) + (x^3 - 9x^2 + 4x)$

$= 5x^3 - x + 3 + x^3 - 9x^2 + 4x$

$= 6x^3 - 9x^2 + 3x + 3$

18. $(x^3 + 4x^2 - 5x) - (4x^3 + x^2 - 7)$

$= x^3 + 4x^2 - 5x - 4x^3 - x^2 + 7$

$= -3x^3 + 3x^2 - 5x + 7$

19. $(x - 6)(5x^2 + x - 8) = (x - 6)5x^2 + (x - 6)x - (x - 6)8$

$= 5x^3 - 30x^2 + x^2 - 6x - 8x + 48$

$= 5x^3 - 29x^2 - 14x + 48$

20. $(x - 4)(x + 7)(5x - 1) = (x^2 + 3x - 28)(5x - 1)$

$= (5x - 1)x^2 + (5x - 1)3x - (5x - 1)28$

$= 5x^3 - x^2 + 15x^2 - 3x - 140x + 28$

$= 5x^3 + 14x^2 - 143x + 28$

21. $64x^3 - 8 = (4x - 2)(16x^2 + 8x + 4)$

$= 8(2x - 1)(4x^2 + 2x + 1)$

22. $2x^5 - 12x^3 + 10x = 2x(x^4 - 6x^2 + 5)$

$= 2x(x^2 - 1)(x^2 - 5)$

$= 2x(x - 1)(x + 1)(x^2 - 5)$

23. $2x^3 - 7x^2 - 8x + 28 = x^2(2x - 7) - 4(2x - 7)$

$= (2x - 7)(x^2 - 4)$

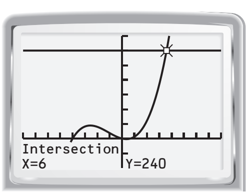
$= (2x - 7)(x - 2)(x + 2)$

24. $V = \ell wh$

When $\ell = x$, $w = x - 4$, and $h = 3x + 2$:

$V = x(x - 4)(3x + 2) = 3x^3 - 20x^2 - 8x$

Graph $y = 3x^2 - 10x^2 - 8x$ and $y = 240$.



From the graph, the volume of the sculpture is 240 cubic inches when $x = 6$. So, the dimensions of the sculpture should be 6 inches long by $6 - 4 = 2$ inches wide, and $3(6) + 2 = 20$ inches high.

25.
$$\begin{array}{r}
x - 6 \\
x^2 + 3x - 1\overline{)x^3 - 3x^2 - x - 10} \\
\underline{x^3 + 3x^2 - x\phantom{{}-10}} \\
-6x^2 + 0x - 10 \\
\underline{-6x^2 - 18x + 6} \\
18x - 16
\end{array}$$

So, $\dfrac{x^3 - 3x^2 - x - 10}{x^2 + 3x - 1} = x - 6 + \dfrac{18x - 16}{x^2 + 3x - 1}$.

26.
$$\begin{array}{r}
2x^2 \phantom{{}+9x} - \frac{13}{2} \\
2x^2 - 2\overline{)4x^4 - 0x^3 - 17x^2 + 9x - 18} \\
\underline{4x^4 \phantom{{}-0x^3} - 4x^2\phantom{{}+9x-18}} \\
-13x^2 + 9x - 18 \\
\underline{-13x^2 + 0x + 13} \\
9x - 31
\end{array}$$

So, $\dfrac{4x^4 - 17x^2 + 9x - 18}{2x^2 - 2} = 2x^2 - \dfrac{13}{2} + \dfrac{9x - 31}{2x^2 - 2}$.

27.
$$\begin{array}{r|rrrr}
5 & 2 & -11 & 13 & -44 \\
 & & 10 & -5 & 40 \\
\hline
 & 2 & -1 & 8 & -4
\end{array}$$

So, $\dfrac{2x^3 - 11x^2 + 13x - 44}{x - 5} = 2x^2 - x + 8 - \dfrac{4}{x - 5}$.

28.
$$\begin{array}{r|rrrrr}
-2 & 5 & 0 & 2 & -15 & 10 \\
 & & -10 & 20 & -44 & 118 \\
\hline
 & 5 & -10 & 22 & -59 & 128
\end{array}$$

So, $\dfrac{5x^4 + 2x^2 - 15x + 10}{x + 2} =$

$5x^3 - 10x^2 + 22x - 59 + \dfrac{128}{(x + 2)}$.

29.
$$\begin{array}{r|rrrr}
-2 & 1 & -5 & -2 & 24 \\
 & & -2 & 14 & -24 \\
\hline
 & 1 & -7 & 12 & 0
\end{array}$$

$f(x) = x^3 - 5x^2 - 2x + 24 = (x + 2)(x^2 - 7x + 12)$

$= (x + 2)(x - 3)(x - 4)$

30.
$$\begin{array}{r|rrrr}
8 & 1 & -11 & 14 & 80 \\
 & & 8 & -24 & -80 \\
\hline
 & 1 & -3 & -10 & 0
\end{array}$$

$f(x) = x^3 - 11x^2 + 14x + 80$

$= (x - 8)(x^2 - 3x - 10)$

$= (x - 8)(x - 5)(x + 2)$

31. $\begin{array}{r|rrrr} 1 & 9 & -9 & -4 & 4 \\ & & 9 & 0 & -4 \\ \hline & 9 & 0 & -4 & 0 \end{array}$

$f(x) = 9x^3 - 9x^2 - 4x + 4 = (x - 1)(9x^2 - 4)$
$= (x - 1)(3x - 2)(3x + 2)$

32. $\begin{array}{r|rrrr} -6 & 2 & 7 & -33 & -18 \\ & & -12 & 30 & 18 \\ \hline & 2 & -5 & -3 & 0 \end{array}$

$f(x) = 2x^3 + 7x^2 - 33x - 18$
$= (x + 6)(2x^2 - 5x - 3)$
$= (x + 6)(2x + 1)(x - 3)$

33. $f(x) = x^3 - 4x^2 - 11x + 30$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

Test $x = 2$:

$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$

$f(x) = (x - 2)(x^2 - 2x - 15) = (x - 2)(x + 3)(x - 5)$

The zeros are -3, 2, and 5.

34. $f(x) = 2x^4 - x^3 - 42x + 16x + 160$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 16, \pm 20, \pm 32, \pm 40, \pm 80, \pm 160, \pm \frac{1}{2}, \pm \frac{5}{2}$

Test $x = 4$:

$\begin{array}{r|rrrrr} 4 & 2 & -1 & -42 & 16 & 160 \\ & & 8 & 28 & -56 & -160 \\ \hline & 2 & 7 & -14 & -40 & 0 \end{array}$

$f(x) = (x - 4)(2x^3 + 7x^2 - 14x - 40)$

$\begin{array}{r|rrrr} \frac{5}{2} & 2 & 7 & -14 & -40 \\ & & 5 & 30 & 40 \\ \hline & 2 & 12 & 16 & 0 \end{array}$

$f(x) = (x - 4)\left(x - \frac{5}{2}\right)(2x^2 + 12x + 16)$
$= 2(x - 4)(2x - 5)(x^2 + 6x + 8)$
$= 2(x - 4)(2x - 5)(x + 4)(x + 2)$

The zeros are $-4, -2, \frac{5}{2}$, and 4.

35. $f(x) = (x + 4)(x - 1)(x - 5)$
$= (x^2 + 3x - 4)(x - 5)$
$= x^3 - 2x^2 - 19x + 20$

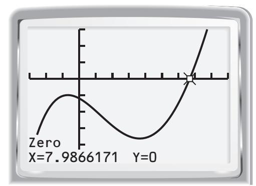
36. $f(x) = (x + 1)(x + 1)(x - 6)(x - 3i)(x + 3i)$
$= (x^2 + 2x + 1)(x - 6)[x^2 - (3i)^2]$
$= (x^3 - 4x^2 - 11x - 6)(x^2 + 9)$
$= x^5 - 4x^4 - 2x^3 - 42x^2 - 99x - 54$

37. $f(x) = (x - 2)(x - 7)[x - (3 - \sqrt{5})][x - (3 + \sqrt{5})]$
$= (x^2 - 9x + 14)[(x - 3) + \sqrt{5}][(x - 3) - \sqrt{5}]$
$= (x^2 - 9x + 14)[(x - 3)^2 - (\sqrt{5})^2]$
$= (x^2 - 9x + 14)(x^2 - 6x + 4)$
$= x^4 - 15x^3 + 72x^2 - 120x + 56$

38. $R = -0.0040t^4 + 0.088t^3 - 0.36t^2 - 0.55t + 5.8$

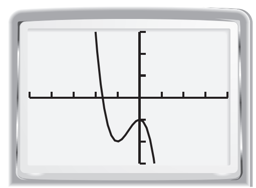
When $R = 7$:

$0 = -0.0040t^4 + 0.088t^3 - 0.3t^2 - 0.55t - 1.2$



From the graph, there is only one real zero: $t \approx 7.99$. So, the revenue became greater than 7 million near the end of the seventh year.

39.



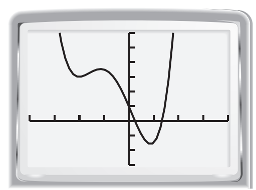
$f(x) = -2x^3 - 3x^2 - 1$

x-intercept: $x \approx -1.68$

Local maximum: $(0, -1)$

Local minimum: $(-1, -2)$

40.



$f(x) = x^4 + 3x^3 - x^2 - 8x + 2$

x-intercepts: $x \approx 0.25$, $x \approx 1.34$

Local maximum: $(-1.13, 7.06)$

Local minimums: $(-2, 6)$, $(0.88, -3.17)$

41.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	
-6	-21	-40	-57	-66	-61	
-15	-19	-17	-9	5		1st order differences
-4	2	8	14			2nd order differences
6	6	6				3rd order differences

Cubic function: $f(x) = ax^3 + bx^2 + cx + d$

$$a(1)^3 + b(1)^2 + c(1) + d = -6$$
$$a + b + c + d = -6$$
$$a(2)^3 + b(2)^2 + c(2) + d = -21$$
$$8a + 4b + 2c + d = -21$$
$$a(3)^3 + b(3)^2 + c(3) + d = -40$$
$$27a + 9b + 3c + d = -40$$
$$a(4)^3 + b(4)^2 + c(4) + d = -57$$
$$64a + 16b + 4c + d = -57$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -6 \\ -21 \\ -40 \\ -57 \end{bmatrix}$$

$$A \qquad X = B$$
$$X = A^{-1}B$$

Using a graphing calculator, the solution is $a = 1$, $b = -8$, $c = 2$, and $d = -1$. A polynomial function is $f(x) = x^3 - 8x^2 + 2x - 1$.

Chapter 5 Test (p. 407)

1. $x^3 \cdot x^2 \cdot x^{-4} = x^{3+2-4}$ Product of powers property

$= x$ Simplify exponents

2. $(2x^{-2}y^3)^{-5}$

$= 2^{-5}(x^{-2})^{-5}(y^3)^{-5}$ Power of a product property

$= 2^{-5}x^{10}y^{-15}$ Power of a power property

$= \dfrac{x^{10}}{2^5y^{15}}$ Negative exponent property

$= \dfrac{x^{10}}{32y^{15}}$ Simplify and evaluate power.

3. $\left(\dfrac{x^{-4}}{y^2}\right)^{-2} = \dfrac{(x^{-4})^{-2}}{(y^2)^{-2}}$ Power of a quotient property

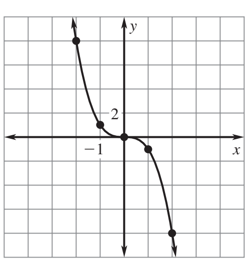
$= \dfrac{x^8}{y^{-4}}$ Power of a power property

$= x^8y^4$ Negative exponent property

4. $\dfrac{3(xy)^3}{27x - 5y^3} = \dfrac{3x^3y^3}{27x - 5y^3}$ Power of a product property

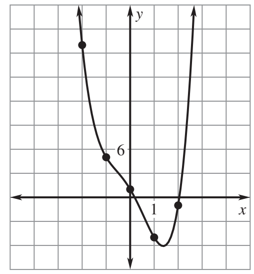
5. $f(x) = -x^3$

x	-2	-1	0	1	2
$f(x)$	8	1	0	-1	-8



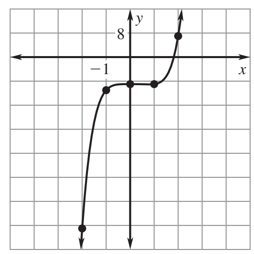
6. $f(x) = x^4 - 2x^2 - 5x + 1$

x	-2	-1	0	1	2
$f(x)$	19	5	1	-5	-1



7. $f(x) = x^5 - x^4 - 9$

x	-2	-1	0	1	2
$f(x)$	-57	-11	-9	-9	7



8. $(2x^3 + 5x^2 - 7x + 4) + (x^3 - 3x^2 - 4x)$

$= 2x^3 + 5x^2 - 7x + 4 + x^3 - 3x^2 - 4x$

$= 3x^3 + 2x^2 - 11x + 4$

9. $(3x^3 - 4x^2 + 3x - 5) - (x^2 + 4x - 8)$

$= 3x^3 - 4x^2 + 3x - 5 - x^2 - 4x + 8$

$= 3x^3 - 5x^2 - x + 3$

10. $(3x - 2)(x^2 + 4x - 7)$

$= (3x - 2)x^2 + (3x - 2)4x - (3x - 2)7$

$= 3x^3 - 2x^2 + 12x^2 - 8x - 21x + 14$

$= 3x^3 + 10x^2 - 29x + 14$

11. $(3x - 5)^3 = (3x)^3 - 3(3x)^2(5) + 3(3x)(5^2) - 5^3$

$= 27x^3 - 15(9x^2) + 9x(25) - 125$

$= 27x^3 - 135x^2 + 225x - 125$

12.
$$\begin{array}{r|rrrr} 4 & 3 & -14 & 16 & -22 \\ & & 12 & -8 & 32 \\ \hline & 3 & -2 & 8 & 10 \end{array}$$

So, $\dfrac{3x^3 - 14x^2 + 16x - 22}{x - 4} = 3x^2 - 2x + 8 + \dfrac{10}{x - 4}$.

13.
$$\begin{array}{r} 2x^2 + 2x + 3 \\ 3x^2 - 3x + 2 \overline{\big)\, 6x^4 + 0x^3 + 7x^2 + 4x - 17} \\ \underline{6x^4 - 6x^3 + 4x^2} \\ 6x^3 + 3x^2 + 4x \\ \underline{6x^3 - 6x^2 + 4x} \\ 9x^2 + 0x - 17 \\ \underline{9x^2 - 9x + 6} \\ 9x - 23 \end{array}$$

So, $\dfrac{6x^4 + 7x^2 + 4x - 17}{3x^2 - 3x + 2} = 2x^2 + 2x + 3 + \dfrac{9x - 23}{3x^2 - 3x + 2}$.

14. $8x^3 + 27 = (2x)^3 + 3^3 = (2x + 3)(4x^2 - 6x + 9)$

15. $x^4 + 5x^2 - 6 = (x^2 - 1)(x^2 + 6)$
$= (x - 1)(x + 1)(x^2 + 6)$

16. $x^3 - 3x^2 - 4x + 12 = x^2(x - 3) - 4(x - 3)$
$= (x - 3)(x^2 - 4)$
$= (x - 3)(x - 2)(x + 2)$

17. $f(x) = x^3 + x^2 - 22x - 40$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm8, \pm10, \pm20, \pm40$

Test $x = -2$:
$$\begin{array}{r|rrrr} -2 & 1 & 1 & -22 & -40 \\ & & -2 & 2 & 40 \\ \hline & 1 & -1 & -20 & 0 \end{array}$$

$f(x) = (x + 2)(x^2 - x - 20) = (x + 2)(x - 5)(x + 4)$

The real zeros are -2, 5, and -4.

18. $f(x) = 4x^4 - 8x^3 - 19x^2 + 23x - 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

Test $x = -2$:
$$\begin{array}{r|rrrrr} -2 & 4 & -8 & -19 & 23 & -6 \\ & & -8 & 32 & -26 & 6 \\ \hline & 4 & -16 & 13 & -3 & 0 \end{array}$$

$f(x) = (x + 2)(4x^3 - 16x^2 + 13x - 3)$

Test $x = 3$:
$$\begin{array}{r|rrrr} 3 & 4 & -16 & 13 & -3 \\ & & 12 & -12 & 3 \\ \hline & 4 & -4 & 1 & 0 \end{array}$$

$f(x) = (x + 2)(x - 3)(4x^2 - 4x + 1)$
$= 4(x + 2)(x - 3)\left(x - \frac{1}{2}\right)^2$

The real zeros are -2, $\frac{1}{2}$, and 3.

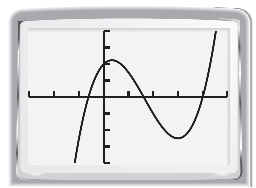
19. $f(x) = (x + 1)(x - 3)(x - 4)$
$= (x^2 - 2x - 3)(x - 4)$
$= x^3 - 2x^2 - 3x - 4x^2 + 8x + 12$
$= x^3 - 6x^2 + 5x + 12$

20. $f(x) = (x - 6)(x - 2i)(x + 2i)$
$= (x - 6)[x^2 - (2i)^2]$
$= (x - 6)(x^2 + 4)$
$= x^3 - 6x^2 + 4x - 24$

21. $f(x) = (x + 3)(x + 1)[x - (1 - \sqrt{5})][x - (1 + \sqrt{5})]$
$= (x^2 + 4x + 3)[(x - 1) + \sqrt{5}][(x - 1) - \sqrt{5}]$
$= (x^2 + 4x + 3)[(x - 1)^2 - (\sqrt{5})^2]$
$= (x^2 + 4x + 3)(x^2 - 2x - 4)$
$= x^4 + 2x^3 - 9x^2 - 22x - 12$

22. $f(x) = [x - (1 + 3i)][x - (1 - 3i)][x - (4 + \sqrt{10})][x - (4 - \sqrt{10})]$
$= [(x - 1) - 3i][(x - 1) + 3i][(x - 4) - \sqrt{10}][(x - 4) + \sqrt{10}]$
$= [(x - 1)^2 - (3i)^2][(x - 4)^2 - (\sqrt{10})^2]$
$= (x^2 - 2x + 10)(x^2 - 8x + 6)$
$= x^4 - 10x^3 + 32x^2 - 92x + 60$

23. $f(x) = x^3 - 5x^2 + 3x + 4$

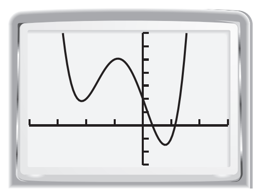


x-intercepts: $x \approx -0.6$, $x \approx 1.6$, $x = 4$

Local maximum: (0.3, 4.5)

Local minimum: $(3, -5)$

24. $f(x) = x^4 + 3x^3 - x^2 - 6x + 2$



x-intercepts: $x \approx 0.34$, $x \approx 1.14$

Local maximum: $(-0.88, 5.06)$

Local minimums: $(-2.16, 1.83)$, $(0.79, -1.50)$

25.

	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
	3	1	1	3	7	13
1st order differences		-2	0	2	4	6
2nd order differences			2	2	2	2

Quadratic function: $f(x) = ax^2 + bx + c$

$a(1)^2 + b(1) + c = 3 \rightarrow a + b + c = 3$

$a(2)^2 + b(2) + c = 1 \rightarrow 4a + 2b + c = 1$

$a(3)^2 + b(3) + c = 1 \rightarrow 9a + 3b + c = 1$

$a + b + c = 3 \rightarrow c = 3 - a - b$

$4a + 2b + c = 1$

$9a + 3b + c = 1$

$4a + 2b + (3 - a - b) = 1$

$3a + b = -2$

$9a + 3b + (3 - a - b) = 1$

$8a + 2b = -2$

$3a + b = -2 \rightarrow b = -3a - 2$

$8a + 2b = -2$

$8a + 2(-3a - 2) = -2 \rightarrow a = 1$

$b = -3(1) - 2 = -5$

$c = 3 - 1 - (-5) = 7$

The quadratic function is $f(x) = x^2 - 5x + 7$.

26.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	
0	-7	-4	21	80	185	
-7	3	25	59	105		1st order differences
10	22	34	46			2nd order differences
12	12	12				3rd order differences

Cubic function: $f(x) = ax^3 + bx^2 + cx + d$

$a(1)^3 + b(1)^2 + c(1) + d = 0 \rightarrow$

$a + b + c + d = 0$

$a(2)^3 + b(2)^2 + c(2) + d = -7 \rightarrow$

$8a + 4b + 2c + d = -7$

$a(3)^3 + b(3)^2 + c(3) + d = -4 \rightarrow$

$27a + 9b + 3c + d = -4$

$a(4)^3 + b(4)^2 + c(4) + d = 21 \rightarrow$

$64a + 16b + 4c + d = 21$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}} \underset{=}{=} \underset{B}{\begin{bmatrix} 0 \\ -7 \\ -4 \\ 21 \end{bmatrix}}$$

$$X = A^{-1}B$$

Using a graphing calculator, the solution is $a = 2$, $b = -7$, $c = 0$, $d = 5$. A cubic function is $f(x) = 2x^3 - 7x^2 + 5$.

27. Per Capita GDP $= \dfrac{\text{GDP}}{\text{Population}}$

$= \dfrac{1.099 \times 10^{13}}{2.91 \times 10^8}$

$= \dfrac{1.099}{2.91} \times \dfrac{10^{13}}{10^8}$

$= 0.378 \times 10^5$

$= 3.78 \times 10^4$

28. Let V represent the total number of U.S. households with both television and VCRs.

$V = T \cdot P$

$$\begin{array}{r} -0.205x^2 + 8.36x + 1.98 \\ \times \qquad\qquad 1.22x + 76.9 \\ \hline -15.76x^2 + 642.9x + 152.26 \\ -0.250x^3 + 10.20x^2 + 2.42x \qquad\qquad \\ \hline -0.250x^3 - 5.56x^2 + 645.3x + 152.26 \end{array}$$

The number of households with televisions and VCRs can be modeled by $V(x) = -0.250x^3 - 5.56x^2 + 645.3x + 152.26$.

29. $\begin{array}{c}\text{Volume} \\ \text{(Cubic} \\ \text{inch)}\end{array} = \begin{array}{c}\text{Length} \\ \text{(inch)}\end{array} \cdot \begin{array}{c}\text{Width} \\ \text{(inch)}\end{array} \cdot \begin{array}{c}\text{Height} \\ \text{(inch)}\end{array}$

$1040 = x \cdot (x + 2) \cdot (2x - 3)$

$1040 = 2x^3 + x^2 - 6x$

$0 = 2x^3 + x^2 - 6x - 1040$

Test possible rational solutions:

$$\begin{array}{r|rrrr} 8 & 2 & 1 & -6 & -1040 \\ & & 16 & 136 & 1040 \\ \hline & 2 & 17 & 130 & 0 \end{array}$$

$0 = (x - 8)(2x^2 + 17x + 130)$

$x - 8 = 0$ or $2x^2 + 17x + 130 = 0$

$x = 8$ or $x = \dfrac{-17 \pm \sqrt{17^2 - 4(2)(130)}}{2(2)}$

$= \dfrac{-17 \pm i\sqrt{751}}{4}$

The only reasonable solution is $x = 8$. So, the volume of the prism can be modeled by $V(x) = 2x^3 + x^2 - 6x$. When $V = 1040$, the prism has a length of 8 in., width of $8 + 2 = 10$ in., and a height of $2(8) - 3 = 13$ in.

Standardized Test Preparation (p. 409)

1. The solution would receive full credit. The solution is complete, correct, and the steps are clearly shown and described.
2. The solution would receive partial credit. The solution is complete and has errors. There are errors in the values in the table. The solution is incorrect because of those values.

Standardized Test Practice (pp. 410–411)

1. When $x = 0$:

$y = -0.0006(0)^3 + 0.0383(0)^2 + 0.383(0) + 68.6$

$= 68.6$

When $x = 20$:

$y = -0.0006(20)^3 + 0.0383(20)^2 + 0.383(20) + 68.6$

$= -4.8 + 15.32 + 7.66 + 68.6 = 86.78$

The difference of the number of voters in 1980 and 1960 can be found by subtracting the y-value when $x = 0$ from the y-value when $x = 20$.

Difference $= 86.78 - 68.6 = 18.18$

There were 18.18 million more people voting in 1980 than in 1960.

2. $f(x) = -3x^4 - 2x^3 + 7x^2 - 9$

Synthetic substitution:

$$\begin{array}{r|rrrrr} -4 & -3 & -2 & 7 & 0 & -9 \\ & & 12 & -40 & 132 & -528 \\ \hline & -3 & 10 & -33 & 132 & -537 \end{array}$$

$f(-4) = -537$

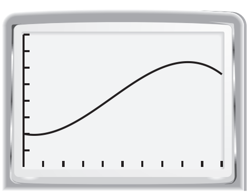
Direct substitution:

$f(-4) = -3(-4)^4 - 2(-4)^3 + 7(-4)^2 - 9$

$= -768 + 128 + 112 - 9 = -537$

3. The graph shows the degree of the function is odd and the leading coefficient is negative because $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. The graph crosses the x-axis five times, so the function has five real zeros.

4. $E = -0.0007t^3 + 0.0278t^2 - 0.0843t + 12.0$



Turning points:

Local minimum: (1.61, 11.93)

Local maximum: (24.86, 16.33)

The fuel efficiency increased steadily after 1971 and then began to decrease after 1995.

5. No. The rational zero theorem can be used only when the coefficients of the terms are integers. The coefficient of the x^2-term is 1.5.

6. $P = -x^3 + 3x^2 + 15x$

When $P = 25$: $25 = -x^3 + 3x^2 + 15x$

$0 = -x^3 + 3x^2 + 15x - 25$

Because $x = 5$ is a solution, then $x - 5$ is a factor of the equation.

$$\begin{array}{r|rrrr} 5 & -1 & 3 & 15 & -25 \\ & & -5 & -10 & 25 \\ \hline & -1 & -2 & 5 & 0 \end{array}$$

So, $(x - 5)(-x^2 - 2x + 5) = 0$

$x - 5 = 0$ or $-x^2 - 2x + 5 = 0$

$x = 5$ or $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(5)}}{2(-1)}$

$= \dfrac{2 \pm 2\sqrt{6}}{-2} = -1 \pm \sqrt{6}$

The solutions are 5, $-1 - \sqrt{6} \approx -3.4$, and $-1 + \sqrt{6} \approx 1.4$. The negative solution can be rejected because you cannot make a negative number of coats. So, the manufacturer can produce about 1.4 million coats and make the same profit.

7. Volume of the prism $=$ (Area of the base)(Height)

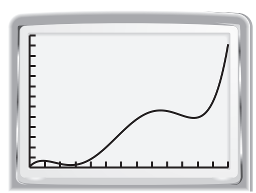
Area of the base $= \dfrac{\text{Volume of the prism}}{\text{Height}}$

$= \dfrac{2x^3 + 5x^2 - 19x - 42}{x - 3}$

$$\begin{array}{r|rrrr} 3 & 2 & 5 & -19 & -42 \\ & & 6 & 33 & 42 \\ \hline & 2 & 11 & 14 & 0 \end{array}$$

So, the area of the base is given by the expression $2x^2 + 11x + 14$.

8. a. $y = 0.014x^5 - 0.40x^4 + 3.8x^3 - 13x^2 + 15x + 1.2$



Using the graph, the most CDs were sold when $x = 13$, or in 2003.

b. No. Factors, such as the introduction of a newer technology, could prevent the model for CD sales from continuing indefinitely.

9.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	
4	9	26	57	104	169	
5	17	31	47	65		1st order differences
12	14	16	18			2nd order differences
2	2	2				3rd order differences

Cubic function: $f(x) = ax^3 + bx^2 + cx + d$

$a(1)^3 + b(1)^2 + c(1) + d = 4 \rightarrow$

$a + b + c + d = 4$

$a(2)^3 + b(2)^2 + c(2) + d = 9 \rightarrow$

$8a + 4b + 2c + d = 9$

$a(3)^3 + b(3)^2 + c(3) + d = 26 \rightarrow$

$27a + 9b + 3c + d = 26$

$a(4)^3 + b(4)^2 + c(4) + d = 57 \rightarrow$

$64a + 16b + 4c + d = 57$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 26 \\ 57 \end{bmatrix}$$

$$A \quad X = B$$

$$X = A^{-1}B$$

Using a graphing calculator, the solution is $a = \frac{1}{3}$, $b = 4$, $c = -\frac{28}{3}$, and $d = 9$. A cubic model is $f(x) = \frac{1}{3}x^3 + 4x^2 - \frac{28}{3}x + 9$.

$g(1)$ $g(2)$ $g(3)$ $g(4)$ $g(5)$ $g(6)$

−2 −2 12 52 130 258

0 14 40 78 128 1st order differences

14 26 38 50 2nd order differences

12 12 12 3rd order differences

Cubic function: $g(x) = ax^3 + bx^2 + cx + d$

$a(1)^3 + b(1)^2 + c(1) + d = -2 \rightarrow$

$a + b + c + d = -2$

$a(2)^3 + b(2)^2 + c(2) + d = -2 \rightarrow$

$8a + 4b + 2c + d = -2$

$a(3)^3 + b(3)^2 + c(3) + d = 12 \rightarrow$

$27a + 9b + 3c + d = 12$

$a(4)^3 + b(4)^2 + c(4) + d = 52 \rightarrow$

$64a + 16b + 4c + d = 52$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 12 \\ 52 \end{bmatrix}$$

$$C \quad X = D$$

$$X = C^{-1}D$$

Using a graphing calculator, the solution is $a = 2$, $b = -5$, $c = 1$, and $d = 0$. A cubic model is $g(x) = 2x^3 - 5x^2 + x$. So, $f(x) + g(x) = \frac{1}{3}x^3 + 4x^2 - \frac{28}{3}x + 9 + (2x^3 - 5x^2 + x) = \frac{7}{3}x^3 - x^2 - \frac{25}{3}x + 9$.

10. Population density $= \dfrac{\text{Population}}{\text{Square miles}}$

In 1800: $Pd = \dfrac{5.308 \times 10^6}{8.647 \times 10^5} = \dfrac{5.308}{8.847} \times \dfrac{10^6}{10^5} \approx 0.6 \times 10^1$

The population density in 1800 was about 6 people per square mile.

In 2000: $Pd = \dfrac{2.814 \times 10^8}{3.537 \times 10^6}$

$= \dfrac{2.814}{3.537} \times \dfrac{10^8}{10^6} = 0.800 \times 10^2$

$= 8.00 \times 10^1$

The population density in 2000 was about 80 people per square mile. The increase in population density is the population density in 2000 minus the population density in 1800. So, the population density increased by $80 - 6 = 74$ people per square mile from 1800 to 2000.

11. D; $\dfrac{(x^2yz)^3}{x^4y^2z^7} = \dfrac{x^6y^3z^3}{x^4y^2z^7} = x^{6-4}y^{3-2}z^{3-7} = x^2y^1z^{-4} = \dfrac{x^2y}{z^4}$

12. C; $x^4 = 125x$

$x^4 - 125x = 0$

$x(x^3 - 125) = 0$

$x = 0$ or $x^3 - 125 = 0$

$x^3 = 125$

$x = \sqrt[3]{125} = 5$

13. A; $f(x) = (x + 1)(x - 3)(x - 4i)(x + 4i)$

$= (x^2 - 2x - 3)[x^2 - (4i)^2]$

$= (x^2 - 2x - 3)(x^2 + 16)$

$= x^4 - 2x^3 + 13x^2 - 32x - 48$

14. Degree 6; the degree of the polynomial is equal to the highest degree of the two polynomials.

15. $f(x) = 2x^4 + 3x^2 - 1$

The coefficients of $f(x)$ have 1 sign change, so f has 1 positive real zero.

$f(-x) = 2(-x)^4 + 3(-x)^2 - 1 = 2x^4 + 3x^2 - 1$

The coefficients of $f(-x)$ have 1 sign change, so f has 1 negative real zero. So, f has two real zeros.

16. $\left(\frac{3}{2}\right)^{-2} = \dfrac{3^{-2}}{2^{-2}} = \dfrac{2^2}{3^2} = \dfrac{4}{9}$

17. $f(x) = 4x^4 - x^3 + 2x^2 - 3$ when $x = 4$:

4	4	−1	2	0	−3
		16	60	248	992
	4	15	62	248	989

So, $f(4) = 989$.

18. A quadratic function has exactly four solutions. The graph crosses the x-axis twice, so the function has two real zeros. The function has $4 - 2 = 2$ imaginary zeros.

19. $f(x) = 15x^4 + 6x^3 - 8x^2 - 9$

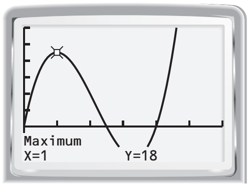
Possible rational zeros: $\pm 1, \pm 3, \pm 9, \pm\frac{1}{3}, \pm\frac{1}{5}, \pm\frac{3}{5}, \pm\frac{9}{5}, \pm\frac{1}{15}$

There are 16 possible rational zeros of the function.

20. a.

Volume (cubic inches)	=	Length (inches)	•	Width (inches)	•	Height (inches)
V	=	$(8 - 2x)$	•	$(5 - 2x)$	•	x

$$= (40 - 26x + 4x^2)x$$
$$= 4x^3 - 26x^2 + 40x$$



Consider only the interval $0 < x < 2.5$ because this describes the physical restrictions on the size of the flaps.

From the graph, you can see that the maximum volume occurs when $x = 1$. So, you should make the cuts 1 inch long.

b. From the graph, the maximum volume is 18 cubic inches.

c. The dimensions of a box with this volume will be $8 - 2(1) = 6$ inches long, $5 - 2(1) = 3$ inches wide, and 1 inch high.

21. a. Let T represent the total number of hospital beds in U.S. hospitals.

$T = H \cdot B$

$$\begin{array}{rrrrr}
 & 0.0066t^3 - & 0.192t^2 - & 0.174t + 196 & \\
\times & & & -58.7t + 7070 & \\
\hline
 & 46.662t^3 - & 1357.44t^2 - & 1230.18t + 1{,}385{,}720 & \\
-0.387t^4 + & 11.3t^3 + & 10.2t^2 - & 11{,}505t & \\
\hline
-0.387t^4 + & 58t^3 - & 1347t^2 - & 12{,}735t + 1{,}385{,}720 &
\end{array}$$

So, a model for the total number of hospital beds is $T(t) = -0.387t^4 + 58t^3 - 1347t^2 - 12{,}735t + 1{,}385{,}720$.

b. When $t = 15$, $T(t) \approx 1{,}070{,}000$.

So, there were 1,070,000 hospital beds in 1995.

c. To find the number of hospital beds in thousands, divide the function by 1000.

$$T(t) = \frac{-0.387t^4 + 58t^3 - 1347t^2 - 12{,}735t + 1{,}385{,}720}{1000}$$

So, $T(t) = -0.000387t^4 + 0.058t^3 - 1.35t^2 - 12.74t + 1386$, where $T(t)$ is the number of hospital beds (in thousands).