Chapter 4

Prerequisite Skills (p. 234)

1. The x-intercept of the line shown is 3.

2. The y-intercept of the line shown is 2.

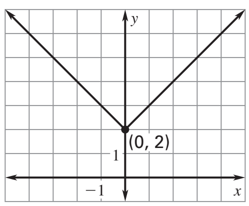
3. $-5x^2 + 1 = -5(-3)^2 + 1$
$= -5(9) + 1$
$= -45 + 1$
$= -44$

4. $x^2 - x - 8 = (-3)^2 - (-3) - 8$
$= 9 - (-3) - 8$
$= 9 + 3 - 8$
$= 4$

5. $(x + 4)^2 = (-3 + 4)^2 = (1)^2 = 1$

6. $-3(x - 7)^2 + 2 = -3(-3 - 7)^2 + 2$
$= -3(-10)^2 + 2$
$= -3(100) + 2$
$= -300 + 2$
$= -298$

7.



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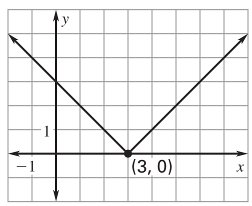

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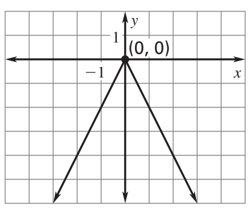

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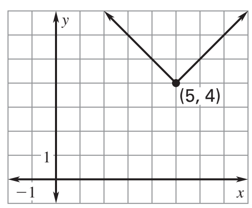


11. $x + 8 = 0$
$x = -8$

12. $3x - 5 = 0$
$3x = 5$
$x = \frac{5}{3}$

13. $2x + 1 = x$
$x + 1 = 0$
$x = -1$

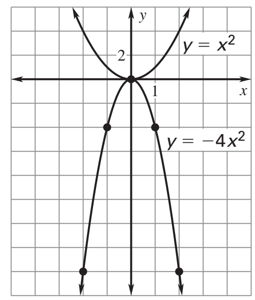
14. $4(x - 3) = x + 9$
$4x - 12 = x + 9$
$3x - 12 = 9$
$3x = 21$
$x = 7$

Lesson 4.1

4.1 Guided Practice (pp. 237–239)

1. $y = -4x^2$

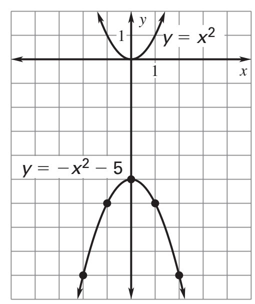
x	−2	−1	0	1	2
y	−16	−4	0	−4	−16



Both graphs have the same vertex and axis of symmetry. However, the graph of $y = -4x^2$ opens down and is narrower than the graph of $y = x^2$.

2. $y = -x^2 - 5$

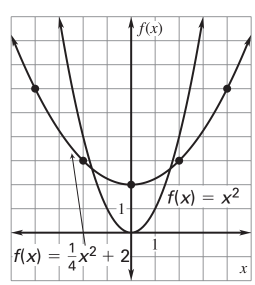
x	−2	−1	0	1	2
y	−9	−6	−5	−6	−9



Both graphs have the same axis of symmetry. However, the graph of $y = -x^2 - 5$ opens down, and its vertex is 5 units lower.

3. $f(x) = \frac{1}{4}x^2 + 2$

x	−4	−2	0	2	4
y	6	3	2	3	6



Both graphs open up and have the same axis of symmetry. However, the graph of $f(x) = \frac{1}{4}x^2 + 2$ is wider than the graph of $f(x) = x^2$, and its vertex is 2 units higher.

4. $y = x^2 - 2x - 1$

$x = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = 1$

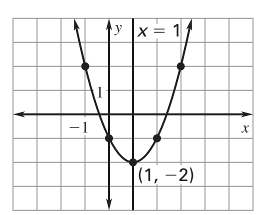
$y = (1)^2 - 2(1) - 1 = -2$

Vertex: $(1, -2)$

Axis of symmetry: $x = 1$

y-intercept: -1; $(0, -1)$

$x = -1$: $y = (-1)^2 - 2(-1) - 1 = 2$; $(-1, 2)$



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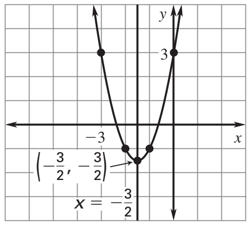
5. $y = 2x^2 + 6x + 3$

$x = -\frac{b}{2a} = -\frac{6}{2(2)} = -\frac{3}{2}$

$y = 2\left(-\frac{3}{2}\right)^2 + 6\left(-\frac{3}{2}\right) + 3$

$= -\frac{3}{2}$



Vertex: $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

Axis of symmetry: $x = -\frac{3}{2}$

y-intercept: 3; (0, 3)

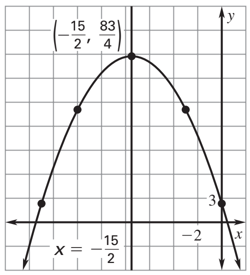
$x = -1$: $y = 2(-1)^2 + 6(-1) + 3 = -1$; $(-1, -1)$

6. $f(x) = -\frac{1}{3}x^2 - 5x + 2$

$x = -\frac{b}{2a} = -\frac{(-5)}{2\left(-\frac{1}{3}\right)} = -\frac{15}{2}$

$y = -\frac{1}{3}\left(-\frac{15}{2}\right)^2 - 5\left(-\frac{15}{2}\right) + 2$

$= \frac{83}{4}$



Vertex: $\left(-\frac{15}{2}, \frac{83}{4}\right)$

Axis of symmetry: $x = -\frac{15}{2}$

y-intercept: 2; (0, 2)

$x = -3$: $y = -\frac{1}{3}(-3)^2 - 5(-3) + 2 = 14$; $(-3, 14)$

7. $y = 4x^2 + 16x - 3$

$x = -\frac{b}{2a} = -\frac{16}{2(4)} = -2$

$y = 4(-2)^2 + 16(-2) - 3 = -19$

The minimum value is $y = -19$.

8. $R(x) = (35 - x) \cdot (380 + 40x)$

$R(x) = 13{,}300 + 1400x - 380x - 40x^2$

$R(x) = -40x^2 + 1020x + 13{,}300$

$x = \frac{b}{2a} = -\frac{1020}{2(-40)} = 12.75$

$R(12.75) = -40(12.75)^2 + 1020(12.75) + 13{,}300$

$= 19{,}802.5$

The vertex is (12.75, 19,802.5), which means the owner should reduce the price per racer by \$12.75 to increase the weekly revenue to \$19,802.50.

4.1 Exercises (pp. 240–243)

Skill Practice

1. The graph of a quadratic function is called a parabola.

2. Look at the value of a in the quadratic function. If $a > 0$, the function has a minimum value. If $a < 0$, the function has a maximum value.

3. $y = 4x^2$

x	−2	−1	0	1	2
y	16	4	0	4	16

4. $y = -3x^2$

x	−2	−1	0	1	2
y	−12	−3	0	−3	−12

5. $y = \frac{1}{2}x^2$

x	−4	−2	0	2	4
y	8	2	0	2	8

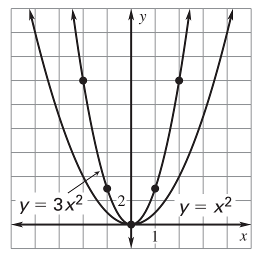
6. $y = -\frac{1}{3}x^2$

x	−6	−3	0	3	6
y	−12	−3	0	−3	−12

7. $y = 3x^2$

x	−2	−1	0	1	2
y	12	3	0	3	12

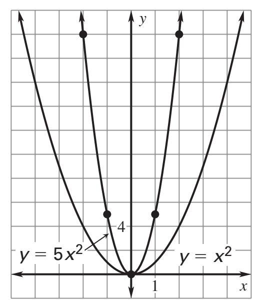
Both graphs open up and have the same vertex and axis of symmetry. However, the graph of $y = 3x^2$ is narrower than the graph of $y = x^2$.



8. $y = 5x^2$

x	−2	−1	0	1	2
y	20	5	0	5	20

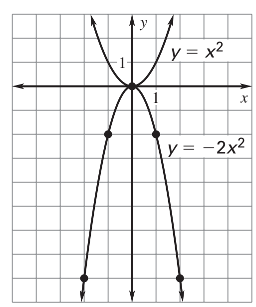
Both graphs open up and have the same vertex and axis of symmetry. However, the graph of $y = 5x^2$ is narrower than the graph of $y = x^2$.



9. $y = -2x^2$

x	−2	−1	0	1	2
y	−8	−2	0	−2	−8

Both graphs have the same vertex and axis of symmetry. However, the graph of $y = -2x^2$ opens down and is narrower than the graph of $y = x^2$.



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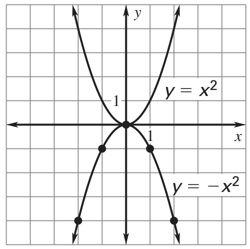
10. $y = -x^2$

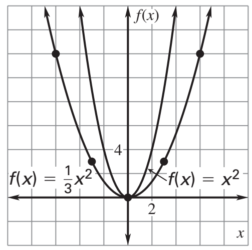
x	-2	-1	0	1	2
y	-4	-1	0	-1	-4



Both graphs have the same vertex and axis of symmetry. However, the graph of $y = -x^2$ opens down.

11. $f(x) = \frac{1}{3}x^2$

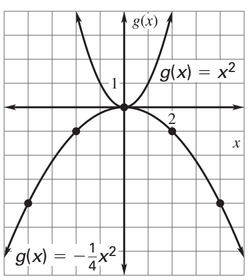
x	-6	-3	0	3	6
$f(x)$	12	3	0	3	12



Both graphs open up and have the same vertex and axis of symmetry. However, the graph of $f(x) = \frac{1}{3}x^2$ is wider than the graph of $f(x) = x^2$.

12. $g(x) = -\frac{1}{4}x^2$

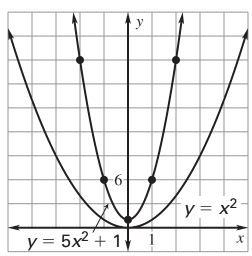
x	-4	-2	0	2	4
$g(x)$	-4	-1	0	-1	-4



Both graphs have the same vertex and axis of symmetry. However, the graph of $g(x) = -\frac{1}{4}x^2$ opens down and is wider than the graph of $g(x) = x^2$.

13. $y = 5x^2 + 1$

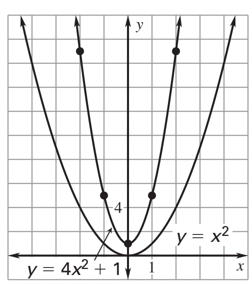
x	-2	-1	0	1	2
y	21	6	1	6	21



Both graphs open up and have the same axis of symmetry. However, the graph of $y = 5x^2 + 1$ is narrower than the graph of $y = x^2$ and its vertex is 1 unit higher.

14. $y = 4x^2 + 1$

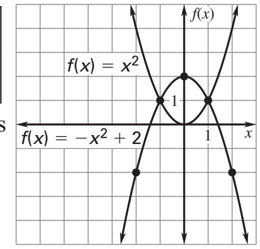
x	-2	-1	0	1	2
y	17	5	1	5	17



Both graphs open up and have the same axis of symmetry. However, the graph of $y = 4x^2 + 1$ is narrower than the graph of $y = x^2$ and its vertex is 1 unit higher.

15. $f(x) = -x^2 + 2$

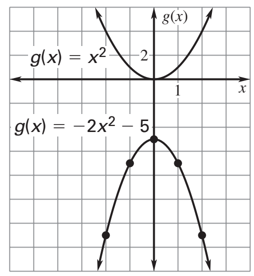
x	-2	-1	0	1	2
$f(x)$	-2	1	2	1	-2



Both graphs have the same axis of symmetry. However, the graph of $f(x) = -x^2 + 2$ opens down and its vertex is 2 units higher.

16. $g(x) = -2x^2 - 5$

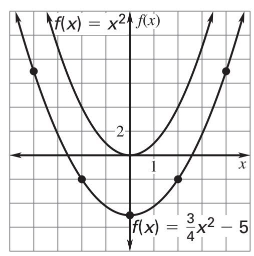
x	-2	-1	0	1	2
$g(x)$	-13	-7	-5	-7	-13



Both graphs have the same axis of symmetry. However, the graph of $g(x) = -2x^2 - 5$ opens down and is narrower than the graph of $g(x) = x^2$. Also, its vertex is 5 units lower.

17. $f(x) = \frac{3}{4}x^2 - 5$

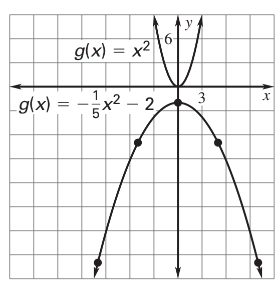
x	-4	-2	0	2	4
$f(x)$	7	-2	-5	-2	7



Both graphs open up and have the same axis of symmetry. However, the graph of $f(x) = \frac{3}{4}x^2 - 5$ is wider than the graph of $f(x) = x^2$ and its vertex is 5 units lower.

18. $g(x) = -\frac{1}{5}x^2 - 2$

x	-10	-5	0	5	10
$g(x)$	-22	-7	-2	-7	-22



Both graphs have the same axis of symmetry. However, the graph of $g(x) = -\frac{1}{5}x^2 - 2$ opens down and is wider than the graph of $g(x) = x^2$. Also, its vertex is 2 units lower.

19. The x-coordinate of the vertex of a parabola is $-\frac{b}{2a}$, not $\frac{b}{2a}$. The x-coordinate of the vertex is:

$$x = -\frac{b}{2a} = -\frac{24}{2(4)} = -3.$$

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20. It is correct that the y-intercept of the graph is the value of c. However, the value of c in $y = 4x^2 + 24x - 7$ is -7.

21. $y = x^2 + 2x + 1$

$x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$

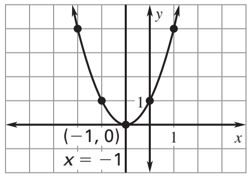
$y = (-1)^2 + 2(-1) + 1 = 0$

Vertex: $(-1, 0)$

Axis of symmetry: $x = -1$

y-intercept: 1; $(0, 1)$

$x = 1$: $y = 1^2 + 2(1) + 1 = 4$; $(1, 4)$



22. $y = 3x^2 - 6x + 4$

$x = -\frac{b}{2a} = -\frac{(-6)}{2(3)} = 1$

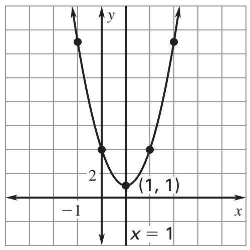
$y = 3(1)^2 - 6(1) + 4 = 1$

Vertex: $(1, 1)$

Axis of symmetry: $x = 1$

y-intercept: 4; $(0, 4)$

$x = -1$: $y = 3(-1)^2 - 6(-1) + 4 = 13$; $(-1, 13)$



23. $y = -4x^2 + 8x + 2$

$x = -\frac{b}{2a} = -\frac{8}{2(-4)} = 1$

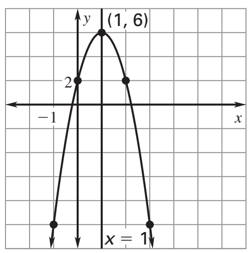
$y = -4(1)^2 + 8(1) + 2 = 6$

Vertex: $(1, 6)$

Axis of symmetry: $x = 1$

y–intercept: 2; $(0, 2)$

$x = -1$: $y = -4(-1)^2 + 8(-1) + 2 = -10$; $(-1, -10)$



24. $y = -2x^2 - 6x + 3$

$x = -\frac{b}{2a} = -\frac{(-6)}{2(-2)} = -\frac{3}{2}$

$y = -2\left(-\frac{3}{2}\right)^2 - 6\left(-\frac{3}{2}\right) + 3$

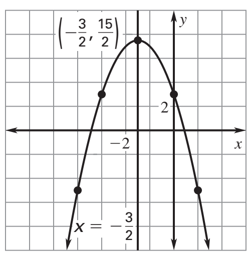
$= \frac{15}{2}$

Vertex: $\left(-\frac{3}{2}, \frac{15}{2}\right)$

Axis of symmetry: $x = -\frac{3}{2}$

y-intercept: 3; $(0, 3)$

$x = 1$: $y = -2(1)^2 - 6(1) + 3 = -5$; $(1, -5)$



25. $g(x) = -x^2 - 2x - 1$

$x = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$

$g(-1) = -(-1)^2 - 2(-1) - 1$

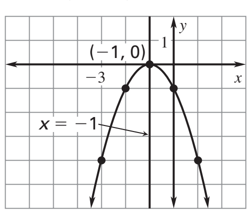
$= 0$

Vertex: $(-1, 0)$

Axis of symmetry: $x = -1$

y-intercept: -1; $(0, -1)$

$x = 1$: $g(1) = -(1)^2 - 2(1) - 1 = -4$; $(1, -4)$



26. $f(x) = -6x^2 - 4x - 5$

$x = -\frac{b}{2a} = -\frac{(-4)}{2(-6)} = -\frac{1}{3}$

$f\left(-\frac{1}{3}\right) = -6\left(-\frac{1}{3}\right)^2 - 4\left(-\frac{1}{3}\right) - 5 = -\frac{13}{3}$

Vertex: $\left(-\frac{1}{3}, -\frac{13}{3}\right)$

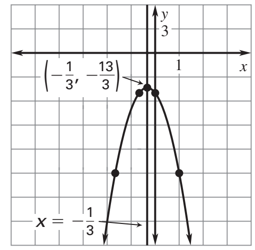
Axis of symmetry: $x = -\frac{1}{3}$

y-intercept: -5; $(0, -5)$

$x = 1$:

$f(1) = -6(1)^2 - 4(1) - 5$

$= -15$; $(1, -15)$



27. $y = \frac{2}{3}x^2 - 3x + 6$

$x = -\frac{b}{2a} = -\frac{(-3)}{2\left(\frac{2}{3}\right)} = \frac{9}{4}$

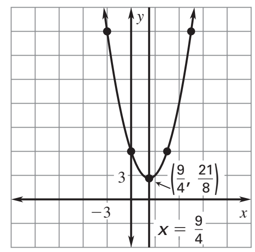
$y = \frac{2}{3}\left(\frac{9}{4}\right)^2 - 3\left(\frac{9}{4}\right) + 6 = \frac{21}{8}$

Vertex: $\left(\frac{9}{4}, \frac{21}{8}\right)$

Axis of symmetry: $x = \frac{9}{4}$

y-intercept: 6; $(0, 6)$

$x = -3$: $y = \frac{2}{3}(-3)^2 - 3(-3) + 6 = 21$; $(-3, 21)$



28. $y = -\frac{3}{4}x^2 - 4x - 1$

$x = -\frac{b}{2a} = -\frac{(-4)}{2\left(-\frac{3}{4}\right)} = -\frac{8}{3}$

$y = -\frac{3}{4}\left(-\frac{8}{3}\right)^2 - 4\left(-\frac{8}{3}\right) - 1$

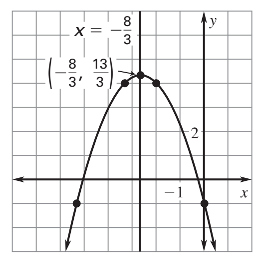
$= \frac{13}{3}$

Vertex: $\left(-\frac{8}{3}, \frac{13}{3}\right)$

Axis of symmetry: $x = -\frac{8}{3}$

y-intercept: -1; $(0, -1)$

$x = -2$: $y = -\frac{3}{4}(-2)^2 - 4(-2) - 1 = 4$; $(-2, 4)$

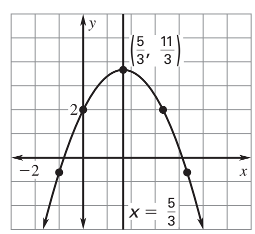


29. $g(x) = -\frac{3}{5}x^2 + 2x + 2$

$x = -\frac{b}{2a} = -\frac{2}{2\left(-\frac{3}{5}\right)} = \frac{5}{3}$

$g\left(\frac{5}{3}\right) = -\frac{3}{5}\left(\frac{5}{3}\right)^2 + 2\left(\frac{5}{3}\right) + 2$

$= \frac{11}{3}$



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Vertex: $\left(\frac{5}{3}, \frac{11}{3}\right)$

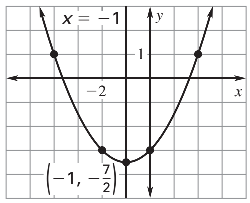
Axis of symmetry: $x = \frac{5}{3}$

y-intercept: 2; (0, 2)

$x = -1$: $g(x) = -\frac{3}{5}(-1)^2 + 2(-1) + 2 = -\frac{3}{5}$;

$\left(-1, -\frac{3}{5}\right)$

30. $f(x) = \frac{1}{2}x^2 + x - 3$



$x = -\frac{b}{2a} = -\frac{1}{2\left(\frac{1}{2}\right)} = -1$

$f(-1) = \frac{1}{2}(-1)^2 + (-1) - 3$

$= -\frac{7}{2}$

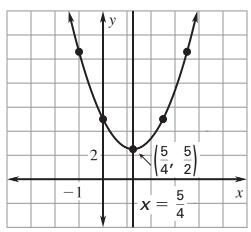
Vertex: $\left(-1, -\frac{7}{2}\right)$

Axis of symmetry: $x = -1$

y-intercept: -3; $(0, -3)$

$x = 2$: $f(2) = \frac{1}{2}(2)^2 + 2 - 3 = 1$; (2, 1)

31. $y = \frac{8}{5}x^2 - 4x + 5$



$x = -\frac{b}{2a} = -\frac{(-4)}{2\left(\frac{8}{5}\right)} = \frac{5}{4}$

$y = \frac{8}{5}\left(\frac{5}{4}\right)^2 - 4\left(\frac{5}{4}\right) + 5 = \frac{5}{2}$

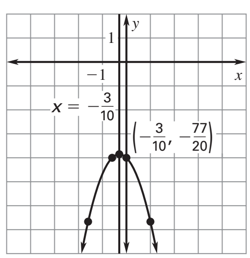
Vertex: $\left(\frac{5}{4}, \frac{5}{2}\right)$

Axis of symmetry: $x = \frac{5}{4}$

y-intercept: 5; (0, 5)

$x = -1$: $y = \frac{8}{5}(-1)^2 - 4(-1) + 5 = \frac{53}{5}$; $\left(-1, \frac{53}{5}\right)$

32. $y = -\frac{5}{3}x^2 - x - 4$



$x = -\frac{b}{2a} = -\frac{(-1)}{2\left(-\frac{5}{3}\right)} = -\frac{3}{10}$

$y = -\frac{5}{3}\left(-\frac{3}{10}\right)^2 - \left(-\frac{3}{10}\right) - 4$

$= -\frac{77}{20}$

Vertex: $\left(-\frac{3}{10}, -\frac{77}{20}\right)$

Axis of symmetry: $x = -\frac{3}{10}$

y-intercept: -4; $(0, -4)$

$x = 1$: $y = -\frac{5}{3}(1)^2 - 1 - 4 = -\frac{20}{3}$; $\left(1, -\frac{20}{3}\right)$

33. $y = -6x^2 - 1$

Because $a < 0$, the function has a maximum value.

$x = -\frac{b}{2a} = -\frac{0}{2(-6)} = 0$

$y = -6(0)^2 - 1 = -1$

The maximum value is $y = -1$.

34. $y = 9x^2 + 7$

Because $a > 0$, the function has a minimum value.

$x = -\frac{b}{2a} = -\frac{0}{2(9)} = 0$

$y = 9(0)^2 + 7 = 7$

The minimum value is $y = 7$.

35. $f(x) = 2x^2 + 8x + 7$

Because $a > 0$, the function has a minimum value.

$x = -\frac{b}{2a} = -\frac{8}{2(2)} = -2$

$f(-2) = 2(-2)^2 + 8(-2) + 7 = -1$

The minimum value is $f(x) = -1$.

36. $g(x) = -3x^2 + 18x - 5$

Because $a < 0$, the function has a maximum value.

$x = -\frac{b}{2a} = -\frac{18}{2(-3)} = 3$

$g(3) = -3(3)^2 + 18(3) - 5 = 22$

The maximum value is $g(x) = 22$.

37. $f(x) = \frac{3}{2}x^2 + 6x + 4$

Because $a > 0$, the function has a minimum value.

$x = -\frac{b}{2a} = -\frac{6}{2\left(\frac{3}{2}\right)} = -2$

$f(-2) = \frac{3}{2}(-2)^2 + 6(-2) + 4 = -2$

The minimum value is $f(x) = -2$.

38. $y = -\frac{1}{4}x^2 - 7x + 2$

Because $a < 0$, the function has a maximum value.

$x = -\frac{b}{2a} = -\frac{(-7)}{2\left(-\frac{1}{4}\right)} = -14$

$y = -\frac{1}{4}(-14)^2 - 7(-14) + 2 = 51$

The maximum value is $y = 51$.

39. D; Because the y-intercept changes from 2 to -3, the vertex moves down the y-axis.

40. C; The graph of $y = ax^2 + bx + c$ is wider than the graph of $y = x^2$ if $|a| < 1$.

41. $y = -0.02x^2 + x + 6$

$a = -0.02$

$b = 1$

$c = 6$

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42. $y = -0.01x^2 + 0.7x + 6$

$a = -0.01$

$b = 0.7$

$c = 6$

43. Vertex: $(4, k)$

$-\frac{b}{2a} = 4 \rightarrow -\frac{b}{a} = 8$

Sample answer: $y = x^2 - 8x + 1$

$y = -2x^2 + 16x - 3$

$y = -\frac{1}{2}x^2 + 4x + 5$

44. C; $y = 0.5x^2 - 2x$

$x = -\frac{b}{2a} = -\frac{(-2)}{2(0.5)} = 2$

$y = 0.5(2)^2 - 2(2) = -2$

Vertex: $(2, -2)$

45. A; $y = 0.5x^2 + 3$

$x = -\frac{b}{2a} = -\frac{0}{2(0.5)} = 0$

$y = 0.5(0)^2 + 3 = 3$

Vertex: $(0, 3)$

46. B; $y = 0.5x^2 - 2x + 3$

$x = -\frac{b}{2a} = -\frac{(-2)}{2(0.5)} = 2$

$y = 0.5(2)^2 - 2(2) + 3 = 1$

Vertex: $(2, 1)$

47. $f(x) = 0.1x^2 + 2$

$x = -\frac{b}{2a} = -\frac{0}{2(0.1)} = 0$

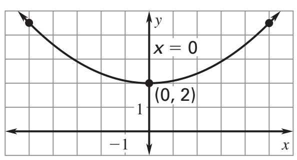
$f(0) = 0.1(0)^2 + 2 = 2$

Vertex: $(0, 2)$

Axis of symmetry: $x = 0$

$x = 5$: $f(5) = 0.1(5)^2 + 2$

$= 4.5$; $(5, 4.5)$



48. $g(x) = -0.5x^2 - 5$

$x = -\frac{b}{2a} = -\frac{0}{2(-0.5)} = 0$

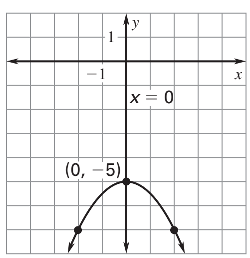
$g(0) = -0.5(0)^2 - 5 = -5$

Vertex: $(0, -5)$

Axis of symmetry: $x = 0$

$x = 2$: $g(2) = -0.5(2)^2 - 5$

$= -7$; $(2, -7)$



49. $y = 0.3x^2 + 3x - 1$

$x = -\frac{b}{2a} = -\frac{3}{2(0.3)} = -5$

$y = 0.3(-5)^2 + 3(-5) - 1$

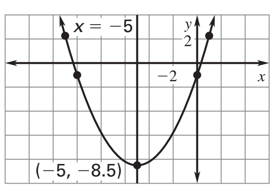
$= -8.5$

Vertex: $(-5, -8.5)$

Axis of symmetry: $x = -5$

y-intercept: -1; $(0, -1)$

$x = 1$: $y = 0.3(1)^2 + 3(1) - 1 = 2.3$; $(1, 2.3)$



50. $y = 0.25x^2 - 1.5x + 3$

$x = -\frac{b}{2a} = -\frac{(-1.5)}{2(0.25)} = 3$

$y = 0.25(3)^2 - 1.5(3) + 3$

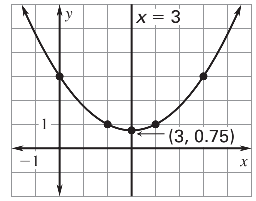
$= 0.75$

Vertex: $(3, 0.75)$

Axis of symmetry: $x = 3$

y-intercept: 3; $(0, 3)$

$x = 2$: $y = 0.25(2)^2 - 1.5(2) + 3 = 1$; $(2, 1)$



51. $f(x) = 4.2x^2 + 6x - 1$

$x = -\frac{b}{2a}$

$= -\frac{6}{2(4.2)} = -\frac{5}{7}$

$f\left(-\frac{5}{7}\right) = 4.2\left(-\frac{5}{7}\right)^2 + 6\left(-\frac{5}{7}\right) - 1$

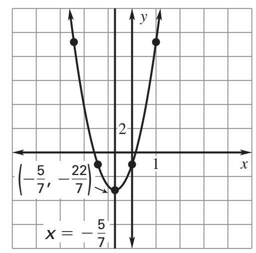
$= -\frac{22}{7}$

Vertex: $\left(-\frac{5}{7}, -\frac{22}{7}\right)$

Axis of symmetry: $x = -\frac{5}{7}$

y-intercept: -1; $(0, -1)$

$x = 1$: $f(1) = 4.2(1)^2 + 6(1) - 1 = 9.2$; $(1, 9.2)$



52. $g(x) = 1.75x^2 - 2.5$

$x = -\frac{b}{2a} = -\frac{0}{2(1.75)} = 0$

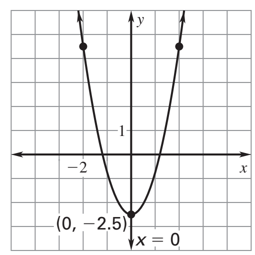
$g(0) = 1.75(0)^2 - 2.5$

$= -2.5$

Vertex: $(0, -2.5)$

Axis of symmetry: $x = 0$

$x = 2$: $g(2) = 1.75(2)^2 - 2.5 = 4.5$; $(2, 4.5)$



53. Because the points $(2, 3)$ and $(-4, 3)$ have the same y-value and lie on the graph of a quadratic function, they are mirror images of each other. The axis of symmetry divides a parabola into mirror images, therefore, the axis of symmetry is halfway between the x-values. The axis of symmetry is $x = -1$.

$x = \frac{2 + -4}{2} = -1$

54. $y = ax^2 + bx + c$

The x-coordinate of the vertex is $-\frac{b}{2a}$.

$y = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c = \frac{ab^2}{4a^2} + \frac{-b^2}{2a} + c$

$= \frac{ab^2 - 2ab^2}{4a^2} + c = \frac{a(b^2 - 2b^2)}{4a^2} + c = -\frac{b^2}{4a} + c$

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Problem Solving

55. $R(x) = (1 + 0.05x) \cdot (4000 - 80x)$

$R(x) = 4000 - 80x + 200x - 4x^2$

$R(x) = -4x^2 + 120x + 4000$

$x = -\frac{b}{2a} = -\frac{120}{2(-4)} = 15$

$R(15) = -4(15)^2 + 120(15) + 4000 = 4900$

Price: $1 + 0.05x \rightarrow$

$1 + 0.05(15) = 1.75$

The store should increase the price per song to \$1.75 to increase the daily revenue to \$4900.

56.

Revenue (dollars)	=	Price (dollars/camera)	•	Sales (cameras)
$R(x)$	=	$(320 - 20x)$	•	$(70 + 5x)$

$R(x) = 22{,}400 + 1600x - 1400x - 100x^2$

$R(x) = -100x^2 + 200x + 22{,}400$

$x = -\frac{b}{2a} = -\frac{200}{2(-100)} = 1$

$R(1) = -100(1)^2 + 200(1) + 22{,}400 = 22{,}500$

Price: $320 - 20x \rightarrow$

$320 - 20(1) = 300$

The store should decrease the price per digital camera to \$300 to increase the monthly revenue to \$22,500.

57. $y = \frac{1}{9000}x^2 - \frac{7}{15}x + 500$

$x = -\frac{b}{2a} = -\frac{\left(-\frac{7}{15}\right)}{2\left(\frac{1}{9000}\right)} = 2100$

$y = \frac{1}{9000}(2100)^2 - \frac{7}{15}(2100) + 500 = 10$

The height above the road of a cable at its lowest point is 10 feet.

58. $y = -0.2x^2 + 1.3x$

$x = -\frac{b}{2a} = -\frac{1.3}{2(-0.2)} = 3.25$

$y = -0.2(3.25)^2 + 1.3(3.25) \approx 2.1$

No, the mouse cannot jump over a fence that is 3 feet high because the maximum height it can jump is about 2.1 feet.

59. a.

Profit (dollars)	=	Price (dollars/ticket)	•	Sales (tickets)	−	Expenses (dollars)
$P(x)$	=	$(20 - x)$	•	$(150 + 10x)$	−	1500

$= 3000 + 200x - 150x - 10x^2 - 1500$

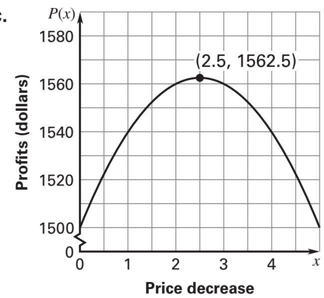
$= -10x^2 + 50x + 1500$

b.

x	0	1	2	3	4	5	6	7
P(x)	1500	1540	1560	1560	1540	1500	1440	1360

x	2.5
P(x)	1562.5

c.



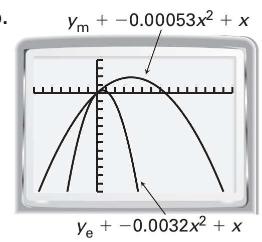
The theater should reduce the price per ticket by \$2.50 to increase the weekly profit to \$1562.50.

60. $y = -\frac{g}{10{,}000}x^2 + x$

a. $y_e = -\frac{32}{10{,}000}x^2 + x = -0.0032x^2 + x$

$y_m = -\frac{5.3}{10{,}000}x^2 + x = -0.00053x^2 + x$

b.



The golf ball travels 312.5 feet on Earth.
The golf ball travels 1886.8 feet on the moon.

c. A golf ball travels $\frac{1886.8}{312.5}$ or about 6 times further on the moon than on Earth. Smaller values of g produce longer distances.

61. $P = 2w + \ell$

$P - 2w = \ell$

$A = \ell w = (P - 2w)w = Pw - 2w^2 = -2w^2 + Pw$

$w = -\frac{b}{2a} = -\frac{P}{2(-2)} = \frac{1}{4}P$

$A = -2\left(\frac{1}{4}P\right)^2 + P\left(\frac{1}{4}P\right) = -\frac{1}{8}P^2 + \frac{1}{4}P^2 = \frac{1}{8}P^2$

In terms of P, the maximum area that the swimming section can have is $\frac{1}{8}P^2$ ft^2.

Mixed Review

62. $x - 3 = 0$

$x = 3$

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63. $3x + 4 = 0$

$3x = -4$

$x = -\frac{4}{3}$

64. $-9x + 7 = -4x - 5$

$7 = 5x - 5$

$12 = 5x$

$\frac{12}{5} = x$

65. $5x - 2 = -2x + 12$

$7x - 2 = 12$

$7x = 14$

$x = 2$

66. $0.7x + 3 = 0.2x - 2$

$0.5x + 3 = -2$

$0.5x = -5$

$x = -10$

67. $0.4x = -0.5x - 5$

$0.9x = -5$

$10 \cdot 0.9x = 10 \cdot (-5)$

$9x = -50$

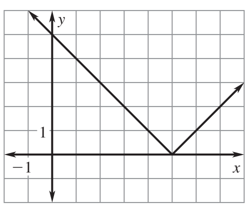
$x = -\frac{50}{9} = -5\frac{5}{9}$

68. $y = |x - 5|$

Vertex: $(5, 0)$

$x = 4$: $y = |4 - 5|$

$= 4; (4, 1)$

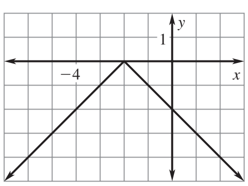


69. $y = -|x + 2|$

Vertex: $(-2, 0)$

$x = -1$: $y = -|-1 + 2|$

$= -1; (-1, -1)$

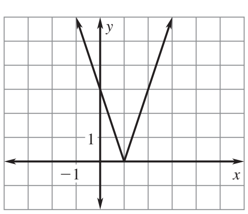


70. $y = 3|x - 1|$

Vertex: $(1, 0)$

$x = 2$: $y = 3|2 - 1|$

$= 3; (2, 3)$

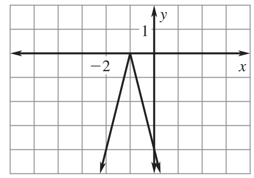


71. $y = -4|x + 1|$

Vertex: $(-1, 0)$

$x = -2$: $y = -4|-2 + 1|$

$= -4; (-2, -4)$

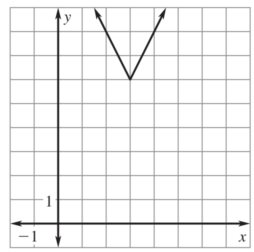


72. $f(x) = 2|x - 3| + 6$

Vertex: $(3, 6)$

$x = 2$: $f(2) = 2|2 - 3| + 6$

$= 8; (2, 8)$

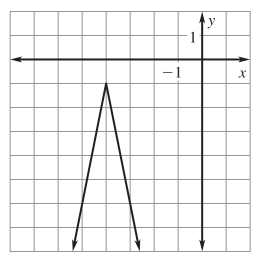


73. $g(x) = -5|x + 4| - 1$

Vertex: $(-4, -1)$

$x = -3$: $y = -5|-3 + 4| - 1$

$= -6; (-3, -6)$



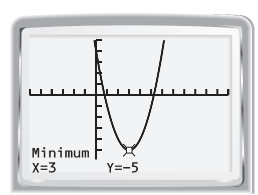
74. Average speed $= \frac{\text{change in distance}}{\text{change in time}}$

$= \frac{340 \text{ mi} - 70 \text{ mi}}{5 \text{ h}} = 54$ miles per hour

Graphing Calculator Activity 4.1 (p. 244)

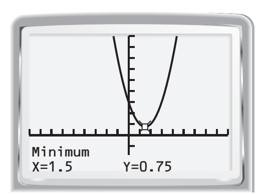
1. $y = x^2 - 6x + 4$

The minimum value of the function is $y = -5$ and occurs at $x = 3$.



2. $f(x) = x^2 - 3x + 3$

The minimum value of the function is $f(x) = 0.75$ and occurs at $x = 1.5$.



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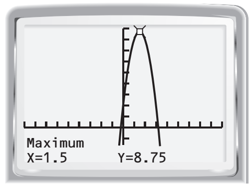
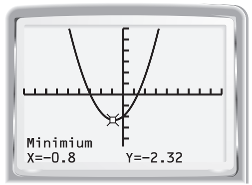
3. $y = -3x^2 + 9x + 2$

The maximum value of the function is $y = 8.75$ and occurs at $x = 1.5$.



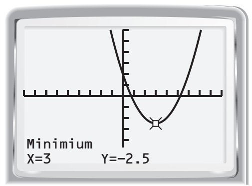
4. $y = 0.5x^2 + 0.8x - 2$

The minimum value of the function is $y = -2.32$ and occurs at $x = -0.8$.



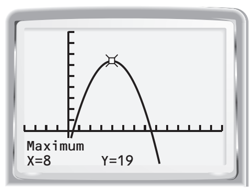
5. $h(x) = \frac{1}{2}x^2 - 3x + 2$

The minimum value of the function is $h(x) = -2.5$ and occurs at $x = 3$.



6. $y = -\frac{3}{8}x^2 + 6x - 5$

The maximum value of the function is $y = 19$ and occurs at $x = 8$.



Lesson 4.2

4.2 Guided Practice (pp. 246–248)

1. $y = (x + 2)^2 - 3$

$a = 1, h = -2, k = -3$

Vertex: $(-2, -3)$

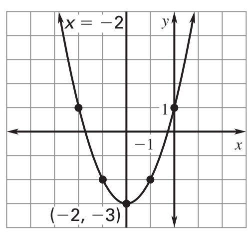
Axis of symmetry: $x = -2$

$x = 0$: $y = (0 + 2)^2 - 3$

$= 1; (0, 1)$

$x = -1$: $y = (-1 + 2)^2 - 3$

$= -2; (-1, -2)$



2. $y = -(x - 1)^2 + 5$

$a = -1, h = 1, k = 5$

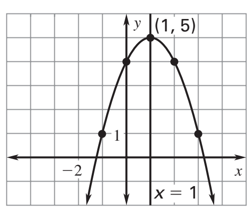
Vertex: $(1, 5)$

Axis of symmetry: $x = 1$

$x = 0$: $y = -(0 - 1)^2 + 5$

$= 4; (0, 4)$

$x = -1$: $y = -(-1 - 1)^2 + 5 = 1; (-1, 1)$



3. $f(x) = \frac{1}{2}(x - 3)^2 - 4$

$a = \frac{1}{2}, h = 3, k = -4$

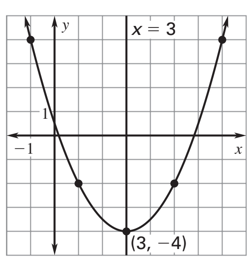
Vertex: $(3, -4)$

Axis of symmetry: $x = 3$

$x = 1$: $f(x) = \frac{1}{2}(1 - 3)^2 - 4$

$= -2; (1, -2)$

$x = -1$: $f(x) = \frac{1}{2}(-1 - 3)^2 - 4 = 4; (-1, 4)$



4. The graphs of both functions open up and have the same vertex and axis of symmetry. However, the a values of the functions differ. The graph of the function $y = \frac{1}{7000}(x - 1400)^2 + 27$ is wider than the graph of the function $y = \frac{1}{6500}(x - 1400)^2 + 27$.

5. $y = (x - 3)(x - 7)$

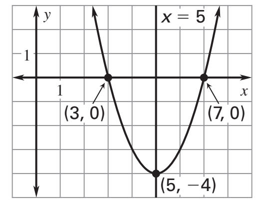
x-intercepts: $p = 3$ and $q = 7$

$x = \frac{p + q}{2} = \frac{3 + 7}{2} = 5$

$y = (5 - 3)(5 - 7) = -4$

Vertex: $(5, -4)$

Axis of symmetry: $x = 5$



6. $f(x) = 2(x - 4)(x + 1)$

x-intercepts: $p = 4$ and $q = -1$

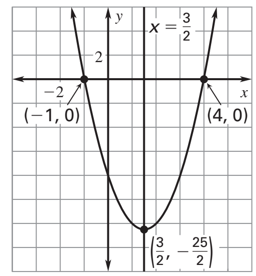
$x = \frac{p + q}{2} = \frac{4 + (-1)}{2} = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2} - 4\right)\left(\frac{3}{2} + 1\right)$

$= -\frac{25}{2}$

Vertex: $\left(\frac{3}{2}, -\frac{25}{2}\right)$

Axis of symmetry: $x = \frac{3}{2}$



7. $y = -(x + 1)(x - 5)$

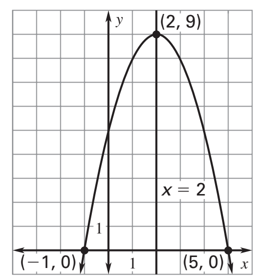
x-intercepts: $p = -1$ and $q = 5$

$x = \frac{p + q}{2} = \frac{-1 + 5}{2} = 2$

$y = -(2 + 1)(2 - 5) = 9$

Vertex: $(2, 9)$

Axis of symmetry: $x = 2$



8. $y = -0.025x(x - 50)$

$y = -0.025(x - 0)(x - 50)$

x-intercepts: $p = 0$ and $q = 50$

$x = \frac{p + q}{2} = \frac{0 + 50}{2} = 25$

$y = -0.025(25)(25 - 50) \approx 15.6$

The maximum height of the football is the y-coordinate of the vertex, or about 15.6 yards.

9. $y = -(x - 2)(x - 7)$

$= -(x^2 - 7x - 2x + 14)$

$= -(x^2 - 9x + 14)$

$= -x^2 + 9x - 14$

10. $y = -4(x - 1)(x + 3)$

$= -4(x^2 + 3x - x - 3)$

$= -4(x^2 + 2x - 3)$

$= -4x^2 - 8x + 12$

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11. $f(x) = 2(x + 5)(x + 4)$
$= 2(x^2 + 4x + 5x + 20)$
$= 2(x^2 + 9x + 20)$
$= 2x^2 + 18x + 40$

12. $y = -7(x - 6)(x + 1)$
$= -7(x^2 + x - 6x - 6)$
$= -7(x^2 - 5x - 6)$
$= -7x^2 + 35x + 42$

13. $y = -3(x + 5)^2 - 1$
$= -3(x + 5)(x + 5) - 1$
$= -3(x^2 + 5x + 5x + 25) - 1$
$= -3(x^2 + 10x + 25) - 1$
$= -3x^2 - 30x - 75 - 1$
$= -3x^2 - 30x - 76$

14. $g(x) = 6(x - 4)^2 - 10$
$= 6(x - 4)(x - 4) - 10$
$= 6(x^2 - 4x - 4x + 16) - 10$
$= 6(x^2 - 8x + 16) - 10$
$= 6x^2 - 48x + 96 - 10$
$= 6x^2 - 48x + 86$

15. $f(x) = -(x + 2)^2 + 4$
$= -(x + 2)(x + 2) + 4$
$= -(x^2 + 2x + 2x + 4) + 4$
$= -(x^2 + 4x + 4) + 4$
$= -x^2 - 4x - 4 + 4$
$= -x^2 - 4x$

16. $y = 2(x - 3)^2 + 9$
$= 2(x - 3)(x - 3) + 9$
$= 2(x^2 - 3x - 3x + 9) + 9$
$= 2(x^2 - 6x + 9) + 9$
$= 2x^2 - 12x + 18 + 9$
$= 2x^2 - 12x + 27$

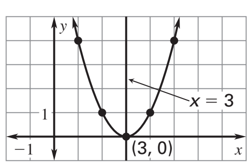
4.2 Exercises (pp. 249–251)

Skill Practice

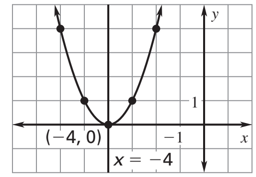
1. A quadratic function in the form $y = a(x - h)^2 + k$ is in vertex form.

2. First identify the x-intercepts. Then use the x-intercepts to calculate the x-coordinate of the vertex. Finally, substitute the x-coordinate of the vertex for x into the original function to find the y-coordinate of the vertex. The y-coordinate of the vertex is the maximum or minimum value.

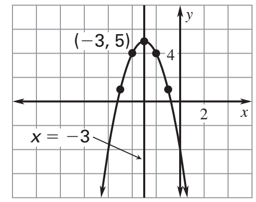
3. $y = (x - 3)^2$
$a = 1, h = 3, k = 0$
Vertex: $(3, 0)$
Axis of symmetry: $x = 3$
$x = 1$: $y = (1 - 3)^2 = 4$; $(1, 4)$
$x = 2$: $y = (2 - 3)^2 = 1$; $(2, 1)$



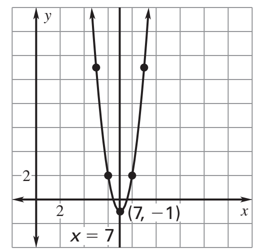
4. $y = (x + 4)^2$
$a = 1, h = -4, k = 0$
Vertex: $(-4, 0)$
Axis of symmetry: $x = -4$
$x = -2$: $y = (-2 + 4)^2$
$= 4$; $(-2, 4)$
$x = -3$: $y = (-3 + 4)^2$
$= 1$; $(-3, 1)$



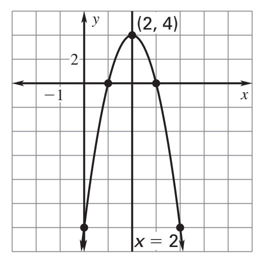
5. $f(x) = -(x + 3)^2 + 5$
$a = -1, h = -3, k = 5$
Vertex: $(-3, 5)$
Axis of symmetry: $x = -3$
$x = -1$:
$f(x) = -(-1 + 3)^2 + 5$
$= 1$; $(-1, 1)$
$x = -2$: $f(x) = -(-2 + 3)^2 + 5 = 4$; $(-2, 4)$



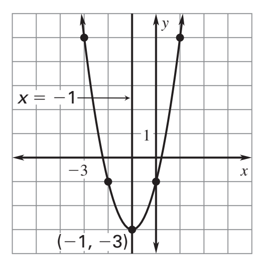
6. $y = 3(x - 7)^2 - 1$
$a = 3, h = 7, k = -1$
Vertex: $(7, -1)$
Axis of symmetry: $x = 7$
$x = 6$: $y = 3(6 - 7)^2 - 1$
$= 2$; $(6, 2)$
$x = 5$: $y = 3(5 - 7)^2 - 1$
$= 11$; $(5, 11)$



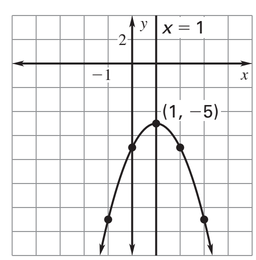
7. $g(x) = -4(x - 2)^2 + 4$
$a = -4, h = 2, k = 4$
Vertex: $(2, 4)$
Axis of symmetry: $x = 2$
$x = 3$: $g(x) = -4(3 - 2)^2 + 4$
$= 0$; $(3, 0)$
$x = 4$: $g(x) = -4(4 - 2)^2 + 4$
$= -12$; $(4, -12)$



8. $y = 2(x + 1)^2 - 3$
$a = 2, h = -1, k = -3$
Vertex: $(-1, -3)$
Axis of symmetry: $x = -1$
$x = 0$: $y = 2(0 + 1)^2 - 3$
$= -1$; $(0, -1)$
$x = 1$: $y = 2(1 + 1)^2 - 3$
$= 5$; $(1, 5)$



9. $f(x) = -2(x - 1)^2 - 5$
$a = -2, h = 1, k = -5$
Vertex: $(1, -5)$
Axis of symmetry: $x = 1$
$x = 0$: $f(x) = -2(0 - 1)^2 - 5$
$= -7$; $(0, -7)$
$x = -1$:
$f(x) = -2(-1 - 1)^2 - 5$
$= -13$; $(-1, -13)$



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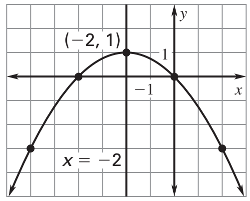
10. $y = -\frac{1}{4}(x + 2)^2 + 1$



$a = -\frac{1}{4}, h = -2, k = 1$

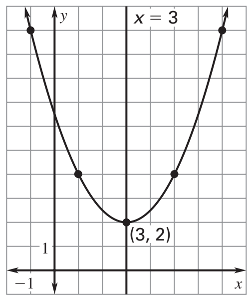
Vertex: $(-2, 1)$

Axis of symmetry: $x = -2$

$x = 0$: $y = -\frac{1}{4}(0 + 2)^2 + 1 = 0$; $(0, 0)$

$x = 2$: $y = -\frac{1}{4}(2 + 2)^2 + 1 = -3$; $(2, -3)$

11. $y = \frac{1}{2}(x - 3)^2 + 2$



$a = \frac{1}{2}, h = 3, k = 2$

Vertex: $(3, 2)$

Axis of symmetry: $x = 3$

$x = 1$: $y = \frac{1}{2}(1 - 3)^2 + 2$

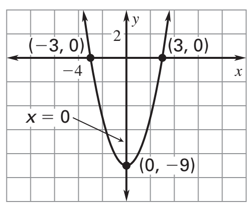
$= 4$; $(1, 4)$

$x = -1$: $y = \frac{1}{2}(-1 - 3)^2 + 2 = 10$; $(-1, 10)$

12. B; $y = 3(x + 2)^2 - 5$

The graph of $y = a(x - h)^2 + k$ has vertex (h, k). The vertex of the graph of the function is $(-2, -5)$.

13. $y = (x + 3)(x - 3)$



x-intercept: $p = -3$ and $q = 3$

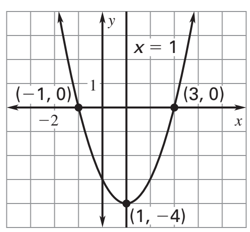
$x = \frac{p + q}{2} = \frac{-3 + 3}{2} = 0$

$y = (0 + 3)(0 - 3) = -9$

Vertex: $(0, -9)$

Axis of symmetry: $x = 0$

14. $y = (x + 1)(x - 3)$



x-intercept: $p = -1$ and $q = 3$

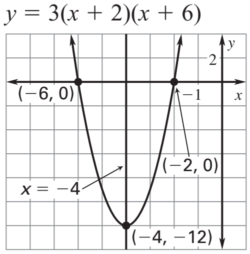
$x = \frac{p + q}{2} = \frac{-1 + 3}{2} = 1$

$y = (1 + 1)(1 - 3) = -4$

Vertex: $(1, -4)$

Axis of symmetry: $x = 1$

15. $y = 3(x + 2)(x + 6)$



x-intercept: $p = -2$ and $q = -6$

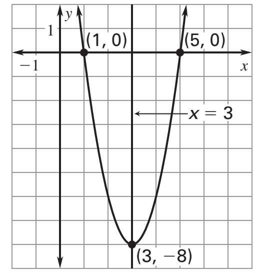
$x = \frac{p + q}{2} = \frac{-2 + (-6)}{2} = -4$

$y = 3(-4 + 2)(-4 + 6) = -12$

Vertex: $(-4, -12)$

Axis of symmetry: $x = -4$

16. $f(x) = 2(x - 5)(x - 1)$



x-intercept: $p = 5$ and $q = 1$

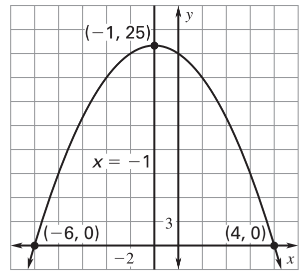
$x = \frac{p + q}{2} = \frac{5 + 1}{2} = 3$

$f(x) = 2(3 - 5)(3 - 1) = -8$

Vertex: $(3, -8)$

Axis of symmetry: $x = 3$

17. $y = -(x - 4)(x + 6)$



x-intercept: $p = 4$ and $q = -6$

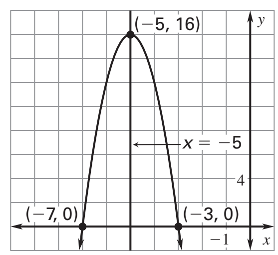
$x = \frac{p + q}{2} = \frac{4 + (-6)}{2} = -1$

$y = -(-1 - 4)(-1 + 6) = 25$

Vertex: $(-1, 25)$

Axis of symmetry: $x = -1$

18. $g(x) = -4(x + 3)(x + 7)$



x-intercept: $p = -3$ and $q = -7$

$x = \frac{p + q}{2} = \frac{-3 + (-7)}{2} = -5$

$g(x) = -4(-5 + 3)(-5 + 7) = 16$

Vertex: $(-5, 16)$

Axis of symmetry: $x = -5$

19. $y = (x + 1)(x + 2)$

x-intercept: $p = -1$ and $q = -2$

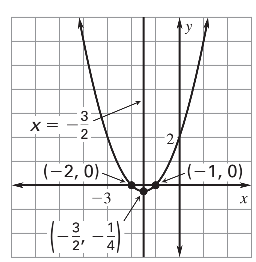
$x = \frac{p + q}{2} = \frac{-1 + (-2)}{2} = -\frac{3}{2}$

$y = \left(-\frac{3}{2} + 1\right)\left(-\frac{3}{2} + 2\right) = -\frac{1}{4}$

Vertex: $\left(-\frac{3}{2}, -\frac{1}{4}\right)$

Axis of symmetry:

$x = -\frac{3}{2}$



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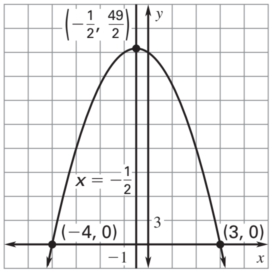
20. $f(x) = -2(x - 3)(x + 4)$



x-intercept: $p = 3$ and $q = -4$

$$x = \frac{p + q}{2} = \frac{3 + (-4)}{2} = -\frac{1}{2}$$

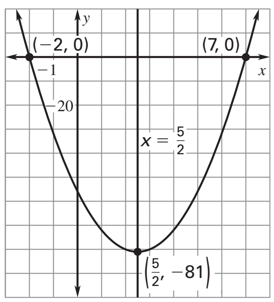
$$f(x) = -2\left(-\frac{1}{2} - 3\right)\left(-\frac{1}{2} + 4\right) = \frac{49}{2}$$

Vertex: $\left(-\frac{1}{2}, \frac{49}{2}\right)$

Axis of symmetry: $x = -\frac{1}{2}$

21. $y = 4(x - 7)(x + 2)$

x-intercept: $p = 7$ and $q = -2$



$$x = \frac{p + q}{2} = \frac{7 + (-2)}{2} = \frac{5}{2}$$

$$y = 4\left(\frac{5}{2} - 7\right)\left(\frac{5}{2} + 2\right) = -81$$

Vertex: $\left(\frac{5}{2}, -81\right)$

Axis of symmetry: $x = \frac{5}{2}$

22. A; $y = -(x - 6)(x + 4)$

x-intercepts: $p = 6$ and $q = -4$

$$x = \frac{p + q}{2} = \frac{6 + (-4)}{2} = 1$$

$y = -(1 - 6)(1 + 4) = 25$

Vertex: $(1, 25)$

23. The x-intercepts of the graph of $y = a(x - p)(x - q)$ are p and q. Therefore, the x-intercepts of the graph of $y = 5(x - 2)(x - (-3))$ are 2 and -3.

24. $y = (x + 4)(x + 3)$
$= x^2 + 3x + 4x + 12$
$= x^2 + 7x + 12$

25. $y = (x - 5)(x + 3)$
$= x^2 + 3x - 5x - 15$
$= x^2 - 2x - 15$

26. $h(x) = 4(x + 1)(x - 6)$
$= 4(x^2 - 6x + x - 6)$
$= 4(x^2 - 5x - 6)$
$= 4x^2 - 20x - 24$

27. $y = -3(x - 2)(x - 4)$
$= -3(x^2 - 4x - 2x + 8)$
$= -3(x^2 - 6x + 8)$
$= -3x^2 + 18x - 24$

28. $f(x) = (x + 5)^2 - 2$
$= (x + 5)(x + 5) - 2$
$= (x^2 + 5x + 5x + 25) - 2$
$= x^2 + 10x + 25 - 2$
$= x^2 + 10x + 23$

29. $y = (x - 3)^2 + 6$
$= (x - 3)(x - 3) + 6$
$= (x^2 - 3x - 3x + 9) + 6$
$= x^2 - 6x + 9 + 6$
$= x^2 - 6x + 15$

30. $g(x) = -(x + 6)^2 + 10$
$= -(x + 6)(x + 6) + 10$
$= -(x^2 + 6x + 6x + 36) + 10$
$= -(x^2 + 12x + 36) + 10$
$= -x^2 - 12x - 36 + 10$
$= -x^2 - 12x - 26$

31. $y = 5(x + 3)^2 - 4$
$= 5(x + 3)(x + 3) - 4$
$= 5(x^2 + 3x + 3x + 9) - 4$
$= 5(x^2 + 6x + 9) - 4$
$= 5x^2 + 30x + 45 - 4$
$= 5x^2 + 30x + 41$

32. $f(x) = 12(x - 1)^2 + 4$
$= 12(x - 1)(x - 1) + 4$
$= 12(x^2 - x - x + 1) + 4$
$= 12(x^2 - 2x + 1) + 4$
$= 12x^2 - 24x + 12 + 4$
$= 12x^2 - 24x + 16$

33. $y = 3(x - 3)^2 - 4$

Because $a > 0$, the function has a minimum value. The minimum value is $y = -4$.

34. $g(x) = -4(x + 6)^2 - 12$

Because $a < 0$, the function has a maximum value. The maximum value is $y = -12$.

35. $y = 15(x - 25)^2 + 130$

Because $a > 0$, the function has a minimum value. The minimum value is $y = 130$.

36. $f(x) = 3(x + 10)(x - 8)$

Because $a > 0$, the function has a minimum value.

$$x = \frac{p + q}{2} = \frac{-10 + 8}{2} = -1$$

$f(-1) = 3(-1 + 10)(-1 - 8) = -243$

The minimum value is $f(x) = -243$.

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37. $y = -(x - 36)(x + 18)$

Because $a < 0$, the function has a maximum value.

$$x = \frac{p + q}{2} = \frac{36 + (-18)}{2} = 9$$

$$y = -(9 - 36)(9 + 18) = 729$$

The maximum value is $y = 729$.

38. $y = -12x(x - 9)$

$y = -12(x - 0)(x - 9)$

Because $a < 0$, the function has a maximum value.

$$x = \frac{p + q}{2} = \frac{0 + 9}{2} = \frac{9}{2}$$

$$y = -12\left(\frac{9}{2}\right) = 243$$

The maximum value is $y = 243$.

39. $y = 8x(x + 15)$

$y = 8(x - 0)(x + 15)$

Because $a > 0$, the function has a minimum value.

$$x = \frac{p + q}{2} = \frac{0 + (-15)}{2} = -\frac{15}{2}$$

$$y = 8\left(-\frac{15}{2}\right)\left(-\frac{15}{2} + 15\right) = -450$$

The minimum value is $y = -450$.

40. $y = 2(x - 3)(x - 6)$

Because $a > 0$, the function has a minimum value.

$$x = \frac{p + q}{2} = \frac{3 + 6}{2} = \frac{9}{2}$$

$$y = 2\left(\frac{9}{2} - 3\right)\left(\frac{9}{2} - 6\right) = -\frac{9}{2}$$

The minimum value is $y = -\frac{9}{2}$.

41. $g(x) = -5(x + 9)(x - 4)$

Because $a < 0$, the function has a maximum value.

$$x = \frac{p + q}{2} = \frac{-9 + 4}{2} = -\frac{5}{2}$$

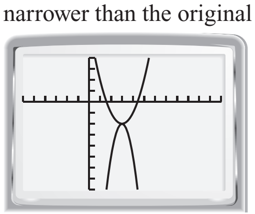
$$g\left(-\frac{5}{2}\right) = -5\left(-\frac{5}{2} + 9\right)\left(-\frac{5}{2} - 4\right) = \frac{845}{4}$$

The maximum value is $g(x) = \frac{845}{4}$.

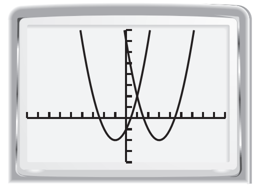
42. $y = a(x - h)^2 + k$

$= (x - 3)^2 - 2$

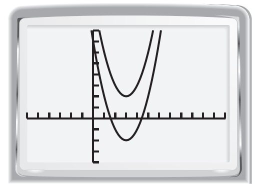
a. If a changes to -3, $a < 0$ so the graph will open down instead of up. Also because $|a| > 1$, the graph will be narrower than the original graph.



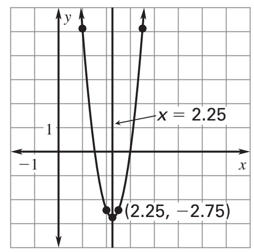
b. If h changes to -1, the graph will be translated horizontally 4 units to the left.



c. If k changes to 2, the graph will be translated vertically 4 units up.



43. $y = 5(x - 2.25)^2 - 2.75$



$a = 5$, $h = 2.25$, $k = -2.75$

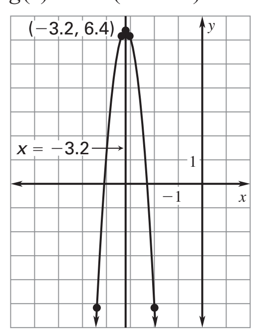
Vertex: $(2.25, -2.75)$

Axis of symmetry: $x = 2.25$

$x = 1$: $y = 5(1 - 2.25)^2 - 2.75 \approx 5.06$; $(1, 5.06)$

$x = 2$: $y = 5(2 - 2.25)^2 - 2.75 \approx -2.44$; $(2, -2.44)$

44. $g(x) = -8(x + 3.2)^2 + 6.4$



$a = -8$, $h = -3.2$, $k = 6.4$

Vertex: $(-3.2, 6.4)$

Axis of symmetry: $x = -3.2$

$x = -3$: $g(x) = -8(-3 + 3.2)^2 + 6.4$

$= 6.08$; $(-3, 6.08)$

$x = -2$: $g(x) = -8(-2 + 3.2)^2 + 6.4$

$= -5.12$; $(-2, -5.12)$

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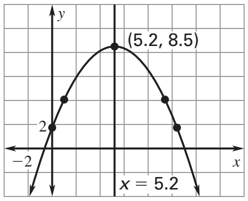
45. $y = -0.25(x - 5.2)^2 + 8.5$



$a = -0.25,\ h = 5.2,\ k = 8.5$

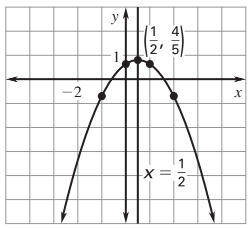
Vertex: $(5.2, 8.5)$

Axis of symmetry: $x = 5.2$

$x = 0$: $y = -0.25(0 - 5.2)^2 + 8.5 = 1.74$; $(0, 1.74)$

$x = 1$: $y = -0.25(1 - 5.2)^2 + 8.5 = 4.09$; $(1, 4.09)$

46. $y = -\frac{2}{3}\left(x - \frac{1}{2}\right)^2 + \frac{4}{5}$



$a = -\frac{2}{3},\ h = \frac{1}{2},\ k = \frac{4}{5}$

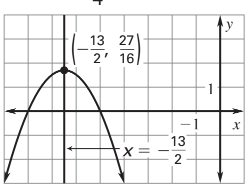
Vertex: $\left(\frac{1}{2}, \frac{4}{5}\right)$

Axis of symmetry: $x = \frac{1}{2}$

$x = -1$: $y = -\frac{2}{3}\left(-1 - \frac{1}{2}\right)^2 + \frac{4}{5} = -\frac{7}{10}$; $\left(-1, -\frac{7}{10}\right)$

$x = 0$: $y = -\frac{2}{3}\left(0 - \frac{1}{2}\right)^2 + \frac{4}{5} = \frac{19}{30}$; $\left(0, \frac{19}{30}\right)$

47. $f(x) = -\frac{3}{4}(x + 5)(x + 8)$



x-intercepts: $p = -5$ and $q = -8$

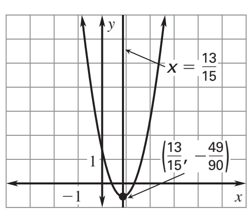
$$x = \frac{p + q}{2} = \frac{-5 + (-8)}{2} = -\frac{13}{2}$$

$$f(x) = -\frac{3}{4}\left(-\frac{13}{2} + 5\right)\left(-\frac{13}{2} + 8\right) = \frac{27}{16}$$

Vertex: $\left(-\frac{13}{2}, \frac{27}{16}\right)$

Axis of symmetry: $x = -\frac{13}{2}$

48. $g(x) = \frac{5}{2}\left(x - \frac{4}{3}\right)\left(x - \frac{2}{5}\right)$



x-intercepts: $p = \frac{4}{3}$ and $q = \frac{2}{5}$

$$x = \frac{p + q}{2} = \frac{\frac{4}{3} + \frac{2}{5}}{2} = \frac{13}{15}$$

$$g(x) = \frac{5}{2}\left(\frac{13}{15} - \frac{4}{3}\right)\left(\frac{13}{15} - \frac{2}{5}\right) = -\frac{49}{90}$$

Vertex: $\left(\frac{13}{15}, -\frac{49}{50}\right)$

Axis of symmetry: $x = \frac{13}{15}$

49. Vertex: $(3, k)$

$\frac{p + q}{2} = 3 \rightarrow p + q = 6$

Sample answer: $y = -(x - 4)(x - 2)$

$y = 3(x + 1)(x - 7)$

50. $y = a(x - h)^2 + k$

$= a(x - h)(x - h) + k$

$= a(x^2 - hx - hx + h^2) + k$

$= a(x^2 - 2hx + h^2) + k$

$= ax^2 - 2ahx + ah^2 + k$

$a = a,\ b = -2ah,\ c = ah^2 + k$

$x = -\frac{b}{2a} = -\frac{(-2ah)}{2(a)} = h$

$y = a(x - p)(x - q)$

$= a(x^2 - qx - px + pq)$

$= ax^2 - apx - aqx + apq$

$= ax^2 + (-ap - aq)x + apq$

$a = a,\ b = -ap - aq,\ c = apq$

$x = \frac{-b}{2a} = -\frac{(-ap - aq)}{2(a)} = -\frac{(-a)(p + q)}{2a} = \frac{p + q}{2}$

Problem Solving

51. $y = -0.03(x - 14)^2 + 6$

The vertex is $(14, 6)$. The maximum height of the kangaroo is 6 feet.

$2(14) = 28$

The kangaroo's jump is 28 feet long.

52. $y = -0.016(x - 52.5)^2 + 45$

The vertex is $(52.5, 45)$.

$2(52.5) = 105$

The width of the arch is 105 meters.

53. **a.** $y = -0.000234x(x - 160)$

$= -0.000234\,(x - 0)(x - 160)$

x-intercepts: $p = 0$ and $q = 160$

The width of the field is 160 feet.

b. $x = \frac{p + q}{2} = \frac{0 + 160}{2} = 80$

$y = -0.000234(80)(80 - 160) \approx 1.5$

The maximum height of the field's surface is about 1.5 feet.

54. $y = -0.5(x - 6)^2 + 18$

The maximum height of the jump with a conventional spring is 18 inches.

$y = -1.17(x - 6)^2 + 42$

The maximum height of the jump with a bow spring is 42 inches.

The jump on the pogo stick with a bow spring is 24 inches higher than the jump on the pogo stick with a conventional spring. The constant k affects the maximum heights of the jumps, while the constants a and h do not.

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55. a. $y = -0.761(x - 5.52)(x - 22.6)$

$p = 5.52,\ q = 22.6$

$x = \frac{p + q}{2} = \frac{5.52 + 22.6}{2} = 14.06$

$y = -0.761(14.06 - 5.52)(14.06 - 22.6) \approx 55.5$

For hot-air popping, a 14.06% moisture content maximizes popping volume. The maximum popping volume is 55.5 cubic centimeters per gram.

b. $y = -0.652(x - 5.35)(x - 21.8)$

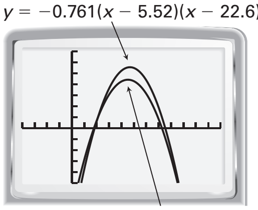
$p = 5.35,\ q = 21.8$

$x = \frac{p + q}{2} = \frac{5.35 + 21.8}{2} \approx 13.58$

$y = -0.652(13.58 - 5.35)(13.58 - 21.8) \approx 44.11$

For hot-oil popping, a 13.58% moisture content maximizes popping volume. The maximum popping volume is 44.11 cubic centimeters per gram.

c. $y = -0.761(x - 5.52)(x - 22.6)$



$y = -0.652(x - 5.35)(x - 21.8)$

hot-air popping: domain: $5.52 \le x \le 22.6$

range: $0 \le y \le 55.5$

hot-oil popping: domain: $5.35 \le x \le 21.8$

range: $0 \le y \le 44.11$

The x-intercepts of the graph of each function determined the domain.

The y-coordinate of the vertex of the graph of each function determined the range. Also, the range did not include any negative values because it does not make sense to have a negative popping volume.

56. $y = a(x - h)^2 + k$

$h = 33$

$k = 5$

$y = a(x - 33)^2 + 5$

At (0, 0): $0 = a(0 - 33)^2 + 5$

$0 = a(1089) + 5$

$-5 = 1089a$

$-\frac{5}{1089} = a$

$y = -\frac{5}{1089}(x - 33)^2 + 5$

Changing the value of a affects the width of the flight path. Changing the value of h affects the horizontal position of the flight path. Changing the value of k affects the height of the flight path.

Mixed Review

57. $x - 5 = 0$

$x = 5$

58. $2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

59. $23x - 14 = -5x - 7$

$28x - 14 = -7$

$28x = 7$

$x = \frac{1}{4}$

60. $-5(3x + 4) = 17x + 2$

$-15x - 20 = 17x + 2$

$-20 = 32x + 2$

$-22 = 32x$

$-\frac{11}{16} = x$

61. $|x - 9| = 16$

$x - 9 = 16$ or $x - 9 = -16$

$x = 25$ or $x = -7$

62. $|4x + 9| = 27$

$4x + 9 = 27$ or $4x + 9 = -27$

$4x = 18$ or $4x = -36$

$x = \frac{9}{2}$ or $x = -9$

63. $|7 - 2x| = 1$

$7 - 2x = 1$ or $7 - 2x = -1$

$-2x = -6$ or $-2x = -8$

$x = 3$ or $x = 4$

64. $|3 - 5x| = 7$

$3 - 5x = 7$ or $3 - 5x = -7$

$-5x = 4$ or $-5x = -10$

$x = -\frac{4}{5}$ or $x = 2$

65. $2A + B = 2\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$

$= \begin{bmatrix} -2 & 6 \\ 4 & -10 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 7 & -2 \end{bmatrix}$

66. $3(B + C) = 3\left(\begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}\right)$

$= 3\begin{bmatrix} 1 & -2 \\ 1 & 11 \end{bmatrix}$

$= \begin{bmatrix} 3 & -6 \\ 3 & 33 \end{bmatrix}$

67. $D - 4E = \begin{bmatrix} 3 & 0 & -1 \\ 6 & 1 & 4 \end{bmatrix} - 4\begin{bmatrix} 1 & -2 & 4 \\ 3 & -1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 & -1 \\ 6 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & -8 & 16 \\ 12 & -4 & 20 \end{bmatrix}$

$= \begin{bmatrix} -1 & 8 & -17 \\ -6 & 5 & -16 \end{bmatrix}$

68. $AB = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$

$= \begin{bmatrix} 7 & 30 \\ -11 & -52 \end{bmatrix}$

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69. $A(B - C) = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\left(\begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 3 & -10 \\ 5 & 5 \end{bmatrix}$

$= \begin{bmatrix} 12 & 25 \\ -19 & -45 \end{bmatrix}$

70. $4(CD) = 4\left(\begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 & -1 \\ 6 & 1 & 4 \end{bmatrix}\right)$

$= 4\begin{bmatrix} 21 & 4 & 17 \\ 12 & 3 & 14 \end{bmatrix}$

$= \begin{bmatrix} 84 & 16 & 68 \\ 48 & 12 & 56 \end{bmatrix}$

Lesson 4.3

4.3 Guided Practice (pp. 252–255)

1.

Factors of −18: *m, n*	1, −18	−1, 18	2, −9
Sum of factors: *m + n*	−17	17	−7

Factors of −18: *m, n*	−2, 9	3, −6	−3, 6
Sum of factors: *m + n*	7	−3	3

$x^2 - 3x - 18 = (x + 3)(x - 6)$

2.

Factors of 9: *p, q*	1, 9	−1, −9	3, 3	−3, −3
Sum of factors: *p + q*	10	−10	6	−6

$n^2 - 3n + 9$

There are no factors of 9, p and q, such that $p + q = -3$. So, $n^2 - 3n + 9$ cannot be factored.

3.

Factors of −63: *m, n*	1, −63	−1, 63	3, −21
Sum of factors: *m + n*	−62	62	−18

Factors of −18: *m, n*	−3, 21	7, −9	−7, 9
Sum of factors: *m + n*	18	−2	2

$r^2 + 2r - 63 = (r - 7)(r + 9)$

4. $x^2 - 9 = x^2 - 3^2$

$= (x + 3)(x - 3)$

5. $q^2 - 100 = q^2 - 10^2$

$= (q + 10)(q - 10)$

6. $y^2 + 16y + 64 = y^2 + 2(y)(8) + 8^2$

$= (y + 8)^2$

7. $w^2 - 18w + 81 = w^2 - 2(w)(9) + 9^2$

$= (w - 9)^2$

8. $x^2 - x - 42 = 0$

$(x - 7)(x + 6) = 0$

$x - 7 = 0$ or $x + 6 = 0$

$x = 7$ or $x = -6$

9. New area (square meters) = New length (meters) • New width (meters)

$2(1000)(300) = (1000 + x) \cdot (300 + x)$

$600{,}000 = 300{,}000 + 1300x + x^2$

$0 = x^2 + 1300x - 300{,}000$

$0 = (x - 200)(x + 1500)$

$x - 200 = 0$ or $x + 1500 = 0$

$x = 200$ or $x = -1500$

Reject the negative value, −1500. The field's length and width should each be increased by 200 meters. The new dimensions are 1200 meters by 500 meters.

10. $y = x^2 + 5x - 14$

$= (x + 7)(x - 2)$

The zeros of the function are −7 and 2.

11. $y = x^2 - 7x - 30$

$= (x + 3)(x - 10)$

The zeros of the function are −3 and 10.

12. $f(x) = x^2 - 10x + 25$

$= (x - 5)(x - 5)$

The zero of the function is 5.

4.3 Exercises (pp. 255–258)

Skill Practice

1. A zero of a function $y = f(x)$ is a number that when substituted for x in the function yields $y = 0$.

2. A monomial is either a number, a variable, or the product of a number and one or more variables, such as x^2. A binomial is the sum of two monomials, such as $x^2 - 5x$. A trinomial is the sum of three monomials, such as $x^2 - 5x + 6$.

3. $x^2 + 6x + 5 = (x + 5)(x + 1)$

4. $x^2 - 7x + 10 = (x - 5)(x - 2)$

5. $a^2 - 13a + 22 = (a - 11)(a - 2)$

6. $r^2 + 15r + 56 = (r + 7)(r + 8)$

7. $p^2 + 2p + 4$ Cannot be factored.

8. $q^2 - 11q + 28 = (q - 7)(q - 4)$

9. $b^2 + 3b - 40 = (b + 8)(b - 5)$

10. $x^2 - 4x - 12 = (x + 2)(x - 6)$

11. $x^2 - 7x - 18 = (x + 2)(x - 9)$

12. $c^2 - 9c - 18$ Cannot be factored.

13. $x^2 + 9x - 36 = (x + 12)(x - 3)$

14. $m^2 + 8m - 65 = (m + 13)(m - 5)$

15. $x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$

16. $b^2 - 81 = b^2 - 9^2 = (b + 9)(b - 9)$

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17. $x^2 - 24x + 144 = x^2 - 2(x)(12) + 12^2$
$= (x - 12)^2$

18. $t^2 - 16t + 64 = t^2 - 2(t)(8) + 8^2$
$= (t - 8)^2$

19. $x^2 + 8x + 16 = x^2 + 2(x)(4) + 4^2$
$= (x + 4)^2$

20. $c^2 + 28c + 196 = c^2 + 2(c)(14) + 14^2$
$= (c + 14)^2$

21. $n^2 + 14n + 49 = n^2 + 2(n)(7) + 7^2$
$= (n + 7)^2$

22. $s^2 - 26s + 169 = s^2 - 2(s)(13) + 13^2$
$= (s - 13)^2$

23. $z^2 - 121 = z^2 - 11^2$
$= (z + 11)(z - 11)$

24. $x^2 - 8x + 12 = 0$
$(x - 6)(x - 2) = 0$
$x - 6 = 0$ or $x - 2 = 0$
$x = 6$ or $x = 2$

25. $x^2 - 11x + 30 = 0$
$(x - 6)(x - 5) = 0$
$x - 6 = 0$ or $x - 5 = 0$
$x = 6$ or $x = 5$

26. $x^2 + 2x - 35 = 0$
$(x + 7)(x - 5) = 0$
$x + 7 = 0$ or $x - 5 = 0$
$x = -7$ or $x = 5$

27. $a^2 - 49 = 0$
$(a - 7)(a + 7) = 0$
$a - 7 = 0$ or $a + 7 = 0$
$a = 7$ or $a = -7$

28. $b^2 - 6b + 9 = 0$
$(b - 3)(b - 3) = 0$
$b - 3 = 0$
$b = 3$

29. $c^2 + 5c + 4 = 0$
$(c + 4)(c + 1) = 0$
$c + 4 = 0$ or $c + 1 = 0$
$c = -4$ or $c = -1$

30. $n^2 - 6n = 0$
$n(n - 6) = 0$
$n = 0$ or $n - 6 = 0$
$n = 0$ or $n = 6$

31. $t^2 + 10t + 25 = 0$
$(t + 5)(t + 5) = 0$
$t + 5 = 0$
$t = -5$

32. $w^2 - 16w + 48 = 0$
$(w - 12)(w - 4) = 0$
$w - 12 = 0$ or $w - 4 = 0$
$w = 12$ or $w = 4$

33. $z^2 - 3z = 54$
$z^2 - 3z - 54 = 0$
$(z - 9)(z + 6) = 0$
$z - 9 = 0$ or $z + 6 = 0$
$z = 9$ or $z = -6$

34. $r^2 + 2r = 80$
$r^2 + 2r - 80 = 0$
$(r + 10)(r - 8) = 0$
$r + 10 = 0$ or $r - 8 = 0$
$r = -10$ or $r = 8$

35. $u^2 = -9u$
$u^2 + 9u = 0$
$u(u + 9) = 0$
$u = 0$ or $u + 9 = 0$
$u = 0$ or $u = -9$

36. $m^2 = 7m$
$m^2 - 7m = 0$
$m(m - 7) = 0$
$m = 0$ or $m - 7 = 0$
$m = 0$ or $m = 7$

37. $14x - 49 = x^2$
$0 = x^2 - 14x + 49$
$0 = (x - 7)(x - 7)$
$x - 7 = 0$
$x = 7$

38. $-3y + 28 = y^2$
$0 = y^2 + 3y - 28$
$0 = (y + 7)(y - 4)$
$y + 7 = 0$ or $y - 4 = 0$
$y = -7$ or $y = 4$

39. The trinomial was factored incorrectly.
$x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x + 2 = 0$ or $x - 3 = 0$
$x = -2$ or $x = 3$

40. The equation must be written in standard form before you factor and use the zero product property.
$x^2 + 7x + 6 = 14$
$x^2 + 7x - 8 = 0$
$(x + 8)(x - 1) = 0$
$x + 8 = 0$ or $x - 1 = 0$
$x = -8$ or $x = 1$

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41. A; $x^2 + 2x - 63 = 0$

$(x + 9)(x - 7) = 0$

$x + 9 = 0$ or $x - 7 = 0$

$x = -9$ or $x = 7$

42.

New area (square feet)	=	New length (feet)	•	New width (feet)
2(24)(10)	=	(24 + x)	•	(10 + x)

$480 = 240 + 34x + x^2$

$0 = x^2 + 34x - 240$

43.

New area (square feet)	=	New length (feet)	•	New width (feet)
3(12)(10)	=	(12 + x)	•	(10 + x)

$360 = 120 + 22x + x^2$

$0 = x^2 + 22x - 240$

44. $y = x^2 + 6x + 8$

$= (x + 4)(x + 2)$

The zeros of the function are -4 and -2.

45. $y = x^2 - 8x + 16$

$= (x - 4)(x - 4)$

The zero of the function is 4.

46. $y = x^2 - 4x - 32$

$=(x + 4)(x - 8)$

The zeros of the function are -4 and 8.

47. $y = x^2 + 7x - 30$

$= (x + 10)(x - 3)$

The zeros of the function are -10 and 3.

48. $f(x) = x^2 + 11x$

$= x(x + 11)$

The zeros of the function are 0 and -11.

49. $g(x) = x^2 - 8x$

$= x(x - 8)$

The zeros of the function are 0 and 8.

50. $y = x^2 - 64$

$= (x + 8)(x - 8)$

The zeros of the function are -8 and 8.

51. $y = x^2 - 25$

$= (x + 5)(x - 5)$

The zeros of the function are -5 and 5.

52. $f(x) = x^2 - 12x - 45$

$= (x + 3)(x - 15)$

The zeros of the function are -3 and 15.

53. $g(x) = x^2 + 19x + 84$

$= (x + 12)(x + 7)$

The zeros of the function are -12 and -7.

54. $y = x^2 + 22x + 121$

$= (x + 11)(x + 11)$

The zero of the function is -11.

55. $y = x^2 + 2x + 1$

$= (x + 1)(x + 1)$

The zero of the function is -1.

56. B; $f(x) = x^2 + 6x - 55$

$= (x + 11)(x - 5)$

The zeros of the function are -11 and 5.

57. $(x - 8)(x - 11) = 0$

$x^2 - 11x - 8x + 88 = 0$

$x^2 - 19x + 88 = 0$

58. $x^2 + bx + 7$

$(x + 7)(x + 1) = x^2 + 8x + 7$

$(x - 7)(x - 1) = x^2 - 8x + 7$

Therefore, when $b = 8$ or $b = -8$ the expression can be factored.

59. Area of rectangle $= \ell w$

$36 = (x + 5)(x)$

$36 = x^2 + 5x$

$0 = x^2 + 5x - 36$

$0 = (x + 9)(x - 4)$

$x + 9 = 0$ or $x - 4 = 0$

$x = -9$ or $x = 4$

The value of x is 4 units.

60. Area of rectangle $= \ell w$

$84 = (x + 7)(x + 2)$

$84 = x^2 + 9x + 14$

$0 = x^2 + 9x - 70$

$0 = (x + 14)(x - 5)$

$x + 14 = 0$ or $x - 5 = 0$

$x = -14$ or $x = 5$

The value of x is 5 units.

61. Area of triangle $= \frac{1}{2}bh$

$42 = \frac{1}{2}(2x + 8)(x + 3)$

$42 = (x + 4)(x + 3)$

$42 = x^2 + 7x + 12$

$0 = x^2 + 7x - 30$

$0 = (x + 10)(x - 3)$

$x + 10 = 0$ or $x - 3 = 0$

$x = -10$ or $x = 3$

The value of x is 3 units.

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62. Area of trapezoid $= \frac{1}{2}(b_1 + b_2)h$

$$\begin{aligned} 32 &= \tfrac{1}{2}(x + 6 + x + 2)x \\ 32 &= \tfrac{1}{2}(2x + 8)x \\ 32 &= (x + 4)x \\ 32 &= x^2 + 4x \\ 0 &= x^2 + 4x - 32 = 0 \\ 0 &= (x + 8)(x - 4) = 0 \end{aligned}$$

$$x + 8 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -8 \quad \text{or} \quad x = 4$$

The value of x is 4 units.

63. *Sample answer:* $y = (x - 8)(x - 12) = x^2 - 20x + 96$

64. a. $x^2 + 16 = (x + m)(x + n)$

$mn = 16$ and $m + n = 0$

b. $m + n = 0$

$$\begin{aligned} m &= -n \\ (-n)n &= 16 \\ -n^2 &= 16 \\ n^2 &= -16 \end{aligned}$$

$n^2 = -16$ has no real-number solutions because the square of any real number n is never negative.

Because there are no integers m and n that satisfy both equations, you can conclude that there is no formula for factoring the sum of two squares.

Problem Solving

65. New area (square feet) = New length (feet) • New width (feet)

$$3(100)(50) = (100 + x) \cdot (50 + x)$$
$$\begin{aligned} 15{,}000 &= 5000 + 150x + x^2 \\ 0 &= x^2 + 150x - 10{,}000 \\ 0 &= (x + 200)(x - 50) \end{aligned}$$
$$x + 200 = 0 \quad \text{or} \quad x - 50 = 0$$
$$x = -200 \quad \text{or} \quad x = 50$$

Reject the negative value, -200. The skate park's length and width should each be increased by 50 feet. The new dimensions are 150 feet by 100 feet.

66. New area (square feet) = New length (feet) • New width (feet)

$$2(35)(18) = (35 + x) \cdot (18 + x)$$
$$\begin{aligned} 1260 &= 630 + 53x + x^2 \\ 0 &= x^2 + 53x - 630 \\ 0 &= (x + 63)(x - 10) \end{aligned}$$
$$x + 63 = 0 \quad \text{or} \quad x - 10 = 0$$
$$x = -63 \quad \text{or} \quad x = 10$$

Reject the negative value, -63. The enclosure's length and width should each be increased by 10 feet. The new dimensions are 45 feet by 28 feet.

67. a. $A = \ell w = 30(20) = 600$

The area of the existing patio is 600 square feet.

b. New area (square feet) = New length (feet) • New width (feet)

$$600 + 464 = (30 + x) \cdot (20 + x)$$
$$\begin{aligned} 1064 &= 600 + 50x + x^2 \\ 0 &= x^2 + 50x - 464 \end{aligned}$$

c.
$$\begin{aligned} 0 &= x^2 + 50x - 464 \\ 0 &= (x + 58)(x - 8) \end{aligned}$$
$$x + 58 = 0 \quad \text{or} \quad x - 8 = 0$$
$$x = -58 \quad \text{or} \quad x = 8$$

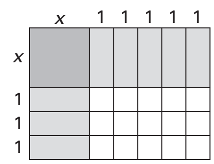
Reject the negative value, -58. The length and width of the patio should each be expanded by 8 feet.

68. a. Area $= x^2 + x + x + x + x + x + 1 + 1 + 1 + 1 + 1 + 1 = x^2 + 5x + 6$

b. $x^2 + 5x + 6 = (x + 3)(x + 2)$

The diagram is a rectangle. The length of the rectangle is represented by $(x + 3)$ and the width is represented by $(x + 2)$. So, the area of the rectangle is $(x + 3)(x + 2)$.

c. $x^2 + 8x + 15 = (x + 5)(x + 3)$



69. New area (square feet) = New length (feet) • New width (feet)

$$3(18)(15) = (18 + x) \cdot (15 + x)$$
$$\begin{aligned} 810 &= 270 + 33x + x^2 \\ 0 &= x^2 + 33x - 540 \\ 0 &= (x + 45)(x - 12) \end{aligned}$$
$$x + 45 = 0 \quad \text{or} \quad x - 12 = 0$$
$$x = -45 \quad \text{or} \quad x = 12$$

Reject the negative value, -45.

New length $= 18 + 12 = 30$

New width $= 15 + 12 = 27$

$P = 2\ell + 2w = 2(30) + 2(27) = 114$

The length of rope needed to enclose the new section is 114 feet.

70. New area (square feet) = New length (feet) • New width (feet)

$$\tfrac{1}{2}(21)(20) = (21 - x) \cdot (20 - x)$$
$$\begin{aligned} 210 &= 420 - 41x + x^2 \\ 0 &= x^2 - 41x + 210 \\ 0 &= (x - 35)(x - 6) \end{aligned}$$
$$x - 35 = 0 \quad \text{or} \quad x - 6 = 0$$
$$x = 35 \quad \text{or} \quad x = 6$$

Reject 35 because it is larger than the original length and width. The new dimensions are 15 feet by 14 feet.

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71. $\underset{\text{(square feet)}}{\text{New area}} = \underset{\text{(feet)}}{\text{New length}} \cdot \underset{\text{(feet)}}{\text{New width}}$

$$2(10)(10) = (10 + x) \cdot (10 + x)$$
$$200 = 100 + 20x + x^2$$
$$0 = x^2 + 20x - 100$$

No, you cannot solve the equation by factoring. There are no integers m and n such that $mn = -100$ and $m + n = 20$.

72. 2 • Area of old lot (square feet) =

Length of store with old lot and new lot (feet) • Width of store with old lot and new lot (feet) − Area of store (square feet)

$$2 \cdot [(375)(240) - (300)(165)] =$$
$$(375 + x) \cdot (240 + x) - (300)(165)$$
$$81{,}000 = 90{,}000 + 615x + x^2 - 49{,}500$$
$$0 = x^2 + 615x - 40{,}500$$
$$0 = (x + 675)(x - 60)$$
$$x + 675 = 0 \quad \text{or} \quad x - 60 = 0$$
$$x = -675 \quad \text{or} \quad x = 60$$

Reject the negative value, −675. The parking lot should be expanded by 60 feet.

Mixed Review

73. $2x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$

74. $3x + 4 = 0$
$3x = -4$
$x = -\frac{4}{3}$

75. $-8x + 7 = 0$
$-8x = -7$
$x = \frac{7}{8}$

76. $6x + 5 = 0$
$6x = -5$
$x = -\frac{5}{6}$

77. $4x - 5 = 0$
$4x = 5$
$x = \frac{5}{4}$

78. $3x + 1 = 0$
$3x = -1$
$x = -\frac{1}{3}$

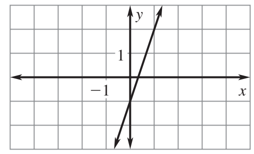
79. $|x - 6| = 7$
$x - 6 = 7$ or $x - 6 = -7$
$x = 13$ or $x = -1$

80. $|2x - 5| = 10$
$2x - 5 = 10$ or $2x - 5 = -10$
$2x = 15$ or $2x = -5$
$x = \frac{15}{2}$ or $x = -\frac{5}{2}$

81. $|4 - 3x| = 8$
$4 - 3x = 8$ or $4 - 3x = -8$
$-3x = 4$ or $-3x = -12$
$x = -\frac{4}{3}$ or $x = 4$

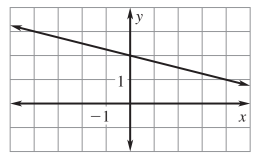
82. y-intercept: 1; (0, −1)

Slope: 3



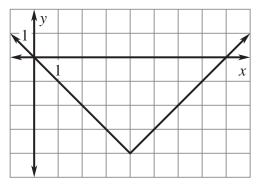
83. y-intercept: 2; (0, 2)

Slope: $-\frac{1}{4}$



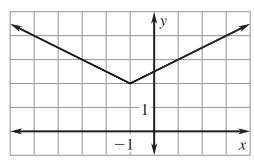
84. Vertex: (4, −4)

$x = 0$: $y = |0 - 4| - 4 = 0$; (0, 0)



85. Vertex: (−1, 2)

$x = 1$: $y = \frac{1}{2}|1 + 1| + 2 = 3$; (1, 3)



86. $y = -2x^2 + 8x + 7$

$x = -\frac{b}{2a} = -\frac{8}{2(-2)} = 2$

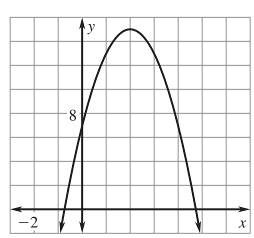
$y = -2(2)^2 + 8(2) + 7 = 15$

Vertex: (2, 15)

Axis of symmetry: $x = 2$

y-intercept: 7 (0, 7)

$x = -1$: $y = -2(-1)^2 + 8(-1) + 7 = -3$; (−1, −3)



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87. $g(x) = -2(x + 1)^2 - 4$

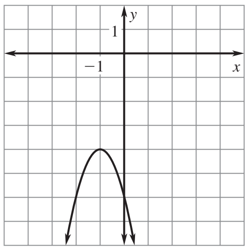
$a = -2, h = -1, k = -4$

Vertex: $(-1, -4)$

Axis of symmetry: $x = -1$

$x = 0$: $g(x) = -2(0 + 1)^2 - 4 = -6$; $(0, -6)$

$x = 1$: $g(x) = -2(1 + 1)^2 - 4 = -12$; $(1, -12)$



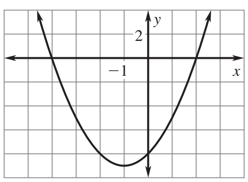
88. $f(x) = (x + 4)(x - 2)$

x-intercepts: $p = -4$ and $q = 2$

$$x = \frac{p + q}{2} = \frac{-4 + 2}{2} = -1$$

$f(x) = (-1 + 4)(-1 - 2) = -9$

Vertex: $(-1, -9)$



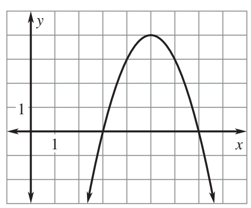
89. $y = -(x - 3)(x - 7)$

x-intercepts: $p = 3$ and $q = 7$

$$x = \frac{p + q}{2} = \frac{3 + 7}{2} = 5$$

$y = -(5 - 3)(5 - 7) = 4$

Vertex: $(5, 4)$



90. $$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 14 & 3 & 1 \\ 6 & 25 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left|\begin{array}{ccc|cc} 0 & 0 & 1 & 0 & 0 \\ 14 & 3 & 1 & 14 & 3 \\ 6 & 25 & 1 & 6 & 25 \end{array}\right.$$

$$= \pm\frac{1}{2}[(0 + 0 + 350) - (18 + 0 + 0)]$$

$$= \pm\frac{1}{2}(332)$$

$$= 166$$

The area of the triangular region is 166 square feet.

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Lesson 4.4

4.4 Guided Practice (pp. 260–262)

1.

k, ℓ	7, 1	7, 1
m, n	$-3, 1$	$-1, 3$
($kx + m$)($\ell x + n$)	$(7x - 3)(x + 1)$	$(7x - 1)(x + 3)$
$ax^2 + bx + c$	$7x^2 + 4x - 3$	$7x^2 + 20x - 3$

k, ℓ	7, 1	7, 1
m, n	$3, -1$	$1, -3$
($kx + m$)($\ell x + n$)	$(7x + 3)(x - 1)$	$(7x + 1)(x - 3)$
$ax^2 + bx + c$	$7x^2 - 4x - 3$	$7x^2 - 20x - 3$

$7x^2 - 20x - 3 = (7x + 1)(x - 3)$

2.

k, ℓ	5, 1	5, 1
m, n	3, 1	1, 3
($kz + m$)($\ell z + n$)	$(5z + 3)(z + 1)$	$(5z + 1)(z + 3)$
$az^2 + bz + c$	$5z^2 + 8z + 3$	$5z^2 + 16z + 3$

$5z^2 + 16z + 3 = (5z + 1)(z + 3)$

3.

k, ℓ	2, 1	2, 1
m, n	3, 1	1, 3
($kw + m$)($\ell w + n$)	$(2w + 3)(w + 1)$	$(2w + 1)(w + 3)$
$aw^2 + bw + c$	$2w^2 + 5w + 3$	$2w^2 + 7w + 3$

$2w^2 + w + 3$ Cannot be factored.

4.

k, ℓ	3, 1	3, 1
m, n	$12, -1$	$-1, 12$
($kx + m$)($\ell x + n$)	$(3x + 12)(x - 1)$	$(3x - 1)(x + 12)$
$ax^2 + bx + c$	$3x^2 + 9x - 12$	$3x^2 + 35x - 12$

k, ℓ	3, 1	3, 1
m, n	$-12, 1$	$1, -12$
($kx + m$)($\ell x + n$)	$(3x - 12)(x + 1)$	$(3x + 1)(x - 12)$
$ax^2 + bx + c$	$3x^2 - 9x - 12$	$3x^2 - 35x - 12$

k, ℓ	3, 1	3, 1
m, n	$6, -2$	$-2, 6$
($kx + m$)($\ell x + n$)	$(3x + 6)(x - 2)$	$(3x - 2)(x + 6)$
$ax^2 + bx + c$	$3x^2 - 12$	$3x^2 + 16x - 12$

k, ℓ	3, 1	3, 1
m, n	−6, 2	2, −6
(kx + m)(ℓx + n)	$(3x - 6)(x + 2)$	$(3x + 2)(x - 6)$
ax² + bx + c	$3x^2 - 12$	$3x^2 - 16x - 12$

k, ℓ	3, 1	3, 1
m, n	4, −3	−3, 4
(kx + m)(ℓx + n)	$(3x + 4)(x - 3)$	$(3x - 3)(x + 4)$
ax² + bx + c	$3x^2 - 5x - 12$	$3x^2 + 9x - 12$

k, ℓ	3, 1	3, 1
m, n	−4, 3	3, −4
(kx + m)(ℓx + n)	$(3x - 4)(x + 3)$	$(3x + 3)(x - 4)$
ax² + bx + c	$3x^2 + 5x - 12$	$3x^2 - 9x - 12$

$3x^2 + 5x - 12 = (3x - 4)(x + 3)$

5.

k, ℓ	4, 1	4, 1
m, n	1, 5	5, 1
(ku + m)(ℓu + n)	$(4u + 1)(u + 5)$	$(4u + 5)(u + 1)$
au² + bu + c	$4u^2 + 21u + 5$	$4u^2 + 9u + 5$

k, ℓ	2, 2
m, n	1, 5
(ku + m)(ℓu + n)	$(2u + 1)(2u + 5)$
au² + bu + c	$4u^2 + 12u + 5$

$4u^2 + 12u + 5 = (2u + 5)(2u + 1)$

6.

k, ℓ	4, 1	4, 1
m, n	−1, −2	−2, −1
(kx + m)(ℓx + n)	$(4x - 1)(x - 2)$	$(4x - 2)(x - 1)$
ax² + bx + c	$4x^2 - 9x + 2$	$4x^2 - 6x + 2$

k, ℓ	2, 2
m, n	−1, −2
(kx + m)(ℓx + n)	$(2x - 1)(2x - 2)$
ax² + bx + c	$4x^2 - 6x + 2$

$4x^2 - 9x + 2 = (4x - 1)(x - 2)$

7. $16x^2 - 1 = (4x)^2 - 1^2 = (4x + 1)(4x - 1)$

8. $9y^2 + 12y + 4 = (3y)^2 + 2(3y)(2) + 2^2 = (3y + 2)^2$

9. $4r^2 - 28r + 49 = (2r)^2 - 2(2r)(7) + 7^2 = (2r - 7)^2$

10. $25s^2 - 80s + 64 = (5s)^2 - 2(5s)(8) + 8^2 = (5s - 8)^2$

11. $49z^2 + 42z + 9 = (7z)^2 + 2(7z)(3) + 3^2 = (7z + 3)^2$

12. $36n^2 - 9 = (6n)^2 - 3^2 = (6n + 3)(6n - 3)$

13. $3s^2 - 24 = 3(s^2 - 8)$

14. $8t^2 + 38t - 10 = 2(4t^2 + 19t - 5) = 2(4t - 1)(t + 5)$

15. $6x^2 + 24x + 15 = 3(2x^2 + 8x + 5)$

16. $12x^2 - 28x - 24 = 4(3x^2 - 7x - 6)$
$= 4(3x + 2)(x - 3)$

17. $-16n^2 + 12n = -4n(4n - 3)$

18. $6z^2 + 33z + 36 = 3(2z^2 + 11z + 12)$
$= 3(2z + 3)(z + 4)$

19. $6x^2 - 3x - 63 = 0$
$2x^2 - x - 21 = 0$
$(2x - 7)(x + 3) = 0$
$2x - 7 = 0$ or $x + 3 = 0$
$x = \frac{7}{2}$ or $x = -3$

20. $12x^2 + 7x + 2 = x + 8$
$12x^2 + 6x - 6 = 0$
$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
$2x - 1 = 0$ or $x + 1 = 0$
$x = \frac{1}{2}$ or $x = -1$

21. $7x^2 + 70x + 175 = 0$
$x^2 + 10x + 25 = 0$
$(x + 5)^2 = 0$
$x + 5 = 0$
$x = -5$

22. $R(x) = (28{,}000 - 2000x) \cdot (11 + x)$
$R(x) = (-2000x + 28{,}000)(x + 11)$
$R(x) = -2000(x - 14)(x + 11)$
$\frac{14 + (-11)}{2} = \frac{3}{2} = 1.5$

To maximize revenue, each subscription should cost \$11 + \$1.50 = \$12.50.

$R(1.5) = -2000(1.5 - 14)(1.5 + 11) = \$312{,}500$

The maximum annual revenue is \$312,500.

4.4 Exercises (pp. 263–265)

Skill Practice

1. $12x^2 + 8x + 20 = 4(3x^2 + 2x + 5)$
The greatest common monomial factor is 4.

2. If a and c are perfect squares, then you may be able to use the perfect square trinomial factoring pattern.

3. $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

4. $3n^2 + 7n + 4 = (3n + 4)(n + 1)$

5. $4r^2 + 5r + 1 = (4r + 1)(r + 1)$

6. $6p^2 + 5p + 1 = (3p + 1)(2p + 1)$

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7. $11z^2 + 2z - 9 = (11z - 9)(z + 1)$

8. $15x^2 - 2x - 8 = (5x - 4)(3x + 2)$

9. The expression $4y^2 - 5y - 4$ cannot be factored.

10. $14m^2 + m - 3 = (7m - 3)(2m + 1)$

11. $9d^2 - 13d - 10 = (9d + 5)(d - 2)$

12. D; $5x^2 + 14x - 3 = (5x - 1)(x + 3)$

13. $9x^2 - 1 = (3x)^2 - 1^2 = (3x + 1)(3x - 1)$

14. $4r^2 - 25 = (2r)^2 - 5^2 = (2r + 5)(2r - 5)$

15. $49n^2 - 16 = (7n)^2 - 4^2 = (7n + 4)(7n - 4)$

16. $16s^2 + 8s + 1 = (4s)^2 + 2(4s)(1) + 1^2 = (4s + 1)^2$

17. $49x^2 + 70x + 25 = (7x)^2 + 2(7x)(5) + 5^2 = (7x + 5)^2$

18. $64w^2 + 144w + 81 = (8w)^2 + 2(8w)(9) + 9^2$
$= (8w + 9)^2$

19. $9p^2 - 12p + 4 = (3p)^2 - 2(3p)(2) + 2^2 = (3p - 2)^2$

20. $25t^2 - 30t + 9 = (5t)^2 - 2(5t)(3) + 3^2 = (5t - 3)^2$

21. $36x^2 - 84x + 49 = (6x)^2 - 2(6x)(7) + 7^2 = (6x - 7)^2$

22. $12x^2 - 4x - 40 = 4(3x^2 - x - 10) = 4(3x + 5)(x - 2)$

23. $18z^2 + 36z + 16 = 2(9z^2 + 18z + 8)$
$= 2(3z + 4)(3z + 2)$

24. $32v^2 - 2 = 2(16v^2 - 1) = 2(4v + 1)(4v - 1)$

25. $6u^2 - 24u = 6u(u - 4)$

26. $12m^2 - 36m + 27 = 3(4m^2 - 12m + 9)$
$= 3(2m - 3)(2m - 3) = 3(2m - 3)^2$

27. $20x^2 + 124x + 24 = 4(5x^2 + 31x + 6)$
$= 4(5x + 1)(x + 6)$

28. $21x^2 - 77x - 28 = 7(3x^2 - 11x - 4)$
$= 7(3x + 1)(x - 4)$

29. $-36n^2 + 48n - 15 = -3(12n^2 - 16n + 5)$
$= -3(6n - 5)(2n - 1)$

30. $-8y^2 + 28y - 60 = -4(2y^2 - 7y + 15)$

31. When factoring out a common monomial, you must factor it from all the terms of the expression, not just the first term.

$4x^2 - 36 = 4(x^2 - 9)$
$= 4(x + 3)(x - 3)$

32. $16x^2 - 1 = 0$
$(4x + 1)(4x - 1) = 0$
$4x + 1 = 0$ or $4x - 1 = 0$
$x = -\frac{1}{4}$ or $x = \frac{1}{4}$

33. $11q^2 - 44 = 0$
$q^2 - 4 = 0$
$(q + 2)(q - 2) = 0$
$q + 2 = 0$ or $q - 2 = 0$
$q = -2$ or $q = 2$

34. $14s^2 - 21s = 0$
$2s^2 - 3s = 0$
$s(2s - 3) = 0$
$s = 0$ or $2s - 3 = 0$
$s = 0$ or $s = \frac{3}{2}$

35. $45n^2 + 10n = 0$
$9n^2 + 2n = 0$
$n(9n + 2) = 0$
$n = 0$ or $9n + 2 = 0$
$n = 0$ or $n = -\frac{2}{9}$

36. $4x^2 - 20x + 25 = 0$
$(2x - 5)(2x - 5) = 0$
$(2x - 5)^2 = 0$
$2x - 5 = 0$
$x = \frac{5}{2}$

37. $4p^2 + 12p + 9 = 0$
$(2p + 3)(2p + 3) = 0$
$(2p + 3)^2 = 0$
$2p + 3 = 0$
$p = -\frac{3}{2}$

38. $15x^2 + 7x - 2 = 0$
$(5x - 1)(3x + 2) = 0$
$5x - 1 = 0$ or $3x + 2 = 0$
$x = \frac{1}{5}$ or $x = -\frac{2}{3}$

39. $6r^2 - 7r - 5 = 0$
$(3r - 5)(2r + 1) = 0$
$3r - 5 = 0$ or $2r + 1 = 0$
$r = \frac{5}{3}$ or $r = -\frac{1}{2}$

40. $36z^2 + 96z + 15 = 0$
$(6z + 1)(6z + 15) = 0$
$6z + 1 = 0$ or $6z + 15 = 0$
$z = \frac{1}{6}$ or $z = -\frac{5}{2}$

41. $y = 4x^2 - 19x - 5$
$= (4x + 1)(x - 5)$

The zeros of the function are $-\frac{1}{4}$ and 5.

42. $g(x) = 3x^2 - 8x + 5$
$= (3x - 5)(x - 1)$

The zeros of the function $\frac{5}{3}$ and 1.

43. $y = 5x^2 - 27x - 18$
$= (5x + 3)(x - 6)$

The zeros of the function are $-\frac{3}{5}$ and 6.

44. $f(x) = 3x^2 - 3x$
$= 3x(x - 1)$

The zeros of the function are 0 and 1.

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45. $y = 11x^2 - 19x - 6$
$= (11x + 3)(x - 2)$

The zeros of the function are $-\frac{3}{11}$ and 2.

46. $y = 16x^2 - 2x - 5$
$= (8x - 5)(2x + 1)$

The zeros of the function are $\frac{5}{8}$ and $-\frac{1}{2}$.

47. $y = 15x^2 - 5x - 20$
$= 5(3x^2 - x - 4)$
$= 5(3x - 4)(x + 1)$

The zeros of the function are $\frac{4}{3}$ and -1.

48. $y = 18x^2 - 6x - 4$
$= 2(9x^2 - 3x - 2)$
$= 2(3x - 2)(3x + 1)$

The zeros of the function are $\frac{2}{3}$ and $-\frac{1}{3}$.

49. $g(x) = 12x^2 + 5x - 7$
$= (12x - 7)(x + 1)$

The zeros of the function are $\frac{7}{12}$ and -1.

50. Area of square $= s^2$

$36 = (2x)^2$
$36 = 4x^2$
$0 = 4x^2 - 36$
$0 = x^2 - 9$
$0 = (x + 3)(x - 3)$
$x + 3 = 0$ or $x - 3 = 0$
$x = -3$ or $x = 3$

The value of x is 3 units.

51. Area of rectangle $= \ell w$

$30 = (3x + 1)(x)$
$30 = 3x^2 + x$
$0 = 3x^2 + x - 30$
$0 = (3x + 10)(x - 3)$
$3x + 10 = 0$ or $x - 3 = 0$
$x = -\frac{10}{3}$ or $x = 3$

The value of x is 3 units.

52. Area of triangle $= \frac{1}{2}bh$

$115 = \frac{1}{2}(5x - 2)(2x)$
$115 = 5x^2 - 2x$
$0 = 5x^2 - 2x - 115$
$0 = (5x + 23)(x - 5)$
$5x + 23 = 0$ or $x - 5 = 0$
$x = -\frac{23}{5}$ or $x = 5$

The value of x is 5 units.

53. $2x^2 - 4x - 8 = -x^2 + x$
$3x^2 - 5x - 8 = 0$
$(3x - 8)(x + 1) = 0$
$3x - 8 = 0$ or $x + 1 = 0$
$x = \frac{8}{3}$ or $x = -1$

54. $24x^2 + 8x + 2 = 5 - 6x$
$24x^2 + 14x - 3 = 0$
$(6x - 1)(4x + 3) = 0$
$6x - 1 = 0$ or $4x + 3 = 0$
$x = \frac{1}{6}$ or $x = -\frac{3}{4}$

55. $18x^2 - 22x = 28$
$18x^2 - 22x - 28 = 0$
$9x^2 - 11x - 14 = 0$
$(9x + 7)(x - 2) = 0$
$9x + 7 = 0$ or $x - 2 = 0$
$x = -\frac{7}{9}$ or $x = 2$

56. $13x^2 + 21x = -5x^2 + 22$
$18x^2 + 21x - 22 = 0$
$(6x + 11)(3x - 2) = 0$
$6x + 11 = 0$ or $3x - 2 = 0$
$x = -\frac{11}{6}$ or $x = \frac{2}{3}$

57. $x = 4x^2 - 15x$
$0 = 4x^2 - 16x$
$0 = x^2 - 4x$
$0 = x(x - 4)$
$x = 0$ or $x = 4$

58. $(x + 8)^2 = 16 - x^2 + 9x$
$x^2 + 16x + 64 = 16 - x^2 + 9x$
$2x^2 + 7x + 48 = 0$

The expression $2x^2 + 7x + 48$ does not factor and has no solution.

59. $2x^3 - 5x^2 + 3x$
$x(2x^2 - 5x + 3)$
$x(2x - 3)(x - 1)$

60. $8x^4 - 8x^3 - 6x^2$
$2x^2(4x^2 - 4x - 3)$
$2x^2(2x + 1)(2x - 3)$

61. $9x^3 - 4x$
$x(9x^2 - 4)$
$x(3x + 2)(3x - 2)$

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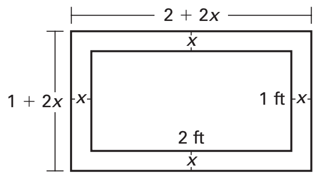
Problem Solving

62.

Area of border (square feet)	=	Area of window and border (square feet)	−	Area of window (square feet)
4	=	$(2 + 2x)(1 + 2x)$	−	$2(1)$

$$4 = 2 + 6x + 4x^2 - 2$$
$$0 = 4x^2 + 6x - 4$$
$$0 = 2x^2 + 3x - 2$$
$$0 = (2x - 1)(x + 2)$$
$$2x - 1 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = \frac{1}{2} \quad \text{or} \quad x = -2$$

Reject the negative value, -2. The width of the border should be $\frac{1}{2}$ foot, or 6 inches.



63.

Area of flower bed and border (square feet)	−	Area of flower bed (square feet)	=	Area of flower bed (square feet)
$(12 + 2x)(8 + 2x)$	−	$12(8)$	=	$12(8)$

$$96 + 40x + 4x^2 - 96 = 96$$
$$4x^2 + 40x - 96 = 0$$
$$x^2 + 10x - 24 = 0$$
$$(x + 12)(x - 2) = 0$$
$$x + 12 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -12 \quad \text{or} \quad x = 2$$

Reject the negative value, -12.

The width of the border of yellow roses should be 2 feet.

64. A;

Monthly revenue (dollars)	=	Price (dollars/ surfboard)	•	Sales (surfboards)
$R(x)$	=	$(500 - 20x)$	•	$(45 + 5x)$

$$R(x) = -20(x - 25)(5)(x + 9)$$
$$R(x) = -100(x - 25)(x + 9)$$
$$x = \frac{p + q}{2} = \frac{25 + (-9)}{2} = 8$$

To maximize revenue, each surfboard should cost $500 - 20(8) = \$340$.

65.

Daily revenue (dollars)	=	Sales (sandwiches)	•	Price (dollars/ sandwich)
$R(x)$	=	$(330 + 15x)$	•	$(6 - 0.25x)$

$$R(x) = 15(x + 22)(-0.25x + 6)$$
$$R(x) = 15(x + 22)(\,-\,0.25)(x - 24)$$
$$R(x) = -3.75(x + 22)(x - 24)$$

The zeros of the revenue function are -22 and 24, and their average is $\frac{-22 + 24}{2} = 1$. To maximize revenue, each sandwich should cost $6 - 0.25(1) = \$5.75$. The maximum daily revenue is $R(1) = -3.75(1 + 22)(1 - 24) = \$1983.75$.

66.

Area of mat (square in.)	=	Area of painting and mat (square in.)	−	Area of painting (square in.)
714	=	$(25 + 4x)(21 + 2x)$	−	$(25)(21)$

$$714 = 525 + 134x + 8x^2 - 525$$
$$0 = 8x^2 + 134x - 714$$
$$0 = 4x^2 + 67x - 357$$
$$0 = (4x - 17)(x + 21)$$
$$4x - 17 = 0 \quad \text{or} \quad x + 21 = 0$$
$$x = \frac{17}{4} \quad \text{or} \quad x = -21$$

Reject the negative value, -21.

Left and right: $2x = 2\left(\frac{17}{4}\right) = \frac{17}{2} = 8.5$

Top and bottom: $x = \frac{17}{4} = 4.25$

The mat should be 8.5 inches wide to the left and right of the painting and 4.25 inches wide at the top and bottom of the painting.

67. a. $36 + x = 108$

$x = 72$

The girth of the package is 72 inches.

b. $2w + 2h = 72$

$$2w = 72 - 2h$$
$$w = 36 - h$$
$$V = \ell wh$$
$$= (36)(36 - h)(h)$$
$$= 36h(36 - h)$$
$$= -36h(h - 36)$$

c. $h = \frac{0 + 36}{2} = 18$ $\qquad w = 36 - 18 = 18$

Vertex: (18, 18)

A height of 18 inches and a width of 18 inches maximize the volume of the package.

Using these maximum dimensions, the maximum volume of the package is $V = \ell wh = 36(18)(18) =$ 11,664 cubic inches.

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68. Theorem: If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

$$(3x + 2)(x + 1) = (2x)(5x - 4)$$
$$3x^2 + 5x + 2 = 10x^2 - 8x$$
$$0 = 7x^2 - 13x - 2$$
$$0 = (7x + 1)(x - 2)$$
$$7x + 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -\frac{1}{7} \quad \text{or} \quad x = 2$$

Reject the negative value, $-\frac{1}{7}$. The value of x is 2 units.

Mixed Review

69. $11 + 2x = 3(4x + 7)$
$$11 + 2x = 12x + 21$$
$$-10 = 10x$$
$$-1 = x$$

70. $6x - 19 = 5(3 + 2x)$
$$6x - 19 = 15 + 10x$$
$$-34 = 4x$$
$$-\frac{17}{2} = x$$

71. $-9(5x + 3) = 9x - 42$
$$-45x - 27 = 9x - 42$$
$$15 = 54x$$
$$\frac{5}{18} = x$$

72. $6(x - 7) = 15(2x - 4)$
$$6x - 42 = 30x - 60$$
$$18 = 24x$$
$$\frac{3}{4} = x$$

73. $9(x - 3) = 3(5x - 17)$
$$9x - 27 = 15x - 51$$
$$24 = 6x$$
$$4 = x$$

74. $4(3x - 11) = 3(11 - x) + x$
$$12x - 44 = 33 - 3x + x$$
$$12x - 44 = 33 - 2x$$
$$14x = 77$$
$$x = \frac{11}{2}$$

75. $4x + 9y = -14$

$3x + 5y = -7$

$$\begin{vmatrix} 4 & 9 \\ 3 & 5 \end{vmatrix} = 20 - 27 = -7$$

$$x = \frac{\begin{vmatrix} -14 & 9 \\ -7 & 5 \end{vmatrix}}{-7} = \frac{-70 - (-63)}{-7} = \frac{-7}{-7} = 1$$

$$y = \frac{\begin{vmatrix} 4 & -14 \\ 3 & -7 \end{vmatrix}}{-7} = \frac{-28 - (-42)}{-7} = \frac{14}{-7} = -2$$

The solution is $(1, -2)$.

76. $8x + 5y = -2$

$2x + 3y = 14$

$$\begin{vmatrix} 8 & 5 \\ 2 & 3 \end{vmatrix} = 24 - 10 = 14$$

$$x = \frac{\begin{vmatrix} -2 & 5 \\ 14 & 3 \end{vmatrix}}{14} = \frac{-6 - 70}{14} = -\frac{76}{14} = -\frac{38}{7}$$

$$y = \frac{\begin{vmatrix} 8 & -2 \\ 2 & 14 \end{vmatrix}}{14} = \frac{112 - (-4)}{14} = \frac{116}{14} = \frac{58}{7}$$

The solution is $\left(-\frac{38}{7}, \frac{58}{7}\right)$.

77. $5x - 8y = -50$

$2x - 3y = -25$

$$\begin{vmatrix} 5 & -8 \\ 2 & -3 \end{vmatrix} = -15 - (-16) = 1$$

$$x = \frac{\begin{vmatrix} -50 & -8 \\ -25 & -3 \end{vmatrix}}{1} = \frac{150 - 200}{1}$$

$$y = \frac{\begin{vmatrix} 5 & -50 \\ 2 & -25 \end{vmatrix}}{1} = \frac{-125 - (-100)}{1} = -25$$

The solution is $(-50, -25)$.

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78. $y = x^2 - 3x - 18$

$x = -\frac{b}{2a} = -\frac{(-3)}{2(1)} = \frac{3}{2}$

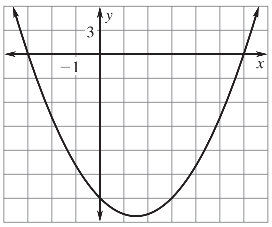
$y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 18 = -\frac{81}{4}$

Vertex: $\left(\frac{3}{2}, -\frac{81}{4}\right)$

Axis of symmetry: $x = \frac{3}{2}$

y-intercept: -18; $(0, -18)$

$x = -3$: $y = (-3)^2 - 3(-3) - 18 = 0$; $(-3, 0)$



79. $f(x) = 2x^2 + 11x + 5$

$x = -\frac{b}{2a} = -\frac{11}{2(2)} = -\frac{11}{4}$

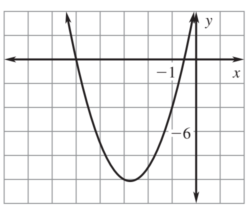
$f(x) = 2\left(-\frac{11}{4}\right)^2 + 11\left(-\frac{11}{4}\right) + 5 = -\frac{81}{8}$

Vertex: $\left(-\frac{11}{4}, -\frac{81}{8}\right)$

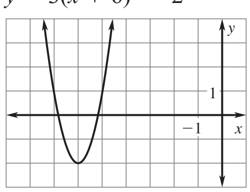
Axis of symmetry: $x = -\frac{11}{4}$

y-intercept: 5; $(0, 5)$

$x = -1$: $f(x) = 2(-1)^2 + 11(-1) + 5 = -4$; $(-1, -4)$



80. $y = 3(x + 6)^2 - 2$



$a = 3, h = -6, k = -2$

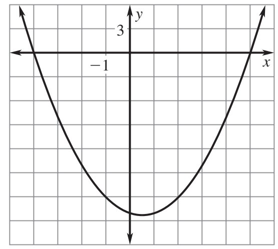
Vertex: $(-6, -2)$

Axis of symmetry: $x = -6$

$x = -5$: $y = 3(-5 + 6)^2 - 2 = 1$; $(-5, 1)$

$x = -4$: $y = 3(-4 + 6)^2 - 2 = 10$; $(-4, 10)$

81. $g(x) = (x + 4)(x - 5)$



x-intercepts: $p = -4$ and $q = 5$

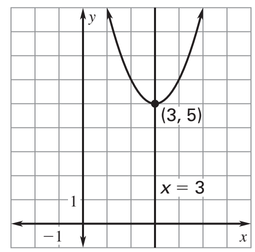
$x = \frac{p + q}{2} = \frac{-4 + 5}{2} = \frac{1}{2}$

$g\left(\frac{1}{2}\right) = \left(\frac{1}{2} + 4\right)\left(\frac{1}{2} - 5\right) = -\frac{81}{4}$

Vertex: $\left(\frac{1}{2}, -\frac{81}{4}\right)$

Quiz 4.1–4.4 (p. 265)

1. $y = x^2 - 6x + 14$



$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$

$y = (3)^2 - 6(3) + 14 = 5$

Vertex: $(3, 5)$

Axis of symmetry: $x = 3$

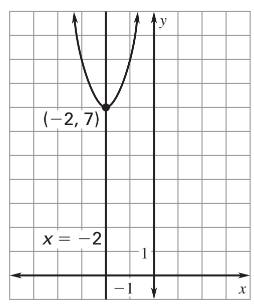
y-intercept: 14; $(0, 14)$

$x = 2$: $y = (2)^2 - 6(2) + 14 = 6$; $(2, 6)$

2. $y = 2x^2 + 8x + 15$

$x = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$

$y = 2(-2)^2 + 8(-2) + 15 = 7$



Vertex: $(-2, 7)$

Axis of symmetry: $x = -2$

y-intercept: 15; $(0, 15)$

$x = -1$: $y = 2(-1)^2 + 8(-1) + 15 = 9$; $(-1, 9)$

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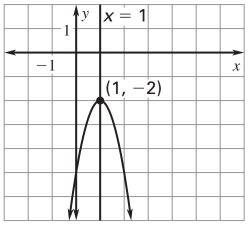
3. $f(x) = -3x^2 + 6x - 5$

$$x = \frac{-b}{2a} = \frac{-6}{2(-3)} = 1$$

$f(1) = -3(1)^2 + 6(1) - 5 = -2$



Vertex: $(1, -2)$

Axis of symmetry: $x = 1$

y-intercept: -5; $(0, -5)$

$x = -1$: $f(-1) = -3(-1)^2 + 6(-1) - 5 = -14$; $(-1, -14)$

4. $y = (x - 4)(x - 8)$
$= x^2 - 8x - 4x + 32$
$= x^2 - 12x + 32$

5. $g(x) = -2(x + 3)(x - 7)$
$= -2(x^2 - 7x + 3x - 21)$
$= -2(x^2 - 4x - 21)$
$= -2x^2 + 8x + 42$

6. $y = 5(x + 6)^2 - 2$
$= 5(x + 6)(x + 6) - 2$
$= 5(x^2 + 6x + 6x + 36) - 2$
$= 5(x^2 + 12x + 36) - 2$
$= 5x^2 + 60x + 180 - 2$
$= 5x^2 + 60x + 178$

7. $x^2 + 9x + 20 = 0$
$(x + 5)(x + 4) = 0$
$x + 5 = 0$ or $x + 4 = 0$
$x = -5$ or $x = -4$

8. $n^2 - 11n + 24 = 0$
$(n - 8)(n - 3) = 0$
$n - 8 = 0$ or $n - 3 = 0$
$n = 8$ or $n = 3$

9. $z^2 - 3z - 40 = 0$
$(z + 5)(z - 8) = 0$
$z + 5 = 0$ or $z - 8 = 0$
$z = -5$ or $z = 8$

10. $5s^2 - 14s - 3 = 0$
$(5s + 1)(s - 3) = 0$
$5s + 1 = 0$ or $s - 3 = 0$
$s = -\frac{1}{5}$ or $s = 3$

11. $7a^2 - 30a + 8 = 0$
$(7a - 2)(a - 4) = 0$
$7a - 2 = 0$ or $a - 4 = 0$
$a = \frac{2}{7}$ or $a = 4$

12. $4x^2 + 20x + 25 = 0$
$(2x + 5)(2x + 5) = 0$
$2x + 5 = 0$
$x = -\frac{5}{2}$

13. Monthly revenue (dollars) = Price (dollars/DVD player) • Sales (DVD players)

$R(x) = (50 + 5x) \cdot (140 - 10x)$
$R(x) = (-10x + 140)(5x + 50)$
$R(x) = -10(x - 14)(5x + 50)$
$R(x) = -10 \cdot 5(x - 14)(x + 10)$
$R(x) = -50(x - 14)(x + 10)$

$$x = \frac{p + q}{2} = \frac{14 + (-10)}{2} = 2$$

To maximize revenue, each DVD player should cost $140 - 10(2) = \$120$.

$R(2) = -50(2 - 14)(2 + 10) = 7200$

The maximum monthly revenue is \$7200.

Lesson 4.5

4.5 Guided Practice (pp. 266–269)

1. $\sqrt{27} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$

2. $\sqrt{98} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$

3. $\sqrt{10} \cdot \sqrt{15} = \sqrt{150} = \sqrt{25} \cdot \sqrt{6} = 5\sqrt{6}$

4. $\sqrt{8} \cdot \sqrt{28} = \sqrt{224} = \sqrt{16} \cdot \sqrt{14} = 4\sqrt{14}$

5. $\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$

6. $\sqrt{\frac{15}{4}} = \frac{\sqrt{15}}{\sqrt{4}} = \frac{\sqrt{15}}{2}$

7. $\sqrt{\frac{11}{25}} = \frac{\sqrt{11}}{\sqrt{25}} = \frac{\sqrt{11}}{5}$

8. $\sqrt{\frac{36}{49}} = \frac{\sqrt{36}}{\sqrt{49}} = \frac{6}{7}$

9. $\sqrt{\frac{6}{5}} = \frac{\sqrt{6}}{\sqrt{5}} = \frac{\sqrt{6}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{30}}{5}$

10. $\sqrt{\frac{9}{8}} = \frac{\sqrt{9}}{\sqrt{8}} = \frac{\sqrt{9}}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{72}}{8} = \frac{\sqrt{36} \cdot \sqrt{2}}{8} = \frac{6\sqrt{2}}{8} = \frac{3\sqrt{2}}{4}$

11. $\sqrt{\frac{17}{12}} = \frac{\sqrt{17}}{\sqrt{12}} = \frac{\sqrt{17}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{204}}{12} = \frac{\sqrt{4} \cdot \sqrt{51}}{12}$
$= \frac{2\sqrt{51}}{12} = \frac{\sqrt{51}}{6}$

12. $\sqrt{\frac{19}{21}} = \frac{\sqrt{19}}{\sqrt{21}} = \frac{\sqrt{19}}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{\sqrt{399}}{21}$

13. $\frac{-6}{7 - \sqrt{5}} = \frac{-6}{7 - \sqrt{5}} \cdot \frac{7 + \sqrt{5}}{7 + \sqrt{5}} = \frac{-42 - 6\sqrt{5}}{49 + 7\sqrt{5} - 7\sqrt{5} - 5}$
$= \frac{-42 - 6\sqrt{5}}{44} = \frac{-21 - 3\sqrt{5}}{22}$

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14. $\frac{2}{4+\sqrt{11}} = \frac{2}{4+\sqrt{11}} \cdot \frac{4-\sqrt{11}}{4-\sqrt{11}} = \frac{8-2\sqrt{11}}{16-4\sqrt{11}+4\sqrt{11}-11}$

$= \frac{8-2\sqrt{11}}{5}$

15. $\frac{-1}{9+\sqrt{7}} = \frac{-1}{9+\sqrt{7}} \cdot \frac{9-\sqrt{7}}{9-\sqrt{7}} = \frac{-9+\sqrt{7}}{81-9\sqrt{7}+9\sqrt{7}-7}$

$= \frac{-9+\sqrt{7}}{74}$

16. $\frac{4}{8-\sqrt{3}} = \frac{4}{8-\sqrt{3}} \cdot \frac{8+\sqrt{3}}{8+\sqrt{3}} = \frac{32+4\sqrt{3}}{64+8\sqrt{3}-8\sqrt{3}-3}$

$= \frac{32+4\sqrt{3}}{61}$

17. $5x^2 = 80$

$x^2 = 16$

$x = \pm\sqrt{16}$

$x = \pm 4$

18. $z^2 - 7 = 29$

$z^2 = 36$

$z = \pm\sqrt{36}$

$z = \pm 6$

19. $3(x-2)^2 = 40$

$(x-2)^2 = \frac{40}{3}$

$x - 2 = \pm\sqrt{\frac{40}{3}}$

$x - 2 = \pm\frac{\sqrt{40}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x - 2 = \pm\frac{\sqrt{120}}{3}$

$x - 2 = \pm\frac{\sqrt{4} \cdot \sqrt{30}}{3}$

$x - 2 = \pm\frac{2\sqrt{30}}{3}$

$x = 2 \pm \frac{2\sqrt{30}}{3}$

20. $h = -16t^2 + h_0$

$0 = -16t^2 + 30$

$-30 = -16t^2$

$\frac{30}{16} = t^2$

$\pm\sqrt{\frac{30}{16}} = t$

$\pm 1.4 \approx t$

Reject the negative solution, -1.4, because time must be positive. The container will fall for about 1.4 seconds before it hits the ground.

4.5 Exercises (pp. 269–271)

Skill Practice

1. In the expression $\sqrt{72}$, 72 is called the radicand of the expression.

2. To "rationalize the denominator" of a quotient containing square roots means to eliminate the radical from the denominator.

3. $\sqrt{28} = \sqrt{4} \cdot \sqrt{7} = 2\sqrt{7}$

4. $\sqrt{192} = \sqrt{64} \cdot \sqrt{3} = 8\sqrt{3}$

5. $\sqrt{150} = \sqrt{25} \cdot \sqrt{6} = 5\sqrt{6}$

6. $\sqrt{3} \cdot \sqrt{27} = \sqrt{81} = 9$

7. $4\sqrt{6} \cdot \sqrt{6} = 4\sqrt{36} = 4(6) = 24$

8. $5\sqrt{24} \cdot 3\sqrt{10} = 15\sqrt{240} = 15(\sqrt{16} \cdot \sqrt{15})$

$= 15(4\sqrt{15}) = 60\sqrt{15}$

9. $\sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{\sqrt{16}} = \frac{\sqrt{5}}{4}$

10. $\sqrt{\frac{35}{36}} = \frac{\sqrt{35}}{\sqrt{36}} = \frac{\sqrt{35}}{6}$

11. $\frac{8}{\sqrt{3}} = \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$

12. $\frac{7}{\sqrt{12}} = \frac{7}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{7\sqrt{12}}{12} = \frac{7\sqrt{3}}{6}$

13. $\sqrt{\frac{18}{11}} = \frac{\sqrt{18}}{\sqrt{11}} = \frac{\sqrt{18}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{198}}{11} = \frac{\sqrt{9} \cdot \sqrt{22}}{11} = \frac{3\sqrt{22}}{11}$

14. $\sqrt{\frac{13}{28}} = \frac{\sqrt{13}}{\sqrt{28}} = \frac{\sqrt{13}}{\sqrt{28}} \cdot \frac{\sqrt{28}}{\sqrt{28}} = \frac{\sqrt{364}}{28} = \frac{\sqrt{4} \cdot \sqrt{91}}{28}$

$= \frac{2\sqrt{91}}{28} = \frac{\sqrt{91}}{14}$

15. $\frac{2}{1-\sqrt{3}} = \frac{2}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{2+2\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3} = \frac{2+2\sqrt{3}}{-2}$

$= -1 - \sqrt{3}$

16. $\frac{1}{5+\sqrt{6}} = \frac{1}{5+\sqrt{6}} \cdot \frac{5-\sqrt{6}}{5-\sqrt{6}} = \frac{5-\sqrt{6}}{25-5\sqrt{6}+5\sqrt{6}-6}$

$= \frac{5-\sqrt{6}}{19}$

17. $\frac{\sqrt{2}}{4+\sqrt{5}} = \frac{\sqrt{2}}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{4\sqrt{2}-\sqrt{10}}{16-4\sqrt{5}+4\sqrt{5}-5}$

$= \frac{4\sqrt{2}-\sqrt{10}}{11}$

18. $\frac{3+\sqrt{7}}{2-\sqrt{10}} = \frac{3+\sqrt{7}}{2-\sqrt{10}} \cdot \frac{2+\sqrt{10}}{2+\sqrt{10}} = \frac{6+3\sqrt{10}+2\sqrt{7}+\sqrt{70}}{4+2\sqrt{10}-2\sqrt{10}-10}$

$= -\frac{6+3\sqrt{10}+2\sqrt{7}+\sqrt{70}}{6}$

19. C; $\sqrt{108} = \sqrt{36} \cdot \sqrt{3} = 6\sqrt{3}$

20. The expression was not completely simplified.

$\sqrt{96} = \sqrt{4} \cdot \sqrt{24} = 2\sqrt{24} = 2\sqrt{4} \cdot \sqrt{6} = 4\sqrt{6}$

or $\sqrt{96} = \sqrt{16} \cdot \sqrt{6} = 4\sqrt{6}$

21. Because $81 > 0$, the equation $x^2 = 81$ has two real-number solutions: $x = \sqrt{81}$, and $x = -\sqrt{81}$.

$5x^2 = 405$

$x^2 = 81$

$x = \pm\sqrt{81}$

$x = \pm 9$

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22. $s^2 = 169$

$s = \pm\sqrt{169}$

$s = \pm 13$

23. $a^2 = 50$

$a = \pm\sqrt{50}$

$a = \pm\sqrt{25} \cdot \sqrt{2}$

$a = \pm 5\sqrt{2}$

24. $x^2 = 84$

$x = \pm\sqrt{84}$

$x = \pm\sqrt{4} \cdot \sqrt{21}$

$x = \pm 2\sqrt{21}$

25. $6z^2 = 150$

$z^2 = 25$

$z = \pm\sqrt{25}$

$z = \pm 5$

26. $4p^2 = 448$

$p^2 = 112$

$p = \pm\sqrt{112}$

$p = \pm\sqrt{16} \cdot \sqrt{7}$

$p = \pm 4\sqrt{7}$

27. $-3w^2 = -213$

$w^2 = 71$

$w = \pm\sqrt{71}$

28. $7r^2 - 10 = 25$

$7r^2 = 35$

$r^2 = 5$

$r = \pm\sqrt{5}$

29. $\frac{x^2}{25} - 6 = -2$

$\frac{x^2}{25} = 4$

$x^2 = 100$

$x = \pm\sqrt{100}$

$x = \pm 10$

30. $\frac{t^2}{20} + 8 = 15$

$\frac{t^2}{20} = 7$

$t^2 = 140$

$t = \pm\sqrt{140}$

$t = \pm\sqrt{4} \cdot \sqrt{35}$

$t = \pm 2\sqrt{35}$

31. $4(x - 1)^2 = 8$

$(x - 1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

32. $7(x - 4)^2 - 18 = 10$

$7(x - 4)^2 = 28$

$(x - 4)^2 = 4$

$x - 4 = \pm\sqrt{4}$

$x - 4 = \pm 2$

$x = 4 \pm 2 = 6, 2$

33. $2(x + 2)^2 - 5 = 8$

$2(x + 2)^2 = 13$

$(x + 2)^2 = \frac{13}{2}$

$x + 2 = \pm\sqrt{\frac{13}{2}}$

$x + 2 = \pm\frac{\sqrt{13}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}}$

$x + 2 = \pm\frac{\sqrt{26}}{2}$

$x = -2 \pm \frac{\sqrt{26}}{2}$

34. C; $3(x + 2)^2 + 4 = 13$

$3(x + 2)^2 = 9$

$(x + 2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x = -2 \pm \sqrt{3}$

35. One method for solving the equation is to use the special factoring pattern known as the difference of two squares.

$x^2 - 4 = 0$

$(x + 2)(x - 2) = 0$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ or $x = 2$

Another method for solving the equation is to use square roots.

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

36. *Sample answer:*

a. $x^2 = 121$

b. $x^2 = 0$

c. $x^2 = -36$

37. $a(x + b)^2 = c$

$(x + b)^2 = \frac{c}{a}$

$x + b = \pm\sqrt{\frac{c}{a}}$

$x + b = \pm\frac{\sqrt{c}}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}}$

$x + b = \pm\frac{\sqrt{ca}}{a}$

$x = -b \pm \frac{\sqrt{ca}}{a}$

Problem Solving

38. $h = -16t^2 + h_0$

$h = -16t^2 + 40$

$0 = -16t^2 + 40$

$-40 = -16t^2$

$\frac{40}{16} = t^2$

$\pm\sqrt{\frac{40}{16}} = t$

$\pm 1.6 \approx t$

Reject the negative solution, -1.6, because time must be positive. The diver is in the air for about 1.6 seconds.

39. $h = -\frac{g}{2}t^2 + h_0$

Earth: $0 = -\frac{32}{2}t^2 + 150$

$0 = -16t^2 + 150$

$-150 = -16t^2$

$\frac{150}{16} = t^2$

$\pm\sqrt{\frac{150}{16}} = t$

$\pm 3.1 \approx t$

It takes the rock about 3.1 seconds to hit the surface of Earth.

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Mars: $0 = -\frac{12}{2}t^2 + 150$

$0 = -6t^2 + 150$

$-150 = -6t^2$

$25 = t^2$

$\pm\sqrt{25} = t^2$

$\pm 5 = t$

It takes the rock 5 seconds to hit the surface of Mars.

Jupiter: $0 = -\frac{76}{2}t^2 + 150$

$0 = -38t^2 + 150$

$-150 = -38t^2$

$\frac{150}{38} = t^2$

$\pm\sqrt{\frac{150}{38}} = t$

$\pm 2 \approx t$

It takes the rock about 2 seconds to hit the surface of Jupiter.

Saturn: $0 = -\frac{30}{2}t^2 + 150$

$0 = -15t^2 + 150$

$-150 = -15t^2$

$10 = t^2$

$\pm\sqrt{10} = t$

$\pm 3.2 \approx t$

It takes the rock about 3.2 seconds to hit the surface of Saturn.

Pluto: $0 = -\frac{2}{2}t^2 + 150$

$0 = -t^2 + 150$

$-150 = -t^2$

$150 = t^2$

$\pm\sqrt{150} = t$

$\pm 12.2 \approx t$

It takes the rock about 12.2 seconds to hit the surface of Pluto.

40. $h = 0.019s^2$

$5 = 0.019s^2$	$20 = 0.019s^2$
$\frac{5}{0.019} = s^2$	$\frac{20}{0.019} = s^2$
$\pm\sqrt{\frac{5}{0.019}} = s$	$\pm\sqrt{\frac{20}{0.019}} = s$
$\pm 16.2 \approx s$	$\pm 32.4 \approx s$
16.2 knots	32.4 knots

The wind speed required to generate 20 foot waves is twice the wind speed required to generate 5 foot waves.

41. a. Area of circle = Area of square

$\pi r^2 = 10^2$

$\pi r^2 = 100$

b. $\pi r^2 = 100$

$r^2 = \frac{100}{\pi}$

$r = \pm\sqrt{\frac{100}{\pi}}$

$r \approx \pm 5.6$

The radius of the circular lot should be about 5.6 feet.

c. $\pi r^2 = s^2$

$r^2 = \frac{s^2}{\pi}$

$r = \pm\sqrt{\frac{s^2}{\pi}}$

42. a. $R = 0.00829s^2$

$5 = 0.00829s^2$

$\frac{5}{0.00829} = s^2$

$\pm\sqrt{\frac{5}{0.00829}} = s$

$\pm 24.6 \approx s$

Reject the negative solution, -24.6, because speed must be positive. The speed of the racing cyclist is about 24.6 miles per hour.

b. $R = 0.00829s^2$

$R = 0.00829(2s)^2$

$R = 0.00829(4s^2)$

$R = 4(0.00829s^2)$

The air resistance quadruples when the cyclist's speed doubles.

43. $h = \left(\sqrt{h_0} - \frac{2\pi d^2\sqrt{3}}{\ell w}t\right)^2$

The pool is completely drained when $h = 0$.

$0 = \left(\sqrt{h_0} - \frac{2\pi d^2\sqrt{3}}{\ell w}t\right)^2$

$0 = \sqrt{h_0} - \frac{2\pi d^2\sqrt{3}}{\ell w}t$

$\frac{2\pi d^2\sqrt{3}}{\ell w}t = \sqrt{h_0}$

$t = \frac{\ell w\sqrt{h_0}}{2\pi d^2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\ell w\sqrt{3h_0}}{6\pi d^2}$

Mixed Review

44. $(-5)^2 = 25$

45. $(-4)^2 = 16$

46. $(-8)^2 = 64$

47. $(-13)^2 = 169$

48. $-3^2 = -9$

49. $-11^2 = -121$

50. $-15^2 = -225$

51. $-7^2 = -49$

52. $x - 8 = 2$

$x = 10$

53. $3x + 4 = 13$

$3x = 9$

$x = 3$

54. $2x - 1 = 6x + 3$

$-1 = 4x + 3$

$-4 = 4x$

$-1 = x$

55. $x + 9 > 5$

$x > -4$

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56. $-7x - 15 \geq 6$

$-7x \geq 21$

$x \leq -3$

57. $3 - 6x \leq 23 - 10x$

$3 + 4x \leq 23$

$4x \leq 20$

$x \leq 5$

58. $|x + 12| = 5$

$x + 12 = 5$ or $x + 12 = -5$

$x = -7$ or $x = -17$

59. $|-2 + 3x| = 10$

$-2 + 3x = 10$ or $-2 + 3x = -10$

$3x = 12$ or $3x = -8$

$x = 4$ or $x = -\frac{8}{3}$

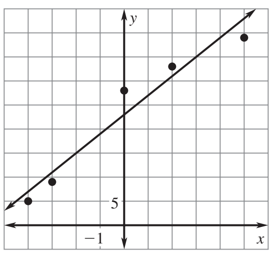
60. $\left|\frac{1}{2}x + 9\right| \geq 4$

$\frac{1}{2}x + 9 \leq -4$ or $\frac{1}{2}x + 9 \geq 4$

$\frac{1}{2}x \leq -13$ or $\frac{1}{2}x \geq -5$

$x \leq -26$ or $x \geq -10$

61. a.



b. *Sample answer:* Using $(-4, 5)$ and $(-3, 9)$,

$$m = \frac{9 - 5}{-3 - (-4)} = \frac{4}{1} = 4.$$

$y - y_1 = m(x - x_1)$

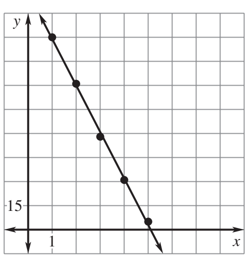
$y - 5 = 4(x + 4)$

$y - 5 = 4x + 16$

$y = 4x + 21$

c. *Sample answer:* When $x = 20$, $y = 4(20) + 21 = 101$.

62. a.



b. *Sample answer:* Using $(1, 120)$ and $(2, 91)$,

$$m = \frac{91 - 120}{2 - 1} = -29.$$

$y - y_1 = m(x - x_1)$

$y - 120 = -29(x - 1)$

$y - 120 = -29x + 29$

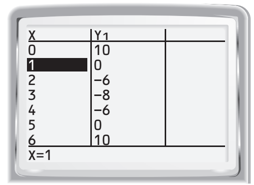
$y = -29x + 149$

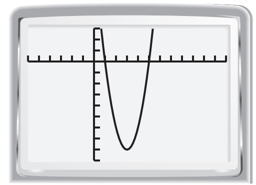
c. *Sample answer:*
When $x = 20$, $y = -29(20) + 149 = -431$.

Problem Solving Workshop 4.5 (p. 273)

1. $2x^2 - 12x + 10 = 0$

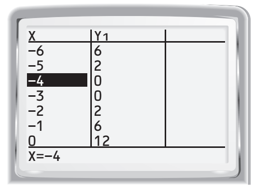
$x = 1$ or $x = 5$

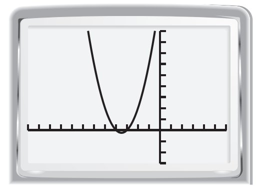




2. $x^2 + 7x + 12 = 0$

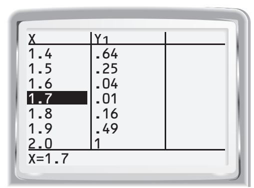
$x = -4$ or $x = -3$

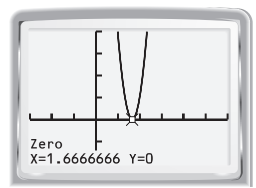




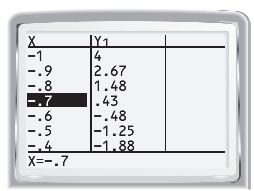
3. $9x^2 - 30x + 25 = 0$

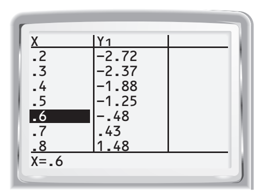
$x \approx 1.7$

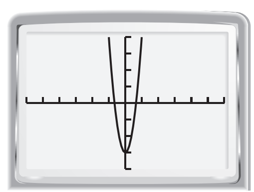




4. $7x^2 - 3 = 0$







The tables show that x is between -0.7 and -0.6 or x is between 0.6 and 0.7.

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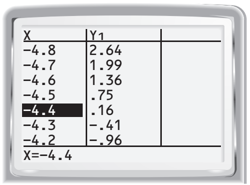
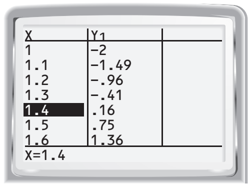
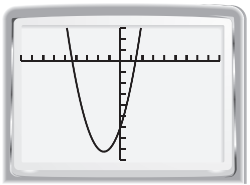
5. $x^2 + 3x - 6 = 0$



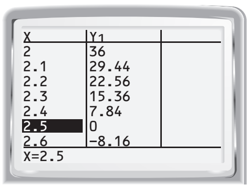


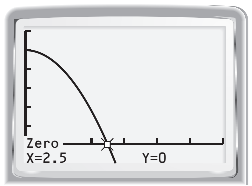


The tables show that x is between -4.4 and -4.3 or x is between 1.3 and 1.4.

6. $h = -16t^2 + h_0$

$0 = -16t^2 + 100$



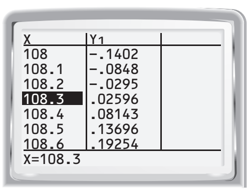


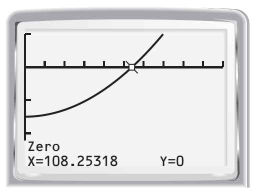
The container hits the ground 2.5 seconds after it is dropped.

7. $P = 0.00256s^2$

$30 = 0.00256s^2$

$0 = 0.00256s^2 - 30$



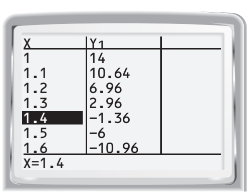


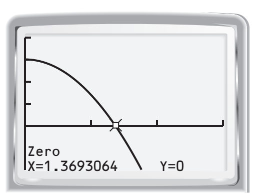
A pressure of 30 lb/ft^2 is produced by a wind speed between 108.2 and 108.3 miles per hour.

A pressure of 30 lb/ft^2 is produced by a wind speed of about 108.3 miles per hour.

8. $h = -16t^2 + h_0$

$0 = -16t^2 + 30$





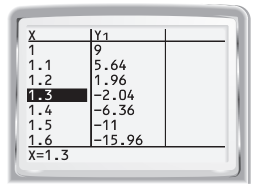
The shellfish hits the ground between 1.3 and 1.4 seconds after it is dropped.

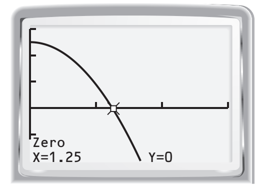
The shellfish hits the ground about 1.4 seconds after it is dropped.

9. $h = -16t^2 + h_0$

$4 = -16t^2 + 29$

$0 = -16t^2 + 25$





The ball is in the air between 1.2 and 1.3 seconds before your friend catches it.

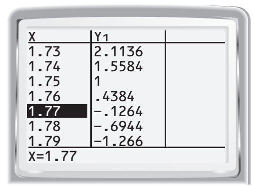
The ball is in the air 1.25 seconds before your friend catches it.

10. $h = 16t^2 + h_0$

$0 = -16t^2 + 50$

The table feature of the graphing calculator must be set so that the x-values start at 1 and increase in increments of 0.01. Scroll through the table to find the time x at which the height y of the container is 0 feet.

The container hits the ground between 1.76 and 1.77 seconds after it is dropped.



Mixed Review of Problem Solving (p. 274)

1. a. $h = -16t^2 + h_0$

$h = -16t^2 + 20$

b. $t = -\frac{b}{2a} = \frac{0}{2(-16)} = 0$

$h = -16(0)^2 + 20 = 20$

Vertex: (0, 20)

$t = 1$: $h = -16(1)^2 + 20$

$= 4$; (1, 4)

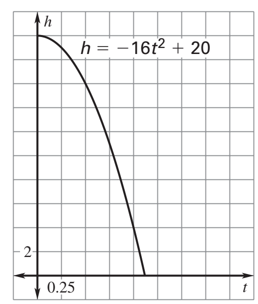
$h = 0$: $0 = -16t^2 + 20$

$-20 = -16t^2$

$\frac{20}{16} = t^2$

$\pm\sqrt{\frac{20}{16}} = t$

$\pm 1.12 \approx t$



c. $h = -16t^2 + 20$

$0 = -16t^2 + 20$

$-20 = -16t^2$

$\frac{20}{16} = t^2$

$\pm\sqrt{\frac{20}{16}} = t$

$\pm 1.1 \approx t$

The pinecone hits the ground after about 1.1 seconds.

d. domain: $0 \le t \le 1.1$

range: $0 \le h \le 20$

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2. a. $y = -0.0035x(x - 143.9)$

$y = -0.0035(x - 0)(x - 143.9)$

$p = 0$ and $q = 143.9$

Because the x-intercepts are 0 and 143.9 the domain is $0 \le x \le 143.9$.

$x = \frac{p + q}{2} = \frac{0 + 143.9}{2} = 71.95$

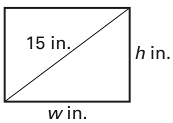
$y = -0.0035(71.95)(71.95 - 143.9) \approx 18.1$

Because the maximum height is the y-coordinate of the vertex, the range is $0 \le y \le 18.1$.

b. From part (a), you can see that the water cannon can shoot 143.9 feet.

c. From part (a), you can see that the maximum height of the water is 18.1 feet.

3.



a. $\frac{w}{h} = \frac{4}{3}$

$w = \frac{4}{3}h$

b. $w^2 + h^2 = 15^2$

$\left(\frac{4}{3}h\right)^2 + h^2 = 15^2$

c. $\frac{16}{9}h^2 + h^2 = 225$

$\frac{25}{9}h^2 = 225$

$\frac{9}{25} \cdot \frac{25}{9}h^2 = \frac{9}{25} \cdot 225$

$h^2 = 81$

$h = \pm\sqrt{81}$

$h = \pm 9$

You must reject the negative solution because the height of a laptop computer screen is never a negative measurement.

d. $h = 9$

$w = \frac{4}{3}h = \frac{4}{3}(9) = 12$

$A = \ell w = 9(12) = 108$

The laptop screen is 9 inches high and 12 inches wide. The area of the laptop screen is 108 in.^{2}

4. $416 = (20 + 2x)(24 + 2x) - 20(24)$

$416 = 480 + 88x + 4x^2 - 480$

$0 = 4x^2 + 88x - 416$

$0 = x^2 + 22x - 104$

$0 = (x + 26)(x - 4)$

$x + 26 = 0$ or $x - 4 = 0$

$x = -26$ or $x = 4$

Reject the negative value, -26. The width of the border is 4 inches.

If $x = 2(4) = 8$, the area of the border would be $4x^2 + 88x = 4(8)^2 + 88(8) = 960$ square inches. So, doubling the width requires more than twice as much metal.

5. a. $\underset{\text{(dollars)}}{\text{Daily revenue}} = \underset{\text{(dollars/slice)}}{\text{Price}} \cdot \underset{\text{(slices)}}{\text{Sales}}$

$R(x) = (2 + 0.25x) \cdot (80 - 5x)$

$R(x) = (0.25x + 2)(-5x + 80)$

$R(x) = 0.25(x + 8)(-5)(x - 16)$

$R(x) = -1.25(x + 8)(x - 16)$

b. $x = \frac{-8 + 16}{2} = 4$; when $x = 4$, R is maximized.

To maximize revenue, each slice of pizza should cost $0.25(4) + 2 = \$3$.

c. $R(x) = (2 - 0.25x)(80 + 5x)$

$R(x) = (-0.25x + 2)(5x + 80)$

$R(x) = -0.25(x - 8)(5)(x + 16)$

$R(x) = -1.25(x - 8)(x + 16)$

$x = \frac{8 + (-16)}{2} = -4$; when $x = -4$, R is maximized.

This means that there is no way to decrease the price and increase revenue. The only way to maximize revenue is to increase the price to $-0.25(-4) + 2 = \$3$ per slice. This confirms the answer in part (b).

6. $\underset{\text{(square feet)}}{\text{New area}} = \underset{\text{(feet)}}{\text{New length}} \cdot \underset{\text{(feet)}}{\text{New width}}$

$2(42)(8) = (42 + x) \cdot (8 + x)$

$672 = 336 + 50x + x^2$

$0 = x^2 + 50x - 336$

$0 = (x + 56)(x - 6)$

$x + 56 = 0$ or $x - 6 = 0$

$x = -56$ or $x = 6$

Reject the negative value, -56.

The value of x is 6 feet.

7. *Sample answer:* $y = -x^2 - 6x - 7$

$y = 3x^2 + 18x + 29$

$y = -5x^2 - 30x - 43$

Lesson 4.6

4.6 Guided Practice (pp. 275–279)

1. $x^2 = -13$

$x = \pm\sqrt{-13}$

$x = \pm i\sqrt{13}$

2. $x^2 = -38$

$x = \pm\sqrt{-38}$

$x = \pm i\sqrt{38}$

3. $x^2 + 11 = 3$

$x^2 = -8$

$x = \pm\sqrt{-8}$

$x = \pm i\sqrt{8}$

$x = \pm 2i\sqrt{2}$

4. $x^2 - 8 = -36$

$x^2 = -28$

$x = \pm\sqrt{-28}$

$x = \pm i\sqrt{28}$

$x = \pm 2i\sqrt{7}$

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5. $3x^2 - 7 = -31$

$3x^2 = -24$

$x^2 = -8$

$x = \pm\sqrt{-8}$

$x = \pm i\sqrt{8}$

$x = \pm 2i\sqrt{2}$

6. $5x^2 + 33 = 3$

$5x^2 = -30$

$x^2 = -6$

$x = \pm\sqrt{-6}$

$x = \pm i\sqrt{6}$

7. $(9 - i) + (-6 + 7i) = [9 + (-6)] + (-1 + 7)i$

$= 3 + 6i$

8. $(3 + 7i) - (8 - 2i) = (3 - 8) + [7 - (-2)]i$

$= -5 + 9i$

9. $-4 - (1 + i) - (5 + 9i) = [(-4 - 1)(i)] - (5 + 9i)$

$= (-5 - i) - (5 + 9i)$

$= (-5 - 5) + (-1 - 9)i$

$= -10 - 10i$

10. $5 + 3i + (-7i) = 5 - 4i$

The impedance of the circuit is $5 - 4i$ ohms.

11. $i(9 - i) = 9i - i^2$

$= 9i - (-1)$

$= 9i + 1$

$= 1 + 9i$

12. $(3 + i)(5 - i) = 15 - 3i + 5i - i^2$

$= 15 + 2i - (-1)$

$= 15 + 2i + 1$

$= 16 + 2i$

13. $\dfrac{5}{1 + i} = \dfrac{5}{1 + i} \cdot \dfrac{1 - i}{1 - i}$

$= \dfrac{5 - 5i}{1 - i + i - i^2}$

$= \dfrac{5 - 5i}{1 - (-1)}$

$= \dfrac{5 - 5i}{2}$

$= \dfrac{5}{2} - \dfrac{5}{2i}$

14. $\dfrac{5 + 2i}{3 - 2i} = \dfrac{5 + 2i}{3 - 2i} \cdot \dfrac{3 + 2i}{3 + 2i}$

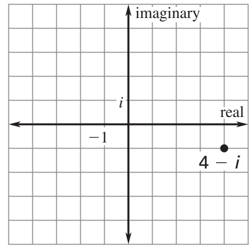
$= \dfrac{15 + 10i + 6i + 4i^2}{9 + 6i - 6i - 4i^2}$

$= \dfrac{15 + 16i + 4(-1)}{9 - 4(-1)}$

$= \dfrac{11 + 16i}{13}$

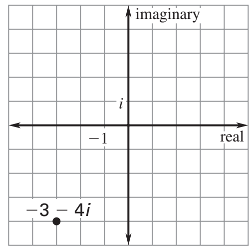
$= \dfrac{11}{13} + \dfrac{16}{13}i$

15.



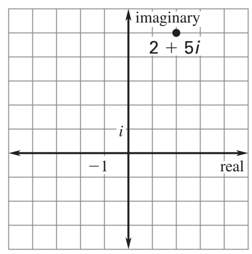
$|4 - i| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$

16.



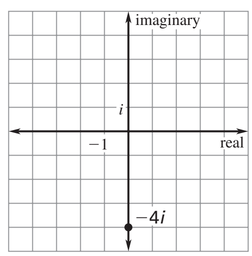
$|-3 - 4i| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$

17.



$|2 + 5i| = \sqrt{2^2 + 5^2} = \sqrt{29}$

18.



$|-4i| = |0 + (-4i)| \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$

4.6 Exercises (pp. 279–282)

Skill Practice

1. The complex conjugate of $a - bi$ is $a + bi$.

2. Not every complex number is an imaginary number. A complex number can also be a real number. For example, $a + 0i$.

3. $x^2 = -28$

$x = \pm\sqrt{-28}$

$x = \pm i\sqrt{28}$

$x = \pm 2i\sqrt{7}$

4. $r^2 = -624$

$r = \pm\sqrt{-624}$

$r = \pm i\sqrt{624}$

$r = \pm 4i\sqrt{39}$

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5. $z^2 + 8 = 4$
$z^2 = -4$
$z = \pm\sqrt{-4}$
$z = \pm i\sqrt{4}$
$z = \pm 2i$

6. $s^2 - 22 = -112$
$s^2 = -90$
$s = \pm\sqrt{-90}$
$s = \pm i\sqrt{90}$
$s = \pm 3i\sqrt{10}$

7. $2x^2 + 31 = 9$
$2x^2 = -22$
$x^2 = -11$
$x = \pm\sqrt{-11}$
$x = \pm i\sqrt{11}$

8. $9 - 4y^2 = 57$
$-4y^2 = 48$
$y^2 = -12$
$y = \pm\sqrt{-12}$
$y = \pm i\sqrt{12}$
$y = \pm 2i\sqrt{3}$

9. $6t^2 + 5 = 2t^2 + 1$
$4t^2 = -4$
$t^2 = -1$
$t = \pm\sqrt{-1}$
$t = \pm i\sqrt{1}$
$t = \pm i$

10. $3p^2 + 7 = -9p^2 + 4$
$12p^2 = -3$
$p^2 = -\frac{3}{12}$
$p^2 = -\frac{1}{4}$
$p = \pm\sqrt{-\frac{1}{4}}$
$p = \pm i\sqrt{\frac{1}{4}}$
$p = \pm\frac{1}{2}i$

11. $-5(n - 3)^2 = 10$
$(n - 3)^2 = -2$
$n - 3 = \pm\sqrt{-2}$
$n - 3 = \pm i\sqrt{2}$
$n = 3\pm\sqrt{2}i$

12. $(6 - 3i) + (5 + 4i) = (6 + 5) + (-3 + 4)i$
$= 11 + i$

13. $(9 + 8i) + (8 - 9i) = (9 + 8) + (8 - 9)i$
$= 17 - i$

14. $(-2 - 6i) - (4 - 6i) = (-2 - 4) + [-6 - (-6)]i$
$= -6$

15. $(-1 + i) - (7 - 5i) = (-1 - 7) + [1 - (-5)]i$
$= -8 + 6i$

16. $(8 + 20i) - (-8 + 12i) = [8 - (-8)] + (20 - 12)i$
$= 16 + 8i$

17. $(8 - 5i) - (-11 + 4i) = [8 - (-11)] + (-5 - 4)i$
$= 19 - 9i$

18. $(10 - 2i) + (-11 - 7i) = (10 - 11) + (-2 - 7)i$
$= -1 - 9i$

19. $(14 + 3i) + (7 + 6i) = (14 + 7) + (3 + 6)i$
$= 21 + 9i$

20. $(-1 + 4i) + (-9 - 2i) = (-1 - 9) + (4 - 2)i$
$= -10 + 2i$

21. C;
$(2 + 3i) - (7 + 4i) = (2 - 7) + (3 - 4)i$
$= -5 - i$

22. $6i(3 + 2i) = 18i + 12i^2$
$= 18i + 12(-1)$
$= -12 + 18i$

23. $-i(4 - 8i) = -4i + 8i^2$
$= -4i + 8(-1)$
$= -8 - 4i$

24. $(5 - 7i)(-4 - 3i) = -20 - 15i + 28i + 21i^2$
$= -20 + 13i + 21(-1)$
$= -41 + 13i$

25. $(-2 + 5i)(-1 + 4i) = 2 - 8i - 5i + 20i^2$
$= 2 - 13i + 20(-1)$
$= -18 - 13i$

26. $(-1 - 5i)(-1 + 5i) = 1 - 5i + 5i - 25i^2$
$= 1 - 25i^2$
$= 1 - 25(-1)$
$= 26$

27. $(8 - 3i)(8 + 3i) = 64 + 24i - 24i - 9i^2$
$= 64 - 9i^2$
$= 64 - 9(-1)$
$= 73$

28. $\frac{7i}{8 + i} = \frac{7i}{8 + i} \cdot \frac{8 - i}{8 - i}$
$= \frac{56i - 7i^2}{64 - 8i + 8i - i^2}$
$= \frac{56i - 7(-1)}{64 - (-1)}$
$= \frac{56i + 7}{65}$
$= \frac{7}{65} + \frac{56}{65}i$

29. $\frac{6i}{3 - i} = \frac{6i}{3 - i} \cdot \frac{3 + i}{3 + i}$
$= \frac{18i + 6i^2}{9 + 3i - 3i - i^2}$
$= \frac{18i + 6(-1)}{9 - (-1)}$
$= \frac{18i - 6}{10}$
$= -\frac{6}{10} + \frac{18}{10}i$
$= -\frac{3}{5} + \frac{9}{5}i$

30. $\frac{-2 - 5i}{3i} = \frac{-2 - 5i}{3i} \cdot \frac{i}{i}$
$= \frac{-2i - 5i^2}{3i^2}$
$= \frac{-2i - 5(-1)}{3(-1)}$
$= \frac{-2i + 5}{-3}$
$= -\frac{5}{3} + \frac{2}{3}i$

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31. $\dfrac{4+9i}{12i} = \dfrac{4+9i}{12i} \cdot \dfrac{i}{i}$

$= \dfrac{4i + 9i^2}{12i^2}$

$= \dfrac{4i + 9(-1)}{12(-1)}$

$= \dfrac{4i - 9}{-12}$

$= \dfrac{9}{12} - \dfrac{4i}{12}$

$= \dfrac{3}{4} - \dfrac{1}{3}i$

32. $\dfrac{7+4i}{2-3i} = \dfrac{7+4i}{2-3i} \cdot \dfrac{2+3i}{2+3i}$

$= \dfrac{14 + 21i + 8i + 12i^2}{4 + 6i - 6i - 9i^2}$

$= \dfrac{14 + 29i + 12i^2}{4 - 9i^2}$

$= \dfrac{14 + 29i + 12(-1)}{4 - 9(-1)}$

$= \dfrac{2 + 29i}{13}$

$= \dfrac{2}{13} + \dfrac{29}{13}i$

33. $\dfrac{-1-6i}{5+9i} = \dfrac{-1-6i}{5+9i} \cdot \dfrac{5-9i}{5-9i}$

$= \dfrac{-5 + 9i - 30i + 54i^2}{25 - 45i + 45i - 81i^2}$

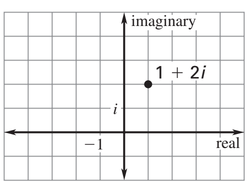
$= \dfrac{-5 - 21i + 54i^2}{25 - 81i^2}$

$= \dfrac{-5 - 21i + 54(-1)}{25 - 81(-1)}$

$= \dfrac{-59 - 21i}{106}$

$= -\dfrac{59}{106} - \dfrac{21}{106}i$

34.



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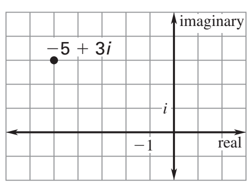
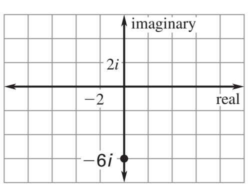


36.



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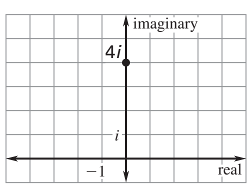
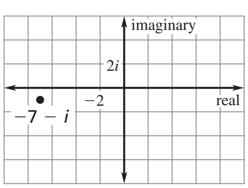
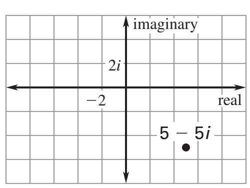


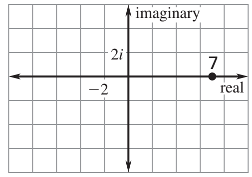
38.



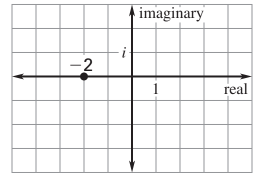
39.



40.



41.



42. $|4 + 3i| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

43. $|-3 + 10i| = \sqrt{(-3)^2 + 10^2} = \sqrt{109}$

44. $|10 - 7i| = \sqrt{10^2 + (-7)^2} = \sqrt{149}$

45. $|-1 - 6i| = \sqrt{(-1)^2 + (-6)^2} = \sqrt{37}$

46. $|-8i| = |0 + (-8i)| = \sqrt{0^2 + (-8)^2} = \sqrt{64} = 8$

47. $|4i| = |0 + 4i| = \sqrt{0^2 + 4^2} = \sqrt{16} = 4$

48. $|-4 + i| = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$

49. $|7 + 7i| = \sqrt{7^2 + 7^2} = \sqrt{98} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$

50. B;

$|9 + 12i| = \sqrt{9^2 + 12^2} = \sqrt{225} = 15$

51. $-8 - (3 + 2i) - (9 - 4i) = -8 - 3 - 2i - 9 + 4i$

$= (-8 - 3 - 9) + (-2 + 4)i$

$= -20 + 2i$

52. $(3 + 2i) + (5 - i) + 6i = (3 + 5) + (2 - 1 + 6)i$

$= 8 + 7i$

53. $5i(3 + 2i)(8 + 3i) = (15i + 10i^2)(8 + 3i)$

$= (-10 + 15i)(8 + 3i)$

$= -80 - 30i + 120i + 45i^2$

$= -80 + 90i + 45(-1)$

$= -125 + 90i$

54. $(1 - 9i)(1 - 4i)(4 - 3i) = (1 - 4i - 9i + 36i^2)(4 - 3i)$

$= [1 - 13i + 36(-1)](4 - 3i)$

$= (-35 - 13i)(4 - 3i)$

$= -140 + 105i - 52i + 39i^2$

$= -140 + 53i + 39(-1)$

$= -179 + 53i$

55. $\dfrac{(5 - 2i) + (5 + 3i)}{(1 + i) - (2 - 4i)} = \dfrac{(5 + 5) + (-2 + 3)i}{(1 - 2) + (1 + 4)i}$

$= \dfrac{10 + i}{-1 + 5i} \cdot \dfrac{-1 - 5i}{-1 - 5i}$

$= \dfrac{-10 - 50i - i - 5i^2}{1 + 5i - 5i - 25i^2}$

$= \dfrac{-10 - 51i - 5i^2}{1 - 25i^2}$

$= \dfrac{-10 - 51i - 5(-1)}{1 - 25(-1)}$

$= \dfrac{-5 - 51i}{26}$

$= \dfrac{-5}{26} - \dfrac{51}{26}i$

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56. $\dfrac{(10+4i)-(3-2i)}{(6-7i)(1-2i)} = \dfrac{(10-3)+(4+2)i}{6-12i-7i+14i^2}$

$= \dfrac{7+6i}{6-19i+14(-1)}$

$= \dfrac{7+6i}{-8-19i} \cdot \dfrac{-8+19i}{-8+19i}$

$= \dfrac{-56+133i-48i+114i^2}{64-152i+152i-361i^2}$

$= \dfrac{-56+85i+114i^2}{64-361i^2}$

$= \dfrac{-56+85i+114(-1)}{64-361(-1)}$

$= \dfrac{-170+85i}{425}$

$= -\dfrac{2}{5}+\dfrac{1}{5}i$

57. The expression is not written in standard form. The term $-2i^2$ can be simplified using $i^2 = -1$.

$(1+2i)(4-i) = 4 - i + 8i - 2i^2$

$= 4 + 7i - 2(-1)$

$= 6 + 7i$

58. The absolute value of a complex number $z = a + bi$, denoted $|z|$ is a nonnegative real number defined as $|z| = \sqrt{a^2+b^2}$. In $2 - 3i$, $a = 2$ and $b = -3$. Therefore, $|2-3i| = \sqrt{2^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$.

59. a. $z = 2 + i$

$z + z_a = 0$

$2 + i + z_a = 0$

$z_a = -2 - i$

$z \cdot z_m = 1$

$(2+i) \cdot z_m = 1$

$z_m = \dfrac{1}{2+i} \cdot \dfrac{2-i}{2-i}$

$z_m = \dfrac{2-i}{4-2i+2i-i^2}$

$z_m = \dfrac{2-i}{4-(-1)}$

$z_m = \dfrac{2-i}{5}$

$z_m = \dfrac{2}{5} - \dfrac{1}{5}i$

b. $z = 5 - i$

$z + z_a = 0$

$5 - i + z_a = 0$

$z_a = -5 + i$

$z \cdot z_m = 1$

$(5-i) \cdot z_m = 1$

$z_m = \dfrac{1}{5-i} \cdot \dfrac{5+i}{5+i}$

$z_m = \dfrac{5+i}{25+5i-5i-i^2}$

$z_m = \dfrac{5+i}{25-(-1)}$

$z_m = \dfrac{5+i}{26}$

$z_m = \dfrac{5}{26} + \dfrac{1}{26}i$

c. $z = -1 + 3i$

$z + z_a = 0$

$-1 + 3i + z_a = 0$

$z_a = 1 - 3i$

$z \cdot z_m = 1$

$(-1+3i) \cdot z_m = 1$

$z_m = \dfrac{1}{-1+3i} \cdot \dfrac{-1-3i}{-1-3i}$

$z_m = \dfrac{-1-3i}{1+3i-3i-9i^2}$

$z_m = \dfrac{-1-3i}{1-9(-1)}$

$z_m = -\dfrac{1}{10} - \dfrac{3}{10}i$

60. *Sample answer:* $3 - 5i$ and $4 + 5i$; The imaginary parts are opposites.

61. $\dfrac{a+bi}{c+di} = \dfrac{a+bi}{c+di} \cdot \dfrac{c-di}{c-di}$

$= \dfrac{ac - adi + bci - bdi^2}{c^2 - cdi + cdi - d^2i^2}$

$= \dfrac{ac - (ad - bc)i - bd(-1)}{c^2 - d^2(-1)}$

$= \dfrac{(ac+bd) - (ad-bc)i}{c^2+d^2}$

$= \dfrac{ac+bd}{c^2+d^2} - \dfrac{ad-bc}{c^2+d^2}i$

62. $\dfrac{a-bi}{c-di} = \dfrac{a-bi}{c-di} \cdot \dfrac{c+di}{c+di}$

$= \dfrac{ac + adi - bci - bdi^2}{c^2 + cdi - cdi - d^2i^2}$

$= \dfrac{ac + (ad - bc)i - bd(-1)}{c^2 - d^2(-1)}$

$= \dfrac{(ac+bd) + (ad-bc)i}{c^2+d^2}$

$= \dfrac{ac+bd}{c^2+d^2} + \dfrac{ad-bc}{c^2+d^2}i$

63. $\dfrac{a+bi}{c-di} = \dfrac{a+bi}{c-di} \cdot \dfrac{c+di}{c+di}$

$= \dfrac{ac + adi + bci + bdi^2}{c^2 + cdi - cdi - d^2i^2}$

$= \dfrac{ac + (ad + bc)i + bd(-1)}{c^2 - d^2(-1)}$

$= \dfrac{(ac-bd) + (ad+bc)i}{c^2+d^2}$

$= \dfrac{ac-bd}{c^2+d^2} + \dfrac{ad+bc}{c^2+d^2}i$

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64. $\dfrac{a-bi}{c+di} = \dfrac{a-bi}{c+di} \cdot \dfrac{c-di}{c-di}$

$$= \frac{ac - adi - bci + bdi^2}{c^2 - cdi + cdi - d^2i^2}$$

$$= \frac{ac - (ad+bc)i + bd(-1)}{c^2 - d^2(-1)}$$

$$= \frac{(ac-bd) - (ad+bc)i}{c^2 + d^2}$$

$$= \frac{ac-bd}{c^2+d^2} - \frac{ad+bc}{c^2+d^2}i$$

Problem Solving

65. Impedance of circuit $= 4 + 6i - 9i$

$= 4 - 3i$

The impedance of the circuit is $4 - 3i$ ohms.

66. Impedance of circuit $= 14 + 7i - 8i$

$= 14 - i$

The impedance of the circuit is $14 - i$ ohms.

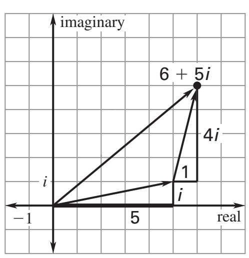
67. Impedance of circuit $= -6i + 12 - 10i + 8i$

$= 12 + (-6 - 10 + 8)i$

$= 12 - 8i$

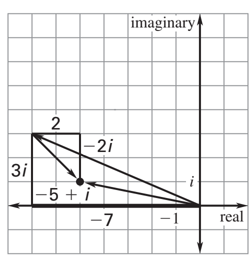
The impedance of the circuit is $12 - 8i$ ohms.

68. **a.** $(5 + i) + (1 + 4i)$



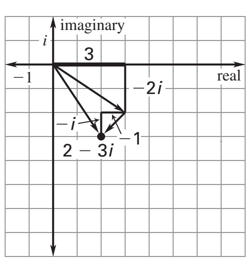
$6 + 5i$

b. $(-7 + 3i) + (2 - 2i)$



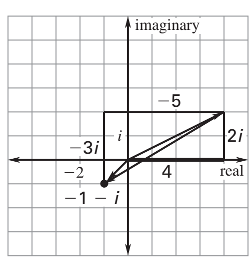
$-5 + i$

c. $(3 - 2i) + (-1 - i)$



$2 - 3i$

d. $(4 + 2i) + (-5 - 3i)$



$-1 - i$

69.

Powers of *i*	i^1	i^2	i^3	i^4	i^5	i^6	i^7	i^8
Simplified form	i	-1	$-i$	1	i	-1	$-i$	1

The pattern in the table is $i, -1, -i, 1, \ldots$

$i^9 = i^8 \cdot i = (1)(i) = i$

$i^{10} = i^8 \cdot i^2 = (1)(-1) = -1$

$i^{11} = i^9 \cdot i^2 = (i)(-1) = -i$

$i^{12} = i^{10} \cdot i^2 = (-1)(-1) = 1$

70. $c = i$

Let $f(z) = z^2 + i$.

$z_0 = 0 \qquad |z_0| = 0$

$z_1 = f(0) = 0^2 + i = i \qquad |z_1| = 1$

$z_2 = f(i) = i^2 + i = -1 + i \qquad |z_2| \approx 1.41$

$z_3 = f(-1 + i) = (-1 + i)^2 + i = -i \qquad |z_3| \approx 1$

$z_4 = f(-i) = (-i)^2 + i = i^2 + i = -1 + i \qquad |z_4| \approx 1.41$

Because the absolute values are all less than some fixed number, $c = i$ belongs to the Mandelbrot Set.

71. $c = -1 + i$

Let $f(z) = z^2 - 1 + i$.

$z_0 = 0 \qquad |z_0| = 0$

$z_1 = f(0) = 0^2 - 1 + i = -1 + i \qquad |z_1| \approx 1.41$

$z_2 = f(-1 + i) = (-1 + i)^2 - 1 + i = -1 - i$

$|z_2| \approx 1.41$

$z_3 = f(-1 - i) = (-1 - i)^2 - 1 + i = -1 + 3i$

$|z_3| \approx 3.16$

$z_4 = f(-1 + 3i) = (-1 + 3i)^2 - 1 + i = -8 - 6i$

$|z_4| \approx 10$

$z_5 = f(-8 - 6i) = (-8 - 6i)^2 - 1 + i = 27 + 97i$

$|z_5| \approx 100.69$

Because the absolute values are becoming infinitely large, $c = -1 + i$ does not belong to the Mandelbrot Set.

72. $c = -1$

Let $f(z) = z^2 - 1$.

$z_0 = 0 \qquad |z_0| = 0$

$z_1 = f(0) = 0^2 - 1 = -1 \qquad |z_1| = 1$

$z_2 = f(-1) = (-1)^2 - 1 = 0 \qquad |z_2| \approx 0$

$z_3 = f(0) = 0^2 - 1 = -1 \qquad |z_3| \approx 1$

$z_4 = f(-1) = (-1)^2 - 1 = 0 \qquad |z_4| \approx 0$

Because the absolute values are all less than some fixed number, $c = -1$ belongs to the Mandelbrot Set.

73. $c = -0.5i$

Let $f(z) = z^2 - 0.5i$.

$z_0 = 0 \qquad |z_0| = 0$

$z_1 = f(0) = -0.5i \qquad |z_1| = 0.5$

$z_2 = f(-0.5i) = (-0.5i)^2 - 0.5i = -0.25 - 0.5i$

$|z_2| \approx 0.56$

$z_3 = f(-0.25 - 0.5i) = (-0.25 - 0.5i)^2 - 0.5i$

$= -0.1875 - 0.25i$

$|z_3| \approx 0.31$

$z_4 = f(-0.1875 - 0.25i) = (-0.1875 - 0.25i)^2 - 0.5i$

$\approx -0.02734 - 0.04375i$

$|z_4| \approx 0.1$

Because the absolute values are all less than some fixed number, $c = -0.5i$ belongs to the Mandelbrot Set.

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74. $\sqrt{-4} \cdot \sqrt{-25} = (2i)(5i) = 10i^2 = 10(-1) = -10$

$\sqrt{100} = 10$

The rule $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ does not hold when a and b are negative numbers.

75. a. $Z_1 = 4 + 5i$ ohms

$Z_2 = 7 - 3i$ ohms

$$Z = \frac{Z_1Z_2}{Z_1 + Z_2} = \frac{(4 + 5i)(7 - 3i)}{(4 + 5i) + (7 - 3i)}$$

$$= \frac{28 - 12i + 35i - 15i^2}{11 + 2i}$$

$$= \frac{28 + 23i - 15(-1)}{11 + 2i}$$

$$= \frac{43 + 23i}{11 + 2i} \cdot \frac{11 - 2i}{11 - 2i}$$

$$= \frac{473 - 86i + 253i - 46i^2}{121 - 22i + 22i - 4i^2}$$

$$= \frac{473 + 167i - 46(-1)}{121 - 4(-1)}$$

$$= \frac{519 + 167i}{125}$$

$$= \frac{519}{125} + \frac{167}{125}i$$

The impedance of the circuit is $\frac{519}{125} + \frac{167}{125}i$ ohms.

b. $Z_1 = 6 + 8i$ ohms

$Z_2 = 10 - 11i$ ohms

$$Z = \frac{Z_1Z_2}{Z_1 + Z_2} = \frac{(6 + 8i)(10 - 11i)}{(6 + 8i) + (10 - 11i)}$$

$$= \frac{60 - 66i + 80i - 88i^2}{16 - 3i}$$

$$= \frac{60 + 14i - 88(-1)}{16 - 3i}$$

$$= \frac{148 + 14i}{16 - 3i} \cdot \frac{16 + 3i}{16 + 3i}$$

$$= \frac{2368 + 444i + 224i + 42i^2}{256 + 48i - 48i - 9i^2}$$

$$= \frac{2368 + 668i + 42(-1)}{256 - 9(-1)}$$

$$= \frac{2326 + 668i}{265}$$

$$= \frac{2326}{265} + \frac{668}{265}i$$

The impedance of the circuit is $\frac{2326}{265} + \frac{668}{265}i$ ohms.

c. $Z_1 = 3 + i$ ohms

$Z_2 = 4 - 6i$ ohms

$$Z = \frac{Z_1Z_2}{Z_1 + Z_2} = \frac{(3 + i)(4 - 6i)}{(3 + i) + (4 - 6i)}$$

$$= \frac{12 - 18i + 4i - 6i^2}{7 - 5i}$$

$$= \frac{12 - 14i - 6(-1)}{7 - 5i}$$

$$= \frac{18 - 14i}{7 - 5i} \cdot \frac{7 + 5i}{7 + 5i}$$

$$= \frac{126 + 90i - 98i - 70i^2}{49 + 35i - 35i - 25i^2}$$

$$= \frac{126 - 8i - 70(-1)}{49 - 25(-1)}$$

$$= \frac{196 - 8i}{74}$$

$$= \frac{98}{37} - \frac{4}{37}i$$

The impedance of the circuit is $\frac{98}{37} - \frac{4}{37}i$ ohms.

76. a. Let $f(z) = z^2 + 1 + i$.

$z_0 = i$ $\quad |z_0| = 1$

$z_1 = f(i) = i^2 + 1 + i = i$ $\quad |z_1| = 1$

$z_2 = f(i) = i^2 + 1 + i = i$ $\quad |z_2| = 1$

$z_3 = f(i) = i^2 + 1 + i = i$ $\quad |z_3| = 1$

$z_4 = f(i) = i^2 + 1 + i = i$ $\quad |z_4| = 1$

Because the absolute values are all less than some fixed number, $z_0 = i$ belongs to the Julia set.

b. Let $F(z) = z^2 + 1 + i$.

$z_0 = 1$ $\quad |z_0| = 1$

$z_1 = f(1) = 1^2 + 1 + i = 2 + i$ $\quad |z_1| \approx 2.24$

$z_2 = f(2 + i) = (2 + i)^2 + 1 + i$

$= 4 + 5i$

$|z_2| \approx 6.4$

$z_3 = f(4 + 5i) = (4 + 5i)^2 + 1 + i$

$= -8 + 41i$

$|z_3| \approx 41.77$

$z_4 = f(-8 + 41i) = (-8 + 41i)^2 + 1 + i$

$= -1616 - 655i$

$|z_4| \approx 1743.7$

Because the absolute values are becoming infinitely large, $z_0 = 1$ does not belong to the Julia set.

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c. Let $F(z) = z^2 + 1 + i$.

$z_0 = 2i \qquad |z_0| = 2$

$z_1 = f(2i) = (2i)^2 + 1 + i = -3 + i$

$|z_1| \approx 3.16$

$z_2 = f(-3 + i) = (-3 + i)^2 + 1 + i$
$= 9 - 5i$

$|z_2| \approx 10.3$

$z_3 = f(9 - 5i) = (9 - 5i)^2 + 1 + i$
$= 57 - 89i$

$|z_3| \approx 105.69$

$z_4 = f(57 - 89i) = (57 - 89i)^2 + 1 + i$
$= -4671 - 10{,}145i$

$|z_4| \approx 11{,}168.67$

Because the absolute values are becoming infinitely large, $z_0 = 2i$ does not belong to the Julia set.

d. Let $F(z) = z^2 + 1 + i$.

$z_0 = 2 + 3i \qquad |z_0| \approx 3.61$

$z_1 = f(2 + 3i) = (2 + 3i)^2 + 1 + i$
$= -4 + 13i$

$|z_1| \approx 13.6$

$z_2 = f(-4 + 13i) = (-4 + 13i)^2 + 1 + i$
$= -152 - 103i$

$|z_2| \approx 183.61$

$z_3 = f(-152 - 103i) = (-152 - 103i)^2 + 1 + i$
$= 12{,}496 + 31{,}313i$

$|z_3| \approx 33{,}714.30$

Because the absolute values are becoming infinitely large, $z_0 = 2 + 3i$ does not belong to the Julia set.

Mixed Review

77. The relation is a function because each input in the domain is mapped onto exactly one output in the range.

78. The relation is not a function because the input 3 is mapped onto both -1 and 0.

79. The relation is not a function because the input 1 is mapped onto both 2 and -5.

80. The relation is a function because each input in the domain is mapped onto exactly one output in the range.

81. $\begin{bmatrix} 5 & -4 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 9 \\ 1 & -8 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ -1 & -2 \end{bmatrix}$

82. $\begin{bmatrix} 6 & 3 \\ -5 & -1 \end{bmatrix} - \begin{bmatrix} 9 & -3 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ -9 & 0 \end{bmatrix}$

83. $\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} -3 & 13 \\ 12 & -20 \end{bmatrix}$

84. $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -14 \end{bmatrix}$

85. $3x^2 - 3x - 36 = 0$
$x^2 - x - 12 = 0$
$(x + 3)(x - 4) = 0$
$x + 3 = 0$ or $x - 4 = 0$
$x = -3$ or $x = 4$

86. $2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$2x - 1 = 0$ or $x - 4 = 0$
$x = \frac{1}{2}$ or $x = 4$

87. $6x^2 = 96$
$x^2 = 16$
$x = \pm\sqrt{16}$
$x = \pm 4$

88. $14x^2 = 91$
$x^2 = \frac{91}{14}$
$x = \pm\sqrt{\frac{91}{14}}$
$x = \pm\frac{\sqrt{91}}{\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}}$
$x = \pm\frac{\sqrt{1274}}{14}$
$x = \pm\frac{7\sqrt{26}}{14}$
$x = \pm\frac{\sqrt{26}}{2}$

89. $2x^2 - 8 = 42$
$2x^2 = 50$
$x^2 = 25$
$x = \pm\sqrt{25}$
$x = \pm 5$

90. $3x^2 + 13 = 121$
$3x^2 = 108$
$x^2 = 36$
$x = \pm\sqrt{36}$
$x = \pm 6$

Lesson 4.7

Investigating Algebra Activity 4.7 (p. 283)

1.

Completing the Square		
Expression	**Number of 1-tiles needed to complete the square**	**Expression written as a square**
$x^2 + 2x + ?$	1	$x^2 + 2x + 1 = (x + 1)^2$
$x^2 + 4x + ?$	4	$x^2 + 4x + 4 = (x + 2)^2$
$x^2 + 6x + ?$	9	$x^2 + 6x + 9 = (x + 3)^2$
$x^2 + 8x + ?$	16	$x^2 + 8x + 16 = (x + 4)^2$
$x^2 + 10x + ?$	25	$x^2 + 10x + 25 = (x + 5)^2$

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2. a. The value of d is one half the value of b.

b. The value of c is the square of the value of d.

c. You can multiply the value of b by one half and then square the result.

4.7 Guided Practice (pp. 285–287)

1. $x^2 + 6x + 9 = 36$

$(x + 3)^2 = 36$

$x + 3 = \pm 6$

$x = -3 \pm 6$

The solutions are $-3 + 6 = 3$ and $-3 - 6 = -9$.

2. $x^2 - 10x + 25 = 1$

$(x - 5)^2 = 1$

$x - 5 = \pm 1$

$x = 5 \pm 1$

The solution are $5 + 1 = 6$ and $5 - 1 = 4$.

3. $x^2 - 24x + 144 = 100$

$(x - 12)^2 = 100$

$x - 12 = \pm 10$

$x = 12 \pm 10$

The solutions are $12 + 10 = 22$ and $12 - 10 = 2$.

4. $x^2 + 14x + c$

$c = \left(\frac{14}{2}\right)^2 = 7^2 = 49$

$x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$

5. $x^2 + 22x + c$

$c = \left(\frac{22}{2}\right)^2 = 11^2 = 121$

$x^2 + 22x + 121 = (x + 11)(x + 11) = (x + 11)^2$

6. $x^2 - 9x + c$

$c = \left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

$x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)\left(x - \frac{9}{2}\right) = \left(x - \frac{9}{2}\right)^2$

7. $x^2 + 6x + 4 = 0$

$x^2 + 6x = -4$

$x^2 + 6x + 9 = -4 + 9$

$(x + 3)^2 = 5$

$x + 3 = \pm\sqrt{5}$

$x = -3 \pm \sqrt{5}$

The solutions are $-3 + \sqrt{5}$ and $-3 - \sqrt{5}$.

8. $x^2 - 10x + 8 = 0$

$x^2 - 10 = -8$

$x^2 - 10x + 25 = -8 + 25$

$(x - 5)^2 = 17$

$x - 5 = \pm\sqrt{17}$

$x = 5 \pm \sqrt{17}$

The solutions are $5 + \sqrt{17}$ and $5 - \sqrt{17}$.

9. $2n^2 - 4n - 14 = 0$

$n^2 - 2n - 7 = 0$

$n^2 - 2n = 7$

$n^2 - 2n + 1 = 7 + 1$

$(n - 1)^2 = 8$

$n - 1 = \pm\sqrt{8}$

$n = 1 \pm \sqrt{8}$

$n = 1 \pm 2\sqrt{2}$

The solutions are $1 + 2\sqrt{2}$ and $1 - 2\sqrt{2}$.

10. $3x^2 + 12x - 18 = 0$

$x^2 + 4x - 6 = 0$

$x^2 + 4x = 6$

$x^2 + 4x + 4 = 6 + 4$

$(x + 2)^2 = 10$

$x + 2 = \pm\sqrt{10}$

$x = -2 \pm \sqrt{10}$

The solutions are $-2 + \sqrt{10}$ and $-2 - \sqrt{10}$.

11. $6x(x + 8) = 12$

$6x^2 + 48x = 12$

$x^2 + 8x = 2$

$x^2 + 8x + 16 = 2 + 16$

$(x + 4)^2 = 18$

$x + 4 = \pm 3\sqrt{2}$

$x = -4 \pm 3\sqrt{2}$

The solutions are $-4 + 3\sqrt{2}$ and $-4 - 3\sqrt{2}$.

12. $4p(p - 2) = 100$

$4p^2 - 8p = 100$

$p^2 - 2p = 25$

$p^2 - 2p + 1 = 25 + 1$

$(p - 1)^2 = 26$

$p - 1 = \pm\sqrt{26}$

$p = 1 \pm \sqrt{26}$

The solutions are $1 + \sqrt{26}$ and $1 - \sqrt{26}$.

13. $y = x^2 - 8x + 17$

$y + 16 = (x^2 - 8x + 16) + 17$

$y + 16 = (x - 4)^2 + 17$

$y = (x - 4)^2 + 1$

The vertex form of the function is $y = (x - 4)^2 + 1$. The vertex is $(4, 1)$.

14. $y = x^2 + 6x + 3$

$y + 9 = (x^2 + 6x + 9) + 3$

$y + 9 = (x + 3)^2 + 3$

$y = (x + 3)^2 - 6$

The vertex form of the function is $y = (x + 3)^2 - 6$. The vertex is $(-3, -6)$.

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15. $f(x) = x^2 - 4x - 4$
$f(x) + 4 = (x^2 - 4x + 4) - 4$
$f(x) + 4 = (x - 2)^2 - 4$
$f(x) = (x - 2)^2 - 8$
The vertex form of the function is $f(x) = (x - 2)^2 - 8$. The vertex is $(2, -8)$.

16. $y = -16(t^2 - 5t) + 2$
$y = -16t^2 + 80t + 2$
$y = (-16)\left(\frac{25}{4}\right) = -16\left(t^2 - 5t + \frac{25}{4}\right) + 2$
$y - 100 = -16\left(t - \frac{5}{2}\right)^2 + 2$
$y = -16\left(t - \frac{5}{2}\right)^2 + 102$
The vertex is $\left(\frac{5}{2}, 102\right)$, so the maximum height of the baseball is 102 feet.

4.7 Exercises (pp. 288–291)

Skill Practice

1. A binomial is the sum of two monomials and a trinomial is the sum of three monomials.
2. For an expression of the form $x^2 + bx$, you complete the square by first finding half of b and squaring the result. Then you add the result to the expression.
3. $x^2 + 4x + 4 = 9$
$(x + 2)^2 = 9$
$x + 2 = \pm 3$
$x = -2 \pm 3$
The solutions are $-2 + 3 = 1$ and $-2 - 3 = -5$.
4. $x^2 + 10x + 25 = 64$
$(x + 5)^2 = 64$
$x + 5 = \pm 8$
$x = -5 \pm 8$
The solutions are $-5 + 8 = 3$ and $-5 - 8 = -13$.
5. $n^2 + 16n + 64 = 36$
$(n + 8)^2 = 36$
$n + 8 = \pm 6$
$n = -8 \pm 6$
The solutions are $-8 + 6 = -2$ and $-8 - 6 = -14$.
6. $m^2 - 2m + 1 = 144$
$(m - 1)^2 = 144$
$m - 1 = \pm 12$
$m = 1 \pm 12$
The solutions are $1 + 12 = 13$ and $1 - 12 = -11$.
7. $x^2 - 22x + 121 = 13$
$(x - 11)^2 = 13$
$x - 11 = \pm\sqrt{13}$
$x = 11 \pm \sqrt{13}$
The solutions are $11 + \sqrt{13}$ and $11 - \sqrt{13}$.
8. $x^2 - 18x + 81 = 5$
$(x - 9)^2 = 5$
$x - 9 = \pm\sqrt{5}$
$x = 9 \pm \sqrt{5}$
The solutions are $9 + \sqrt{5}$ and $9 - \sqrt{5}$.
9. $t^2 + 8t + 16 = 45$
$(t + 4)^2 = 45$
$t + 4 = \pm 3\sqrt{5}$
$t = -4 \pm 3\sqrt{5}$
The solutions are $-4 + 3\sqrt{5}$ and $-4 - 3\sqrt{5}$.
10. $4u^2 + 4u + 1 = 75$
$(2u + 1)^2 = 75$
$2u + 1 = \pm 5\sqrt{3}$
$2u = -1 \pm 5\sqrt{3}$
$u = -\frac{1}{2} \pm \frac{5\sqrt{3}}{2}$
The solutions are $-\frac{1}{2} + \frac{5\sqrt{3}}{2}$ and $-\frac{1}{2} - \frac{5\sqrt{3}}{2}$.
11. $9x^2 - 12x + 4 = -3$
$(3x - 2)^2 = -3$
$3x - 2 = \pm\sqrt{-3}$
$3x = 2 \pm \sqrt{-3}$
$3x = 2 \pm i\sqrt{3}$
$x = \frac{2}{3} \pm \frac{i\sqrt{3}}{3}$
The solutions are $\frac{2}{3} + \frac{i\sqrt{3}}{3}$ and $\frac{2}{3} - \frac{i\sqrt{3}}{3}$.
12. A;
$x^2 - 4x + 4 = -1$
$(x - 2)^2 = -1$
$x - 2 = \pm\sqrt{-1}$
$x = 2 \pm \sqrt{-1}$
$x = 2 \pm i$
13. $x^2 + 6x + c$
$c = \left(\frac{6}{2}\right)^2 = 3^2 = 9$
$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$
14. $x^2 + 12x + c$
$c = \left(\frac{12}{2}\right)^2 = 6^2 = 36$
$x^2 + 12x + 36 = (x + 6)(x + 6) = (x + 6)^2$
15. $x^2 - 24x + c$
$c = \left(\frac{-24}{2}\right)^2 = (-12)^2 = 144$
$x^2 - 24x + 144 = (x - 12)(x - 12) = (x - 12)^2$
16. $x^2 - 30x + c$
$c = \left(\frac{-30}{2}\right)^2 = (-15)^2 = 225$
$x^2 - 30x + 225 = (x - 15)(x - 15) = (x - 15)^2$

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17. $x^2 - 2x + c$

$c = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$x^2 - 2x + 1 = (x - 1)(x - 1) = (x - 1)^2$

18. $x^2 + 50x + c$

$c = \left(\frac{50}{2}\right)^2 = 25^2 = 625$

$x^2 + 50x + 625 = (x + 25)(x + 25) = (x + 25)^2$

19. $x^2 + 7x + c$

$c = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$

$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)\left(x + \frac{7}{2}\right) = \left(x + \frac{7}{2}\right)^2$

20. $x^2 - 13x + c$

$c = \left(\frac{-13}{2}\right)^2 = \frac{169}{4}$

$x^2 - 13x + \frac{169}{4} = \left(x - \frac{13}{2}\right)\left(x - \frac{13}{2}\right) = \left(x - \frac{13}{2}\right)^2$

21. $x^2 - x + c$

$c = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

$x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) = \left(x - \frac{1}{2}\right)^2$

22. $x^2 + 4x = 10$

$x^2 + 4x + 4 = 10 + 4$

$(x + 2)^2 = 14$

$x + 2 = \pm\sqrt{14}$

$x = -2 \pm \sqrt{14}$

The solutions are $-2 + \sqrt{14}$ and $-2 - \sqrt{14}$.

23. $x^2 + 8x = -1$

$x^2 + 8x + 16 = -1 + 16$

$(x + 4)^2 = 15$

$x + 4 = \pm\sqrt{15}$

$x = -4 \pm \sqrt{15}$

The solutions are $-4 + \sqrt{15}$ and $-4 - \sqrt{15}$.

24. $x^2 + 6x - 3 = 0$

$x^2 + 6x = 3$

$x^2 + 6x + 9 = 3 + 9$

$(x + 3)^2 = 12$

$x + 3 = \pm\sqrt{12}$

$x = -3 \pm \sqrt{12}$

$x = -3 \pm 2\sqrt{3}$

The solutions are $-3 + 2\sqrt{3}$ and $-3 - 2\sqrt{3}$.

25. $x^2 + 12x + 18 = 0$

$x^2 + 12x = -18$

$x^2 + 12x + 36 = -18 + 36$

$(x + 6)^2 = 18$

$x + 6 = \pm\sqrt{18}$

$x = -6 \pm \sqrt{18}$

$x = -6 \pm 3\sqrt{2}$

The solutions are $-6 + 3\sqrt{2}$ and $-6 - 3\sqrt{2}$.

26. $x^2 - 18x + 86 = 0$

$x^2 - 18x = -86$

$x^2 - 18x + 81 = -86 + 81$

$(x - 9)^2 = -5$

$x - 9 = \pm\sqrt{-5}$

$x = 9 \pm \sqrt{-5}$

$x = 9 \pm i\sqrt{5}$

The solutions are $9 + i\sqrt{5}$ and $9 - i\sqrt{5}$.

27. $x^2 - 2x + 25 = 0$

$x^2 - 2x = -25$

$x^2 - 2x + 1 = -25 + 1$

$(x - 1)^2 = -24$

$x - 1 = \pm\sqrt{-24}$

$x = 1 \pm\sqrt{-24}$

$x = 1 \pm 2i\sqrt{6}$

The solutions are $1 + 2i\sqrt{6}$ and $1 - 2i\sqrt{6}$.

28. $2k^2 + 16k = -12$

$k^2 + 8k = -6$

$k^2 + 8k + 16 = -6 + 16$

$(k + 4)^2 = 10$

$k + 4 = \pm\sqrt{10}$

$k = -4 \pm \sqrt{10}$

The solutions are $-4 + \sqrt{10}$ and $-4 - \sqrt{10}$.

29. $3x^2 + 42x = -24$

$x^2 + 14x = -8$

$x^2 + 14x + 49 = -8 + 49$

$(x + 7)^2 = 41$

$x + 7 = \pm\sqrt{41}$

$x = -7 \pm \sqrt{41}$

The solutions are $-7 + \sqrt{41}$ and $-7 - \sqrt{41}$.

30. $4x^2 - 40x - 12 = 0$

$x^2 - 10x - 3 = 0$

$x^2 - 10x = 3$

$x^2 - 10x + 25 = 3 + 25$

$(x - 5)^2 = 28$

$x - 5 = \pm\sqrt{28}$

$x = 5 \pm \sqrt{28}$

$x = 5 \pm 2\sqrt{7}$

The solutions are $5 + 2\sqrt{7}$ and $5 - 2\sqrt{7}$.

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31. $3s^2 + 6s + 9 = 0$

$s^2 + 2s + 3 = 0$

$s^2 + 2s = -3$

$4s^2 + 2s + 1 = -3 + 1$

$(s + 1)^2 = -2$

$s + 1 = \pm\sqrt{-2}$

$s = -1 \pm \sqrt{-2}$

$s = -1 \pm i\sqrt{2}$

The solutions are $-1 + i\sqrt{2}$ and $-1 - i\sqrt{2}$.

32. $7t^2 + 28t + 56 = 0$

$t^2 + 4t + 8 = 0$

$t^2 + 4t = -8$

$t^2 + 4t + 4 = -8 + 4$

$(t + 2)^2 = -4$

$t + 2 = \pm\sqrt{-4}$

$t = -2 \pm \sqrt{-4}$

$t = -2 \pm 2i$

The solutions are $-2 + 2i$ and $-2 - 2i$.

33. $6r^2 + 6r + 12 = 0$

$r^2 + r + 2 = 0$

$r^2 + r = -2$

$r^2 + r + \frac{1}{4} = -2 + \frac{1}{4}$

$\left(r + \frac{1}{2}\right)^2 = -\frac{7}{4}$

$r + \frac{1}{2} = \pm\sqrt{-\frac{7}{4}}$

$r = -\frac{1}{2} \pm \sqrt{-\frac{7}{4}}$

$r = -\frac{1}{2} \pm \frac{i\sqrt{7}}{2}$

The solutions are $-\frac{1}{2} + \frac{i\sqrt{7}}{2}$ and $-\frac{1}{2} - \frac{i\sqrt{7}}{2}$.

34. C;

$x^2 + 10x + 8 = -5$

$x^2 + 10x = -13$

$x^2 + 10x + 25 = -13 + 25$

$(x + 5)^2 = 12$

$x + 5 = \pm\sqrt{12}$

$x = -5 \pm \sqrt{12}$

$x = -5 \pm 2\sqrt{3}$

35. Area of rectangle $= \ell w = 50$

$(x + 10)(x) = 50$

$x^2 + 10x = 50$

$x^2 + 10x + 25 = 50 + 25$

$(x + 5)^2 = 75$

$x + 5 = \pm\sqrt{75}$

$x = -5 \pm \sqrt{75}$

$x = -5 \pm 5\sqrt{3}$

The value of x is $-5 + 5\sqrt{3}$.

36. Area of parallelogram $= bh = 48$

$(x + 6)(x) = 48$

$x^2 + 6x = 48$

$x^2 + 6x + 9 = 48 + 9$

$(x + 3)^2 = 57$

$x + 3 = \pm\sqrt{57}$

$x = -3 \pm \sqrt{57}$

The value of x is $-3 + \sqrt{57}$.

37. Area of triangle $= \frac{1}{2}bh = 40$

$\frac{1}{2}(x)(x + 4) = 40$

$\frac{1}{2}x(x + 4) = 40$

$\frac{1}{2}x^2 + 2x = 40$

$x^2 + 4x = 80$

$x^2 + 4x + 4 = 80 + 4$

$(x + 2)^2 = 84$

$x + 2 = \pm\sqrt{84}$

$x = -2 \pm \sqrt{84}$

$x = -2 \pm 2\sqrt{21}$

The value of x is $-2 + 2\sqrt{21}$.

38. Area of trapezoid $= \frac{1}{2}(b_1 + b_2)h = 20$

$\frac{1}{2}(x + 9 + 3x - 1)(x) = 20$

$\frac{1}{2}x(4x + 8) = 20$

$2x^2 + 4x = 20$

$x^2 + 2x = 10$

$x^2 + 2x + 1 = 10 + 1$

$(x + 1)^2 = 11$

$x + 1 = \pm\sqrt{11}$

$x = -1 \pm \sqrt{11}$

The value of x is $-1 + \sqrt{11}$.

39. $h = -16t^2 + 89.6t$

$h = -16(t^2 - 5.6t)$

$h + (-16)(7.84) = -16(t^2 - 5.6t + 7.84)$

$h - 125.44 = -16(t - 2.8)^2$

$h = -16(t - 2.8)^2 + 125.44$

The vertex of the function's graph is (2.8, 125.44). This means that at 2.8 seconds, the water will reach its maximum height of 125.44 feet.

40. $y = 0.0085x^2 - 1.5x + 120$

$y = 0.0085(x^2 - 176.47x) + 120$

$y + (0.0085)(7785.42)$

$= 0.0085(x^2 - 176.47x + 7785.42) + 120$

$y + 66.18 = 0.0085(x - 88.24)^2 + 120$

$y = 0.0085(x - 88.24)^2 + 53.82$

The vertex of the function's graph is (88.24, 53.82). This means that when you walk about 88.24 meters per minute, your rate of energy use will reach a minimum of 53.82 calories per minute.

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41.
$$\begin{aligned} y &= x^2 - 8x + 19 \\ y + 16 &= (x^2 - 8x + 16) + 19 \\ y + 16 &= (x - 4)^2 + 19 \\ y &= (x - 4)^2 + 3 \end{aligned}$$

The vertex form of the function is $y = (x - 4)^2 + 3$. The vertex is $(4, 3)$.

42.
$$\begin{aligned} y &= x^2 - 4x - 1 \\ y + 4 &= (x^2 - 4x + 4) - 1 \\ y + 4 &= (x - 2)^2 - 1 \\ y &= (x - 2)^2 - 5 \end{aligned}$$

The vertex form of the function is $y = (x - 2)^2 - 5$. The vertex is $(2, -5)$.

43.
$$\begin{aligned} y &= x^2 + 12x + 37 \\ y + 36 &= (x^2 + 12x + 36) + 37 \\ y + 36 &= (x + 6)^2 + 37 \\ y &= (x + 6)^2 + 1 \end{aligned}$$

The vertex form of the function is $y = (x + 6)^2 + 1$. The vertex is $(-6, 1)$.

44.
$$\begin{aligned} y &= x^2 + 20x + 90 \\ y + 100 &= (x^2 + 20x + 100) + 90 \\ y + 100 &= (x + 10)^2 + 90 \\ y &= (x + 10)^2 - 10 \end{aligned}$$

The vertex form of the function is $y = (x + 10)^2 - 10$. The vertex is $(-10, -10)$.

45.
$$\begin{aligned} f(x) &= x^2 - 3x + 4 \\ f(x) + \frac{9}{4} &= \left(x^2 - 3x + \frac{9}{4}\right) + 4 \\ f(x) + \frac{9}{4} &= \left(x - \frac{3}{2}\right)^2 + 4 \\ f(x) &= \left(x - \frac{3}{2}\right)^2 + \frac{7}{4} \end{aligned}$$

The vertex form of the function is $f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$.

The vertex is $\left(\frac{3}{2}, \frac{7}{4}\right)$.

46.
$$\begin{aligned} g(x) &= x^2 + 7x + 2 \\ g(x) + \frac{49}{4} &= \left(x^2 + 7x + \frac{49}{4}\right) + 2 \\ g(x) + \frac{49}{4} &= \left(x + \frac{7}{2}\right)^2 + 2 \\ g(x) &= \left(x + \frac{7}{2}\right)^2 - \frac{41}{4} \end{aligned}$$

The vertex form of the function is $g(x) = \left(x + \frac{7}{2}\right)^2 - \frac{41}{4}$.

The vertex is $\left(-\frac{7}{2}, -\frac{41}{4}\right)$.

47.
$$\begin{aligned} y &= 2x^2 + 24x + 25 \\ y &= 2(x^2 + 12x) + 25 \\ y + (2)(36) &= 2(x^2 + 12x + 36) + 25 \\ y + 72 &= 2(x + 6)^2 + 25 \\ y &= 2(x + 6)^2 - 47 \end{aligned}$$

The vertex form of the function is $y = 2(x + 6)^2 - 47$.

The vertex is $(-6, -47)$.

48.
$$\begin{aligned} y &= 5x^2 + 10x + 7 \\ y &= 5(x^2 + 2x) + 7 \\ y + (5)(1) &= 5(x^2 + 2x + 1) + 7 \\ y + 5 &= 5(x + 1)^2 + 7 \\ y &= 5(x + 1)^2 + 2 \end{aligned}$$

The vertex form of the function is $y = 5(x + 1)^2 + 2$. The vertex is $(-1, 2)$.

49.
$$\begin{aligned} y &= 2x^2 - 28x + 99 \\ y &= 2(x^2 - 14x) + 99 \\ y + (2)(49) &= 2(x^2 - 14x + 49) + 99 \\ y + 98 &= 2(x - 7)^2 + 99 \\ y &= 2(x - 7)^2 + 1 \end{aligned}$$

The vertex form of the function is $y = 2(x - 7)^2 + 1$. The vertex is $(7, 1)$.

50. The error was made when simplifying $\sqrt{12}$ in the last step.

$\sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

$$\begin{aligned} x^2 + 10x + 13 &= 0 \\ x^2 + 10x &= -13 \\ x^2 + 10x + 25 &= -13 + 25 \\ (x + 5)^2 &= 12 \\ x + 5 &= \pm\sqrt{12} \\ x &= -5 \pm \sqrt{12} \\ x &= -5 \pm 2\sqrt{3} \end{aligned}$$

51. The method of completing the square was done incorrectly. Because 4(9), or 36, is added to the left side, it must also be added to the right side.

$$\begin{aligned} 4x^2 + 24x - 11 &= 0 \\ 4(x^2 + 6x) &= 11 \\ 4(x^2 + 6x + 9) &= 11 + 36 \\ 4(x + 3)^2 &= 47 \\ (x + 3)^2 &= \frac{47}{4} \\ x + 3 &= \pm\sqrt{\frac{47}{4}} \\ x &= -3 \pm \frac{\sqrt{47}}{2} \end{aligned}$$

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52. $x^2 + 9x + 20 = 0$

$$x^2 + 9x = -20$$
$$x^2 + 9x + \frac{81}{4} = -20 + \frac{81}{4}$$
$$\left(x + \frac{9}{2}\right)^2 = \frac{1}{4}$$
$$x + \frac{9}{2} = \pm\sqrt{\frac{1}{4}}$$
$$x = -\frac{9}{2} \pm \frac{1}{2}$$

The solutions are -4 and -5.

53. $x^2 + 3x + 14 = 0$

$$x^2 + 3x = -14$$
$$x^2 + 3x + \frac{9}{4} = -14 + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = -\frac{47}{4}$$
$$x + \frac{3}{2} = \pm\sqrt{-\frac{47}{4}}$$
$$x = -\frac{3}{2} \pm \frac{i\sqrt{47}}{2}$$

The solutions are $-\frac{3}{2} + \frac{i\sqrt{47}}{2}$ and $-\frac{3}{2} - \frac{i\sqrt{47}}{2}$.

54.
$$7q^2 + 10q = 2q^2 + 155$$
$$5q^2 + 10q = 155$$
$$q^2 + 2q = 31$$
$$q^2 + 2q + 1 = 31 + 1$$
$$(q + 1)^2 = 32$$
$$q + 1 = \pm\sqrt{32}$$
$$q = -1 \pm 4\sqrt{2}$$

The solutions are $-1 + 4\sqrt{2}$ and $-1 - 4\sqrt{2}$.

55.
$$3x^2 + x = 2x - 6$$
$$3x^2 - x = -6$$
$$x^2 - \frac{1}{3}x = -2$$
$$x^2 - \frac{1}{3}x + \frac{1}{36} = -2 + \frac{1}{36}$$
$$\left(x - \frac{1}{6}\right)^2 = -\frac{71}{36}$$
$$x - \frac{1}{6} = \pm\sqrt{-\frac{71}{36}}$$
$$x = \frac{1}{6} \pm \frac{i\sqrt{71}}{6}$$

The solutions are $\frac{1}{6} + \frac{i\sqrt{71}}{6}$ and $\frac{1}{6} - \frac{i\sqrt{71}}{6}$.

56. $0.1x^2 - x + 9 = 0.2x$

$$0.1x^2 - 1.2x = -9$$
$$x^2 - 12x = -90$$
$$x^2 - 12x + 36 = -90 + 36$$
$$(x - 6)^2 = -54$$
$$x - 6 = \pm\sqrt{-54}$$
$$x = 6 \pm 3i\sqrt{6}$$

The solutions are $6 + 3i\sqrt{6}$ and $6 - 3i\sqrt{6}$.

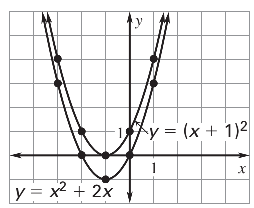
57. $0.4v^2 + 0.7v = 0.3v - 2$

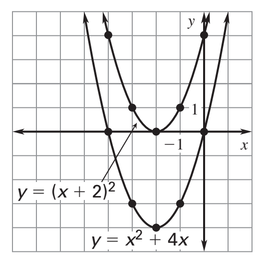
$$0.4v^2 + 0.4v = -2$$
$$v^2 + v = -5$$
$$v^2 + v + \frac{1}{4} = -5 + \frac{1}{4}$$
$$\left(v + \frac{1}{2}\right)^2 = -\frac{19}{4}$$
$$v + \frac{1}{2} = \pm\sqrt{-\frac{19}{4}}$$
$$v = -\frac{1}{2} \pm \frac{i\sqrt{19}}{2}$$

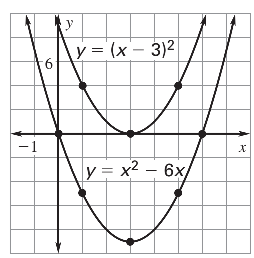
The solutions are $-\frac{1}{2} + \frac{i\sqrt{19}}{2}$ and $-\frac{1}{2} - \frac{i\sqrt{19}}{2}$.

58. *Sample answer:* $x^2 + 6x = 1$

59. a.







b. When you complete the square, there is a vertical translation of the graph of $y = x^2 + bx$, $\left(\frac{b}{2}\right)^2$ units up.

60. 1 real solution: $k = 0$

2 real solutions: $k > 0$

2 imaginary solutions: $k < 0$

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61. $$x^2 + bx + c = 0$$
$$x^2 + bx = -c$$
$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$
$$\left(x + \frac{b}{2}\right)^2 = \frac{b^2}{4} - c$$
$$x + \frac{b}{2} = \pm\sqrt{\frac{b^2}{4} - c}$$
$$x = -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}$$
$$x = -\frac{b}{2} \pm \sqrt{\frac{b^2 - 4c}{4}}$$
$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Problem Solving

62. $$h = -16t^2 + 32t + 6$$
$$h = -16(t^2 - 2t) + 6$$
$$h + (-16)(1) = -16(t^2 - 2t + 1) + 6$$
$$h - 16 = -16(t - 1)^2 + 6$$
$$h = -16(t - 1)^2 + 22$$

The maximum height of the baton is 22 feet.

63. $$h = -16t^2 + 48t + 4$$
$$h = -16(t^2 - 3t) + 4$$
$$h + (-16)\left(\frac{9}{4}\right) = -16\left(t^2 - 3t + \frac{9}{4}\right) + 4$$
$$h - 36 = -16\left(t - \frac{3}{2}\right)^2 + 4$$
$$h = -16\left(t - \frac{3}{2}\right)^2 + 40$$

The maximum height of the volleyball is 40 feet.

64. $$y = (70 - x)(50 + x)$$
$$y = 3500 + 70x - 50x - x^2$$
$$y = 3500 + 20x - x^2$$
$$y = -x^2 + 20x + 3500$$
$$y = -(x^2 - 20x) + 3500$$
$$y + (-1)(100) = -(x^2 - 20x + 100) + 3500$$
$$y - 100 = -(x - 10)^2 + 3500$$
$$y = -(x - 10)^2 + 3600$$

The shop can maximize weekly revenue by decreasing the price per skateboard by \$10. With this decrease in price, the weekly revenue will be \$3600.

65. $$y = (200 + 10x)(40 - x)$$
$$y = 8000 - 200x + 400x - 10x^2$$
$$y = 8000 + 200x - 10x^2$$
$$y = -10x^2 + 200x + 8000$$
$$y = -10(x^2 - 20x) + 8000$$
$$y + (-10)(100) = -10(x^2 - 20x + 100) + 8000$$
$$y - 1000 = -10(x - 10)^2 + 8000$$
$$y = -10(x - 10)^2 + 9000$$

The store can maximize monthly revenue by increasing the price per video game system by $10x = 10(10) = \$100$. With this increase in price, the monthly revenue will be \$9000.

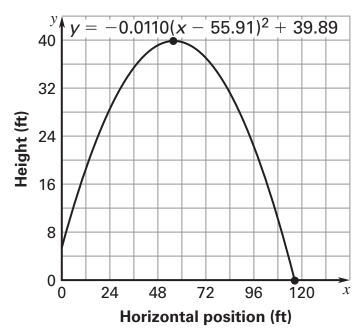
66. a. $$y = -0.0110x^2 + 1.23x + 5.50$$
$$y = -0.0110(x^2 - 111.82x) + 5.50$$
$$y + (-0.0110)(3125.93)$$
$$= -0.0110(x^2 - 111.82x)$$
$$+ 3125.93 + 5.50$$
$$y - 34.39 = -0.0110(x - 55.91)^2 + 5.50$$
$$y = -0.0110(x - 55.91)^2 + 39.89$$

b.

x	0	10	20	30	40	50	60
y	5.50	16.71	25.71	32.51	37.11	39.51	39.71

x	70	80	90	100	110	120
y	37.71	33.51	27.11	18.51	7.71	−5.29

c.



The maximum height of the softball is 39.89 feet. The ball travels a distance of 116.13 feet.

67. a. Area of cutting section $= \ell w$
$$1500 = x(120 - 2x)$$

b. $$x(120 - 2x) = 1500$$
$$120x - 2x^2 = 1500$$
$$-2x^2 + 120x = 1500$$
$$x^2 - 60x = -750$$
$$x^2 - 60x + 900 = -750 + 900$$
$$(x - 30)^2 = 150$$
$$x - 30 = \pm\sqrt{150}$$
$$x = 30 \pm 5\sqrt{6}$$

You must reject $30 - 5\sqrt{6}$, or about 17.75. This value of x gives a width of about 84.5 feet. A width of 84.5 feet is not possible because the side of the school is 70 feet.

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c. Width $= 120 - 2(30 + 5\sqrt{6}) \approx 35.51$

Length $= 30 + 5\sqrt{6} \approx 42.25$

The dimensions of the eating section are 35.51 feet by 42.25 feet.

68. Volume of cylinder $= \pi r^2 h$

Outside cylinder: $r = x + 3$

$h = 9$

$V_{\text{outside}} = \pi(x + 3)^2(9) = 9\pi(x^2 + 6x + 9)$

Inside cylinder: $r = 3$

$h = 9 - x$

$V_{\text{inside}} = \pi(3)^2(9 - x) = 9\pi(9 - x)$

Volume of clay:

$$V_{\text{clay}} = V_{\text{outside}} - V_{\text{inside}}$$

$$200 = 9\pi(x + 6x + 9) - 9\pi(9 - x)$$

$$200 = 9\pi(x^2 + 6x + 9 - 9 + x)$$

$$200 = 9\pi(x^2 + 7x)$$

$$\frac{200}{9\pi} = x^2 + 7x$$

$$\frac{200}{9\pi} + \frac{49}{4} = \left(x^2 + 7x + \frac{49}{4}\right)$$

$$\frac{200}{9\pi} + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$$

$$\pm\sqrt{\frac{200}{9\pi} + \frac{49}{4}} = x + \frac{7}{2}$$

$$-\frac{7}{2} \pm \sqrt{\frac{200}{9\pi} + \frac{49}{4}} = x$$

$$-3.5 \pm 4.4 \approx x$$

Reject the negative value, $-3.5 - 4.4$, or -7.9. The pencil holder should have a thickness of about 0.9 centimeter.

Mixed Review

69. $b^2 - 4ac = 7^2 - 4(2)(5) = 9$

70. $b^2 - 4ac = (-6)^2 - 4(1)(9) = 0$

71. $b^2 - 4ac = (-1)^2 - 4(4)(3) = -47$

72. $b^2 - 4ac = 2^2 - 4(3)(-6) = 76$

73. $b^2 - 4ac = 2^2 - 4(-4)(-7) = -108$

74. $b^2 - 4ac = 3^2 - 4(-5)(2) = 49$

75. $(x_1, y_1) = (2, 5), (x_2, y_2) = (4, 9)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{4 - 2} = \frac{4}{2} = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 2(x - 2)$$
$$y - 5 = 2x - 4$$
$$y = 2x + 1$$

76. $(x_1, y_1) = (3, -1), (x_2, y_2) = (6, -3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-1)}{6 - 3} = -\frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{3}(x - 3)$$
$$y + 1 = -\frac{2}{3}x + 2$$
$$y = -\frac{2}{3}x + 1$$

77. $(x_1, y_1) = (-4, -4), (x_2, y_2) = (-1, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{-1 - (-4)} = \frac{6}{3} = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = 2[x - (-4)]$$
$$y + 4 = 2(x + 4)$$
$$y + 4 = 2x + 8$$
$$y = 2x + 4$$

78. $(x_1, y_1) = (-2, 4), (x_2, y_2) = (1, -2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{1 - (-2)} = \frac{-6}{3} = -2$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -2[x - (-2)]$$
$$y - 4 = -2(x + 2)$$
$$y - 4 = -2x - 4$$
$$y = -2x$$

79. $(x_1, y_1) = (-1, -5), (x_2, y_2) = (1, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{1 - (-1)} = \frac{6}{2} = 3$$

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 3[x - (-1)]$$
$$y + 5 = 3(x + 1)$$
$$y + 5 = 3x + 3$$
$$y = 3x - 2$$

80. $(x_1, y_1) = (6, 3), (x_2, y_2) = (8, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{8 - 6} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{1}{2}(x - 6)$$
$$y - 3 = \frac{1}{2}x - 3$$
$$y = \frac{1}{2}x$$

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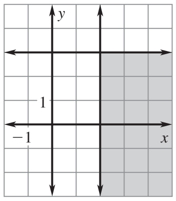
81. $x \geq 2$

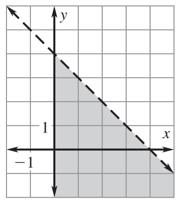
$y \leq 3$



82. $x \geq 0$

$x + y < 4$

$y < -x + 4$



83. $3x - 2y < 8$

$2x + y > 0$

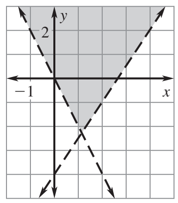
$3x - 2y < 8$

$-2y < -3x + 8$

$y > \frac{3}{2}x - 4$

$2x + y > 0$

$y > -2x$



84. $4x + y \geq 3$

$2x - 3y < 6$

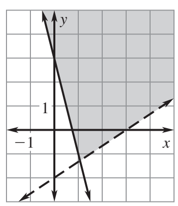
$4x + y \geq 3$

$y \geq -4x + 3$

$2x - 3y < 6$

$-3y < -2x + 6$

$y > \frac{2}{3}x - 2$



Quiz 4.5–4.7 (p. 291)

1. $4x^2 = 64$

$x^2 = 16$

$x = \pm\sqrt{16}$

$x = \pm 4$

2. $3(p - 1)^2 = 15$

$(p - 1)^2 = 5$

$p - 1 = \pm\sqrt{5}$

$p = 1 \pm \sqrt{5}$

3. $16(m + 5)^2 = 8$

$(m + 5)^2 = \frac{1}{2}$

$m + 5 = \pm\sqrt{\frac{1}{2}}$

$m = -5 \pm \sqrt{\frac{1}{2}}$

$m = -5 \pm \frac{\sqrt{2}}{2}$

4. $-2z^2 = 424$

$z^2 = -212$

$z = \pm\sqrt{-212}$

$z = \pm i\sqrt{212}$

$z = \pm 2i\sqrt{53}$

5. $s^2 + 12 = 9$

$s^2 = -3$

$s = \pm\sqrt{-3}$

$s = \pm i\sqrt{3}$

6. $7x^2 - 4 = -6$

$7x^2 = -2$

$x^2 = -\frac{2}{7}$

$x = \pm\sqrt{-\frac{2}{7}}$

$x = \pm i\sqrt{\frac{2}{7}}$

$x = \pm i\frac{\sqrt{14}}{7}$

7. $(5 - 3i) + (-2 + 5i) = [5 + (-2)] + (-3 + 5)i$

$= 3 + 2i$

8. $(-2 + 9i) - (7 + 8i) = (-2 - 7) + (9 - 8)i$

$= -9 + i$

9. $3i(7 - 9i) = 21i - 27i^2$

$= 21i - 27(-1)$

$= 27 + 21i$

10. $(8 - 3i)(-6 - 10i) = -48 - 80i + 18i + 30i^2$

$= -48 - 62i + 30(-1)$

$= -78 - 62i$

11. $\frac{4i}{-6 - 11i} = \frac{4i}{-6 - 11i} \cdot \frac{-6 + 11i}{-6 + 11i}$

$= \frac{-24i + 44i^2}{36 - 66i + 66i - 121i^2}$

$= \frac{-24i + 44(-1)}{36 - 121(-1)}$

$= \frac{-44 - 24i}{157}$

$= -\frac{44}{157} - \frac{24}{157}i$

12. $\frac{3 - 2i}{-8 + 5i} = \frac{3 - 2i}{-8 + 5i} \cdot \frac{-8 - 5i}{-8 - 5i}$

$= \frac{-24 - 15i + 16i + 10i^2}{64 + 40i - 40i - 25i^2}$

$= \frac{-24 + i + 10(-1)}{64 - 25(-1)}$

$= \frac{-34 + i}{89}$

$= -\frac{34}{89} + \frac{1}{89}i$

13. $y = x^2 - 4x + 9$

$y + 4 = (x^2 - 4x + 4) + 9$

$y + 4 = (x - 2)^2 + 9$

$y = (x - 2)^2 + 5$

The vertex form of the function is $y = (x - 2)^2 + 5$. The vertex is (2, 5).

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14.
$$y = x^2 + 14x + 45$$
$$y + 49 = (x^2 + 14x + 49) + 45$$
$$y + 49 = (x + 7)^2 + 45$$
$$y = (x + 7)^2 - 4$$

The vertex form of the function is $y = (x + 7)^2 - 4$. The vertex is $(-7, -4)$.

15.
$$f(x) = x^2 - 10x + 17$$
$$f(x) + 25 = (x^2 - 10x + 25) + 17$$
$$f(x) + 25 = (x - 5)^2 + 17$$
$$f(x) = (x - 5)^2 - 8$$

The vertex form of the function is $f(x) = (x - 5)^2 - 8$. The vertex is $(5, -8)$.

16.
$$g(x) = x^2 - 2x - 7$$
$$g(x) + 1 = (x^2 - 2x + 1) - 7$$
$$g(x) + 1 = (x - 1)^2 - 7$$
$$g(x) = (x - 1)^2 - 8$$

The vertex form of the function is $g(x) = (x - 1)^2 - 8$. The vertex is $(1, -8)$.

17.
$$y = x^2 + x + 1$$
$$y + \frac{1}{4} = \left(x^2 + x + \frac{1}{4}\right) + 1$$
$$y + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2 + 1$$
$$y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

The vertex form of the function is $y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$.

The vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

18.
$$y = x^2 + 9x + 19$$
$$y + \frac{81}{4} = \left(x^2 + 9x + \frac{81}{4}\right) + 19$$
$$y + \frac{81}{4} = \left(x + \frac{9}{2}\right)^2 + 19$$
$$y = \left(x + \frac{9}{2}\right)^2 - \frac{5}{4}$$

The vertex form of the function is $y = \left(x + \frac{9}{2}\right)^2 - \frac{5}{4}$.

The vertex is $\left(-\frac{9}{2}, -\frac{5}{4}\right)$.

19.
$$h = -16t^2 + h_0$$
$$0 = -16t^2 + 45$$
$$-45 = -16t^2$$
$$\frac{45}{16} = t^2$$
$$\pm\sqrt{\frac{45}{16}} = t$$
$$\pm 1.7 \approx t$$

Reject the negative solution, -1.7, because time must be positive. The ball is in the air for about 1.7 seconds.

Lesson 4.8

4.8 Guided Practice (pp. 293–295)

1.
$$x^2 = 6x - 4$$
$$x^2 - 6x + 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$
$$x = \frac{6 \pm \sqrt{20}}{2}$$
$$x = \frac{6 \pm 2\sqrt{5}}{2}$$
$$x = 3 \pm \sqrt{5}$$

The solutions are $x = 3 + \sqrt{5} \approx 5.24$ and $x = 3 - \sqrt{5} \approx 0.76$.

2.
$$4x^2 - 10x = 2x - 9$$
$$4x^2 - 12x + 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$$
$$x = \frac{12 \pm \sqrt{0}}{8}$$
$$x = \frac{3}{2}$$

The solution is $\frac{3}{2}$.

3.
$$7x - 5x^2 - 4 = 2x + 3$$
$$-5x^2 + 5x - 7 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(-5)(-7)}}{2(-5)}$$
$$x = \frac{-5 \pm \sqrt{-115}}{-10}$$
$$x = \frac{-5 \pm i\sqrt{115}}{-10}$$
$$x = \frac{5 \pm i\sqrt{115}}{10}$$

The solutions are $\frac{5 + i\sqrt{115}}{10}$ and $\frac{5 - i\sqrt{115}}{10}$.

4. $2x^2 + 4x - 4 = 0$

$b^2 - 4ac = 4^2 - 4(2)(-4) = 48 > 0$

Two real solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{48}}{2(2)}$$
$$= \frac{-4 \pm 4\sqrt{3}}{4}$$
$$= -1 \pm \sqrt{3} \approx 0.73, -2.73$$

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5. $3x^2 + 12x + 12 = 0$

$b^2 - 4ac = 12^2 - 4(3)(12) = 0$

One real solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{0}}{2(3)}$$

$$= \frac{-12}{6} = -2$$

6. $8x^2 = 9x - 11$

$8x^2 - 9x + 11 = 0$

$b^2 - 4ac = (-9)^2 - 4(8)(11) = -271 < 0$

Two imaginary solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-9) \pm \sqrt{-271}}{2(8)}$$

$$= \frac{9}{16} \pm i\frac{\sqrt{271}}{16}$$

7. $7x^2 - 2x = 5$

$7x^2 - 2x - 5 = 0$

$b^2 - 4ac = (-2)^2 - 4(7)(-5) = 144 > 0$

Two real solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{144}}{2(7)}$$

$$= \frac{2 \pm 12}{14} = \frac{1}{7} \pm \frac{6}{7} = 1, -\frac{5}{7}$$

8. $4x^2 + 3x + 12 = 3 - 3x$

$4x^2 + 6x + 9 = 0$

$b^2 - 4ac = 6^2 - 4(4)(9) = -108 < 0$

Two imaginary solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{-108}}{2(4)}$$

$$= \frac{-6 \pm i6\sqrt{3}}{8} = -\frac{3}{4} \pm \frac{3i\sqrt{3}}{4}$$

9. $3x - 5x^2 + 1 = 6 - 7x$

$-5x^2 + 10x - 5 = 0$

$b^2 - 4ac = 10^2 - 4(-5)(-5) = 0$

One real solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{0}}{2(-5)}$$

$$= \frac{-10}{-10} = 1$$

10. $h = -16t^2 + v_0 t + h_0$

$3 = -16t^2 + 50t + 4$

$0 = -16t^2 + 50t + 1$

$$t = \frac{-50 \pm \sqrt{50^2 - 4(-16)(1)}}{2(-16)}$$

$$t = \frac{-50 \pm \sqrt{2564}}{-32}$$

$t \approx -0.02$ or $t \approx 3.14$

Reject the solution -0.02 because the ball's time in the air cannot be negative. So, the ball is in the air for about 3.14 seconds.

4.8 Exercises (pp. 296–299)

Skill Practice

1. You can use the discriminant of a quadratic equation to determine the equation's number and type of solutions.

2. *Sample answer:*

When hitting a baseball with a bat, you would need to use the model that accounts for initial vertical velocity because the baseball is launched, not dropped.

3. $x^2 - 4x - 5 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{36}}{2}$$

$$x = \frac{4 \pm 6}{2}$$

$x = 2 \pm 3$

$x = 5, -1$

The solutions are 5 and -1.

4. $x^2 - 6x + 7 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{8}}{2}$$

$$x = \frac{6 \pm 2\sqrt{2}}{2}$$

$x = 3 \pm \sqrt{2}$

The solutions are $x = 3 + \sqrt{2} \approx 4.41$ and $x = 3 - \sqrt{2} \approx 1.59$.

5. $t^2 + 8t + 19 = 0$

$$t = \frac{-8 \pm \sqrt{8^2 - 4(1)(19)}}{2(1)}$$

$$t = \frac{-8 \pm \sqrt{-12}}{2}$$

$$t = \frac{-8 \pm 2i\sqrt{3}}{2}$$

$t = -4 \pm i\sqrt{3}$

The solutions are $-4 + i\sqrt{3}$ and $-4 - i\sqrt{3}$.

6. $x^2 - 16x + 7 = 0$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{16 \pm \sqrt{228}}{2}$$

$$x = \frac{16 \pm 2\sqrt{57}}{2}$$

$x = 8 \pm \sqrt{57}$

The solutions are $x = 8 + \sqrt{57} \approx 15.55$ and $x = 8 - \sqrt{57} \approx 0.45$.

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7. $8w^2 - 8w + 2 = 0$

$$w = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(8)(2)}}{2(8)}$$

$$w = \frac{8 \pm \sqrt{0}}{16}$$

$$w = \frac{8}{16}$$

$$w = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

8. $5p^2 - 10p + 24 = 0$

$$p = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(5)(24)}}{2(5)}$$

$$p = \frac{10 \pm \sqrt{-380}}{10}$$

$$p = \frac{10 \pm 2i\sqrt{95}}{10}$$

$$p = 1 \pm i\frac{\sqrt{95}}{5}$$

The solutions are $1 + i\frac{\sqrt{95}}{5}$ and $1 - i\frac{\sqrt{95}}{5}$.

9. $4x^2 - 8x + 1 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{48}}{8}$$

$$x = \frac{8 \pm 4\sqrt{3}}{8}$$

$$x = 1 \pm \frac{\sqrt{3}}{2}$$

The solutions are $x = 1 + \frac{\sqrt{3}}{2} \approx 1.87$ and $x = 1 - \frac{\sqrt{3}}{2} \approx 0.13$.

10. $6u^2 + 4u + 11 = 0$

$$u = \frac{-4 \pm \sqrt{4^2 - 4(6)(11)}}{2(6)}$$

$$u = \frac{-4 \pm \sqrt{-248}}{12}$$

$$u = \frac{-4 \pm 2i\sqrt{62}}{12}$$

$$u = -\frac{1}{3} \pm i\frac{\sqrt{62}}{6}$$

The solutions are $-\frac{1}{3} + i\frac{\sqrt{62}}{6}$ and $-\frac{1}{3} - i\frac{\sqrt{62}}{6}$.

11. $3r^2 - 8r - 9 = 0$

$$r = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-9)}}{2(3)}$$

$$r = \frac{8 \pm \sqrt{172}}{6}$$

$$r = \frac{8 \pm 2\sqrt{43}}{6}$$

$$r = \frac{4}{3} \pm \frac{\sqrt{43}}{3}$$

The solutions are $r = \frac{4}{3} + \frac{\sqrt{43}}{3} \approx 3.52$ and $r = \frac{4}{3} - \frac{\sqrt{43}}{3} \approx -0.85$

12. A;

$2x^2 - 16x + 50 = 0$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(2)(50)}}{2(2)}$$

$$x = \frac{16 \pm \sqrt{-144}}{4}$$

$$x = \frac{16 \pm 12i}{4}$$

$$x = 4 \pm 3i$$

13. $3w^2 - 12w = -12$

$3w^2 - 12w + 12 = 0$

$w^2 - 4w + 4 = 0$

$$w = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$

$$w = \frac{4 \pm \sqrt{0}}{2}$$

$$w = 2$$

The solution is 2.

14. $x^2 + 6x = -15$

$x^2 + 6x + 15 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{-24}}{2}$$

$$x = \frac{-6 \pm 2i\sqrt{6}}{2}$$

$$x = -3 \pm i\sqrt{6}$$

The solutions are $-3 + i\sqrt{6}$ and $-3 - i\sqrt{6}$.

15. $s^2 = -14 - 3s$

$s^2 + 3s + 14 = 0$

$$s = \frac{-3 \pm \sqrt{3^2 - 4(1)(14)}}{2(1)}$$

$$s = \frac{-3 \pm \sqrt{-47}}{2}$$

$$s = \frac{-3 \pm i\sqrt{47}}{2}$$

$$s = \frac{-3}{2} \pm i\frac{\sqrt{47}}{2}$$

The solutions are $-\frac{3}{2} + i\frac{\sqrt{47}}{2}$ and $-\frac{3}{2} - i\frac{\sqrt{47}}{2}$.

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16. $-3y^2 = 6y - 10$

$-3y^2 - 6y + 10 = 0$

$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-3)(10)}}{2(-3)}$$

$$y = \frac{6 \pm \sqrt{156}}{-6}$$

$$y = \frac{6 \pm 2\sqrt{39}}{-6}$$

$$y = -1 \pm \frac{\sqrt{39}}{3}$$

The solutions are $y = -1 + \frac{\sqrt{39}}{3} \approx 1.08$ and $y = -1 - \frac{\sqrt{39}}{3} \approx -3.08$.

17. $3 - 8v - 5v^2 = 2v$

$-5v^2 - 10v + 3 = 0$

$$v = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(-5)(3)}}{2(-5)}$$

$$v = \frac{10 \pm \sqrt{160}}{-10}$$

$$v = \frac{10 \pm 4\sqrt{10}}{-10}$$

$$v = -1 \pm \frac{2\sqrt{10}}{5}$$

The solutions are $v = -1 + \frac{2\sqrt{10}}{5} \approx 0.26$ and $v = -1 - \frac{2\sqrt{10}}{5} \approx -2.26$.

18. $7x - 5 + 12x^2 = -3x$

$12x^2 + 10x - 5 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(12)(-5)}}{2(12)}$$

$$x = \frac{-10 \pm \sqrt{340}}{24}$$

$$x = \frac{-10 \pm 2\sqrt{85}}{24}$$

$$x = -\frac{5}{12} \pm \frac{\sqrt{85}}{12}$$

The solutions are $x = -\frac{5}{12} + \frac{\sqrt{85}}{12} \approx 0.35$ and $x = -\frac{5}{12} - \frac{\sqrt{85}}{12} \approx -1.18$.

19. $4x^2 + 3 = x^2 - 7x$

$3x^2 + 7x + 3 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(3)(3)}}{2(3)}$$

$$x = \frac{-7 \pm \sqrt{13}}{6}$$

$$x = -\frac{7}{6} \pm \frac{\sqrt{13}}{6}$$

The solutions are $x = -\frac{7}{6} + \frac{\sqrt{13}}{6} \approx -0.57$ and $x = -\frac{7}{6} - \frac{\sqrt{13}}{6} \approx -1.77$.

20. $6 - 2t^2 = 9t + 15$

$-2t^2 - 9t - 9 = 0$

$$t = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(-2)(-9)}}{2(-2)}$$

$$t = \frac{9 \pm \sqrt{9}}{-4}$$

$$t = \frac{9 \pm 3}{-4}$$

$$t = -\frac{9}{4} \pm \frac{3}{4}$$

$$t = -\frac{3}{2}, -3$$

The solutions are $-\frac{3}{2}$ and -3.

21. $4 + 9n - 3n^2 = 2 - n$

$-3n^2 + 10n + 2 = 0$

$$n = \frac{-10 \pm \sqrt{10^2 - 4(-3)(2)}}{2(-3)}$$

$$n = \frac{-10 \pm \sqrt{124}}{-6}$$

$$n = \frac{-10 \pm 2\sqrt{31}}{-6}$$

$$n = \frac{5}{3} \pm \frac{\sqrt{31}}{3}$$

The solutions are $n = \frac{5}{3} + \frac{\sqrt{31}}{3} \approx 3.52$ and $n = \frac{5}{3} - \frac{\sqrt{31}}{3} \approx -0.19$.

22. $z^2 + 15z + 24 = -32$

$z^2 + 15z + 56 = 0$

$$z = \frac{-15 \pm \sqrt{15^2 - 4(1)(56)}}{2(1)}$$

$$z = \frac{-15 \pm \sqrt{1}}{2}$$

$$z = \frac{-15 \pm 1}{2}$$

$$z = -7, -8$$

The solutions are -7 and -8.

Check: $z^2 + 15z + 56 = 0$

$(z + 8)(z + 7) = 0$

$z + 8 = 0$ or $z + 7 = 0$

$z = -8$ or $z = -7$

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23. $x^2 - 5x + 10 = 4$

$x^2 - 5x + 6 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{1}}{2}$$

$$x = \frac{5 \pm 1}{2}$$

$$x = 3, 2$$

The solutions are 3 and 2.

Check: $x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x - 3 = 0$ or $x - 2 = 0$

$x = 3$ or $x = 2$

24. $m^2 + 5m - 99 = 3m$

$m^2 + 2m - 99 = 0$

$$m = \frac{-2 \pm \sqrt{2^2 - 4(1)(99)}}{2(1)}$$

$$m = \frac{-2 \pm \sqrt{400}}{2}$$

$$m = \frac{-2 \pm 20}{2}$$

$$m = 9, -11$$

The solutions are 9 and -11.

Check: $m^2 + 2m - 99 = 0$

$(m - 9)(m + 11) = 0$

$m - 9 = 0$ or $m + 11 = 0$

$m = 9$ or $m = -11$

25. $s^2 - s - 3 = s$

$s^2 - 2s - 3 = 0$

$$s = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$s = \frac{2 \pm \sqrt{16}}{2}$$

$$s = \frac{2 \pm 4}{2}$$

$$s = 3, -1$$

The solutions are 3 and -1.

Check: $s^2 - 2s - 3 = 0$

$(s - 3)(s + 1) = 0$

$s - 3 = 0$ or $s + 1 = 0$

$s = 3$ or $s = -1$

26. $r^2 - 4r + 8 = 5r$

$r^2 - 9r + 8 = 0$

$$r = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(8)}}{2(1)}$$

$$r = \frac{9 \pm \sqrt{49}}{2}$$

$$r = \frac{9 \pm 7}{2}$$

$$r = 8, 1$$

The solutions are 8 and 1.

Check: $r^2 - 9r + 8 = 0$

$(r - 8)(r - 1) = 0$

$r - 8 = 0$ or $r - 1 = 0$

$r = 8$ or $r = 1$

27. $3x^2 + 7x - 24 = 13x$

$3x^2 - 6x - 24 = 0$

$x^2 - 2x - 8 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{36}}{2}$$

$$x = \frac{2 \pm 6}{2}$$

$$x = 4, -2$$

The solutions are 4 and -2.

Check: $x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ or $x = -2$

28. $45x^2 + 57x + 1 = 5$

$45x^2 + 57x - 4 = 0$

$$x = \frac{-57 \pm \sqrt{57^2 - 4(45)(-4)}}{2(45)}$$

$$x = \frac{-57 \pm \sqrt{3969}}{90}$$

$$x = \frac{-57 \pm 63}{90}$$

$$x = \frac{1}{15}, -\frac{4}{3}$$

The solutions are $\frac{1}{15}$ and $-\frac{4}{3}$.

Check: $45x^2 + 57x - 4 = 0$

$(15x - 1)(3x + 4) = 0$

$15x - 1 = 0$ or $3x + 4 = 0$

$x = \frac{1}{15}$ or $x = \frac{-4}{3}$

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29. $5p^2 + 40p + 100 = 25$

$5p^2 + 40p + 75 = 0$

$p^2 + 8p + 15 = 0$

$$p = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$$

$$p = \frac{-8 \pm \sqrt{4}}{2} = \frac{-8 \pm 2}{2}$$

$$p = -3, -5$$

The solutions are -3 and -5.

Check: $p^2 + 8p + 15 = 0$

$(p + 3)(p + 5) = 0$

$p + 3 = 0$ or $p + 5 = 0$

$p = -3$ or $p = -5$

30. $9n^2 - 42n - 162 = 21n$

$9n^2 - 63n - 162 = 0$

$n^2 - 7n - 18 = 0$

$$n = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-18)}}{2(1)}$$

$$n = \frac{7 \pm \sqrt{121}}{2} = \frac{7 \pm 11}{2}$$

$$n = 9, -2$$

The solutions are 9 and -2.

Check: $n^2 - 7n - 18 = 0$

$(n - 9)(n + 2) = 0$

$n - 9 = 0$ or $n + 2 = 0$

$n = 9$ or $n = -2$

31. $x^2 - 8x + 16 = 0$

$b^2 - 4ac = (-8)^2 - 4(1)(16) = 0$

One real solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{0}}{2(1)} = 4$$

32. $s^2 + 7s + 11 = 0$

$b^2 - 4ac = 7^2 - 4(1)(11) = 5 > 0$

Two real solutions:

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{5}}{2(1)} \approx -2.38, -4.62$$

33. $8p^2 + 8p + 3 = 0$

$b^2 - 4ac = 8^2 - 4(8)(3) = -32 < 0$

Two imaginary solutions:

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{-32}}{2(8)} = \frac{-8 \pm 4i\sqrt{2}}{16}$$

$$= -\frac{1}{2} \pm i\frac{\sqrt{2}}{4}$$

34. $-4w^2 + w - 14 = 0$

$b^2 - 4ac = 1^2 - 4(-4)(-14) = -223 < 0$

Two imaginary solutions:

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{-223}}{2(-4)}$$

$$= \frac{-1 \pm i\sqrt{223}}{-8}$$

$$= \frac{1}{8} \pm i\frac{\sqrt{223}}{8}$$

35. $5x^2 + 20x + 21 = 0$

$b^2 - 4ac = 20^2 - 4(5)(21) = -20 < 0$

Two imaginary solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{-20}}{2(5)}$$

$$= \frac{-20 \pm 2i\sqrt{5}}{10}$$

$$= -2 \pm i\frac{\sqrt{5}}{5}$$

36. $8z - 10 = z^2 - 7z + 3$

$-z^2 + 15z - 13 = 0$

$b^2 - 4ac = 15^2 - 4(-1)(-13) = 173 > 0$

Two real solutions:

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-15 \pm \sqrt{173}}{2(-1)}$$

$$= \frac{-15 \pm \sqrt{173}}{2(-1)}$$

$$= \frac{15 \pm \sqrt{173}}{2}$$

$$\approx 14.08, 0.92$$

37. $8n^2 - 4n + 2 = 5n - 11$

$8n^2 - 9n + 13 = 0$

$b^2 - 4ac = (-9)^2 - 4(8)(13) = -335 < 0$

Two imaginary solutions:

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-9) \pm \sqrt{-335}}{2(8)}$$

$$= \frac{9 \pm i\sqrt{335}}{16}$$

$$= \frac{9}{16} \pm i\frac{\sqrt{335}}{16}$$

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38. $5x^2 + 16x = 11x - 3x^2$

$8x^2 + 5x = 0$

$b^2 - 4ac = 5^2 - 4(8)(0) = 25 > 0$

Two real solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25}}{2(8)}$$

$$= \frac{-5 \pm 5}{16} = 0, -\frac{5}{8}$$

39. $7r^2 - 5 = 2r + 9r^2$

$-2r^2 - 2r - 5 = 0$

$b^2 - 4ac = (-2)^2 - 4(-2)(-5) = -36 < 0$

Two imaginary solutions:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{-36}}{2(-2)}$$

$$= \frac{2 \pm 6i}{-4}$$

$$= -\frac{1}{2} \pm \frac{3}{2}i$$

40. $16t^2 - 7t = 17t - 9$

$16t^2 - 24t + 9 = 0$

$(4t - 3)(4t - 3) = 0$

$4t - 3 = 0$

$t = \frac{3}{4}$

The solution is $\frac{3}{4}$.

41. $7x - 3x^2 = 85 + 2x^2 + 2x$

$-5x^2 + 5x - 85 = 0$

$x^2 - x + 17 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(17)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-67}}{2}$$

$$x = \frac{1 \pm i\sqrt{67}}{2}$$

$$x = \frac{1}{2} \pm \frac{i\sqrt{67}}{2}$$

The solutions are $\frac{1}{2} + \frac{i\sqrt{67}}{2}$ and $\frac{1}{2} - \frac{i\sqrt{67}}{2}$.

42. $4(x - 1)^2 = 6x + 2$

$4(x^2 - 2x + 1) = 6x + 2$

$4x^2 - 8x + 4 = 6x + 2$

$4x^2 - 14x + 2 = 0$

$2x^2 - 7x + 1 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{41}}{4}$$

The solutions are $x = \frac{7 + \sqrt{41}}{4} \approx 3.35$ and

$x = \frac{7 - \sqrt{41}}{4} \approx 0.15$.

43. $25 - 16v^2 = 12v(v + 5)$

$25 - 16v^2 = 12v^2 + 60v$

$-28v^2 - 60v + 25 = 0$

$28v^2 + 60v - 25 = 0$

$(14v - 5)(2v + 5) = 0$

$14v - 5 = 0$ or $2v + 5 = 0$

$v = \frac{5}{14}$ or $v = -\frac{5}{2}$

The solutions are $\frac{5}{14}$ and $-\frac{5}{2}$.

44. $\frac{3}{2}y^2 - 6y = \frac{3}{4}y - 9$

$6y^2 - 24y = 3y - 36$

$6y^2 - 27y + 36 = 0$

$2y^2 - 9y + 12 = 0$

$$y = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(12)}}{2(2)}$$

$$y = \frac{9 \pm \sqrt{-15}}{4}$$

$$y = \frac{9 \pm i\sqrt{15}}{4}$$

$$y = \frac{9}{4} \pm \frac{i\sqrt{15}}{4}$$

The solutions are $\frac{9}{4} + \frac{i\sqrt{15}}{4}$ and $\frac{9}{4} - \frac{i\sqrt{15}}{4}$.

45. $3x^2 + \frac{9}{2}x - 4 = 5x + \frac{3}{4}$

$12x^2 + 18x - 16 = 20x + 3$

$12x^2 - 2x - 19 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(12)(-19)}}{2(12)}$$

$$x = \frac{2 \pm \sqrt{916}}{24}$$

$$x = \frac{2 \pm 2\sqrt{229}}{24}$$

$$x = \frac{1 \pm \sqrt{229}}{12}$$

The solutions are $x = \frac{1 + \sqrt{229}}{12} \approx 1.34$ and

$x = \frac{1 - \sqrt{229}}{12} \approx -1.18$.

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46. $1.1(3.4x - 2.3)^2 = 15.5$

$11(3.4x - 2.3)^2 = 155$

$\left(\frac{17}{5}x - \frac{23}{10}\right)^2 = \frac{155}{11}$

$\frac{17}{5}x - \frac{23}{10} = \pm\sqrt{\frac{155}{11}}$

$\frac{17}{5}x = \frac{23}{10} \pm \sqrt{\frac{155}{11}}$

$x = \frac{5}{17}\left(\frac{23}{10} \pm \sqrt{\frac{155}{11}}\right)$

$x = \frac{23}{34} \pm \frac{5}{17}\sqrt{\frac{155}{11}}$

$x = \frac{23}{34} \pm \frac{5\sqrt{1705}}{187}$

The solutions are $x = \frac{23}{34} + \frac{5\sqrt{1705}}{187} \approx 1.78$ and $x = \frac{23}{34} - \frac{5\sqrt{1705}}{187} \approx -0.43$.

47. $19.25 = 8.5(2r - 1.75)^2$

$-\frac{77}{34} = \left(2r - \frac{7}{4}\right)^2$

$\pm\sqrt{-\frac{77}{34}} = 2r - \frac{7}{4}$

$\frac{7}{4} \pm i\sqrt{\frac{77}{34}} = 2r$

$\frac{7}{8} \pm \frac{1}{2}i\sqrt{\frac{77}{34}} = r$

$\frac{7}{8} \pm \frac{i\sqrt{2618}}{68} = r$

The solutions are $\frac{7}{8} + \frac{i\sqrt{2618}}{68}$ and $\frac{7}{8} - \frac{i\sqrt{2618}}{68}$.

48. $4.5 = 1.5(3.25 - s)^2$

$3 = (3.25 - s)^2$

$\pm\sqrt{3} = 3.25 - s$

$s = 3.25 \pm \sqrt{3}$

The solutions are $s = 3.25 + \sqrt{3} \approx 4.98$ and $s = 3.25 - \sqrt{3} \approx 1.52$.

49. The solutions are imaginary, not real.

$3x^2 + 6x + 15 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(15)}}{2(3)}$

$= \frac{-6 \pm \sqrt{-144}}{6}$

$= \frac{-6 \pm 12i}{6}$

$= -1 \pm 2i$

The solutions are $-1 + 2i$ and $-1 - 2i$.

50. The quadratic equation must be written in standard form before applying the quadratic formula.

$x^2 + 6x + 8 = 2$

$x^2 + 6x + 6 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{12}}{2}$

$x = \frac{-6 \pm 2\sqrt{3}}{2}$

$x = -3 \pm \sqrt{3}$

The solutions are $x = -3 + \sqrt{3} \approx -1.27$ and $x = -3 - \sqrt{3} \approx -4.73$.

51. $ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Mean of solutions:

$\dfrac{\frac{-b + \sqrt{b^2 - 4ac} + \left(-b - \sqrt{b^2 - 4ac}\right)}{2a}}{2}$

$= \dfrac{\frac{-2b}{2a}}{2} = \frac{-2b}{4a} = -\frac{b}{2a}$, which is the formula for the axis of symmetry.

The axis of symmetry of the graph of $y = ax^2 + bx + c$ is the mean of the x-values of two points that lie on the graph that have the same y-value.

52. Because there are two x-intercepts, the discriminant is positive.

53. Because there are no x-intercepts, the discriminant is negative.

54. Because there is one x-intercept, the discriminant is zero.

55. C;

$2x^2 + 5x + c = 0$

$b^2 - 4ac = 5^2 - 4(2)(c) = -23$

$25 - 8c = -23$

$-8c = -48$

$c = 6$

56. $x^2 - 4x + c = 0$

a. $(-4)^2 - 4(1)(c) > 0$

$16 - 4c > 0$

$-4c > -16$

$c < 4$

b. $(-4)^2 - 4(1)(c) = 0$

$16 - 4c = 0$

$-4c = -16$

$c = 4$

c. $(-4)^2 - 4(1)(c) < 0$

$16 - 4c < 0$

$-4c < -16$

$c > 4$

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57. $x^2 + 8x + c = 0$

a. $8^2 - 4(1)(c) > 0$

$64 - 4c > 0$

$-4c > -64$

$c < 16$

b. $8^2 - 4(1)(c) = 0$

$64 - 4c = 0$

$-4c = -64$

$c = 16$

c. $8^2 - 4(1)(c) < 0$

$64 - 4c < 0$

$-4c < -64$

$c > 16$

58. $-x^2 + 16x + c = 0$

a. $16^2 - 4(-1)(c) > 0$

$256 + 4c > 0$

$4c > -256$

$c > -64$

b. $16^2 - 4(-1)(c) = 0$

$256 + 4c = 0$

$4c = -256$

$c = -64$

c. $16^2 - 4(-1)(c) < 0$

$256 + 4c < 0$

$4c < -256$

$c < -64$

59. $3x^2 + 24x + c = 0$

a. $24^2 - 4(3)(c) > 0$

$576 - 12c > 0$

$-12c > -576$

$c < 48$

b. $24^2 - 4(3)(c) = 0$

$576 - 12c = 0$

$-12c = -576$

$c = 48$

c. $24^2 - 4(3)(c) < 0$

$576 - 12c < 0$

$-12c < -576$

$c > 4$

60. $-4x^2 - 10x + c = 0$

a. $(-10)^2 - 4(-4)(c) > 0$

$100 + 16c > 0$

$16c > -100$

$c > -\frac{25}{4}$

b. $(-10)^2 - 4(-4)(c) = 0$

$100 + 16c = 0$

$16c = -100$

$c = -\frac{25}{4}$

c. $(-10)^2 - 4(-4)(c) < 0$

$100 + 16c < 0$

$16c < -100$

$c < -\frac{25}{4}$

61. $x^2 - x + c = 0$

a. $(-1)^2 - 4(1)(c) > 0$

$1 - 4c > 0$

$-4c > -1$

$c < \frac{1}{4}$

b. $(-1)^2 - 4(1)(c) = 0$

$1 - 4c = 0$

$-4c = -1$

$c = \frac{1}{4}$

c. $(-1)^2 - 4(1)(c) < 0$

$1 - 4c < 0$

$-4c < -1$

$c > \frac{1}{4}$

62. $b^2 - 4ac = -10$

Sample answer: $5^2 - 4\left(\frac{7}{4}\right)(5) = 25 - 35$

$= -10$

$\frac{7}{4}x^2 + 5x + 5 = 0$

63. $ax^2 + bx + 4 = 0$

$a(-4)^2 + b(-4) + 4 = 0$

$16a - 4b + 4 = 0$

$16a = 4b - 4$

$a = \frac{4b - 4}{16}$

$a = \frac{b - 1}{4}$

$a(3)^2 + b(3) + 4 = 0$

$9a + 3b + 4 = 0$

$9\left(\frac{b - 1}{4}\right) + 3b + 4 = 0$

$9(b - 1) + 12b + 16 = 0$

$9b - 9 + 12b + 16 = 0$

$21b = -7$

$b = -\frac{7}{21}$

$b = -\frac{1}{3}$

$a = \frac{b - 1}{4} = \frac{-\frac{1}{3} - 1}{4} = \frac{\frac{-4}{3}}{4} = -\frac{1}{3}$

$ax^2 + bx + 4 = 0$

$-\frac{1}{3}x^2 - \frac{1}{3}x + 4 = 0$

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64. $$ax^2 + bx + 4 = 0$$
$$a(-1)^2 + b(-1) + 4 = 0$$
$$a - b + 4 = 0$$
$$a = b - 4$$
$$a\left(-\frac{4}{3}\right)^2 + b\left(-\frac{4}{3}\right) + 4 = 0$$
$$\frac{16}{9}a - \frac{4}{3}b + 4 = 0$$
$$\frac{16}{9}(b - 4) - \frac{4}{3}b + 4 = 0$$
$$16(b - 4) - 12b + 36 = 0$$
$$16b - 64 - 12b + 36 = 0$$
$$4b - 28 = 0$$
$$4b = 28$$
$$b = 7$$
$$a = b - 4 = 7 - 4 = 3$$
$$ax^2 + bx + 4 = 0$$
$$3x^2 + 7x + 4 = 0$$

65. $$ax^2 + bx + 4 = 0$$
$$a(-1 - i)^2 + b(-1 - i) + 4 = 0$$
$$2ia + (-1 - i)b + 4 = 0$$
$$2ia = -4 - (-1 - i)b$$
$$a = \frac{-4 - (-1 - i)b}{2i}$$
$$a(-1 + i)^2 + b(-1 + i) + 4 = 0$$
$$-2ia + (-1 + i)b + 4 = 0$$
$$-2i\left[\frac{4 + (-1 - i)b}{2i}\right] + (-1 + i)b + 4 = 0$$
$$-4 - (-1 - i)b + (-1 + i)b + 4 = 0$$
$$(-1 - i - 1 + i)b = -8$$
$$-2b = -8$$
$$b = 4$$
$$a = \frac{-4 - (-1 - i)b}{2i} = \frac{-4 - (-1 - i)(4)}{2i}$$
$$= \frac{-4 + 4 + 4i}{2i} = 2$$
$$ax^2 + bx + 4 = 0$$
$$2x^2 + 4x + 4 = 0$$

66. $$ax^2 + bx + c = 0$$
$$a(3i)^2 + b(3i) + c = 0, \qquad a(-2i)^2 + b(-2i) + c = 0$$
$$-9a + (3i)b + c = 0 \qquad -4a + (-2i)b + c = 0$$
$$-9a + (3i)b + c = -4a + (-2i)b + c$$
$$(5i)b = 5a$$
$$ib = a$$
$$-9(ib) + (3i)b + c = 0$$
$$(-9i)b + (3i)b + c = 0$$
$$(-6i)b + c = 0$$
$$c = 6ib$$

You can see from the equations $a = ib$ and $c = 6ib$, that a, b, and c cannot be real numbers.

67. $$ax^2 + bx + c = 0$$
$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$
$$x^2 + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Problem Solving

68. $h = -16t^2 + V_0t + h_0$
$$0 = -16t^2 - 50t + 7$$
$$t = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(-16)(7)}}{2(-16)}$$
$$t = \frac{50 \pm \sqrt{2948}}{-32}$$
$$t \approx -3.26 \text{ or } t \approx 0.13$$

Reject the solution -3.26 because the ball's time in the air cannot be negative. So the defensive player's teammates have about 0.13 second to intercept the ball before it hits the ground.

69. C;
$$s = 858t^2 + 1412t + 4982$$
$$50{,}000 = 858t^2 + 1412t + 4982$$
$$0 = 858t^2 + 1412t - 45{,}018$$
$$t = \frac{-1412 \pm \sqrt{(1412)^2 - 4(858)(-45{,}018)}}{2(858)}$$
$$t = \frac{-1412 \pm \sqrt{156{,}495{,}520}}{1716}$$
$$t \approx 6.47 \text{ or } t \approx -8.11$$

Reject the solution -8.11. The number of subscribers reached 50 million 6 years after 1990, or 1996.

70. a. The motorcycle's height r when it lands on the ramp is 20 feet.

b. $y = -\frac{1}{640}x^2 + \frac{1}{4}x + 20$
$$x = -\frac{b}{2a} = \frac{-\frac{1}{4}}{2\left(-\frac{1}{640}\right)} = 80$$

Because the x-coordinate of the vertex is 80, the distance d between the ramps is 2(80), or 160 feet.

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c. Because the x-coordinate of the vertex is 80, the horizontal distance h the motorcycle has traveled when it reaches its maximum height is 80 feet.

d. $y = -\frac{1}{640}(80)^2 + \frac{1}{4}(80) + 20 = 30$

Because the y-coordinate of the vertex is 30, the motorcycle's maximum height k above the ground is 30 feet.

71. $S = -0.000013E^2 + 0.042E - 21$

$10 = -0.000013E^2 + 0.042E - 21$

$0 = -0.000013E^2 + 0.042E - 31$

$$E = \frac{-0.042 \pm \sqrt{(0.042)^2 - 4(-0.000013)(-31)}}{2(-0.000013)}$$

$$E = \frac{-0.042 \pm \sqrt{0.000152}}{-0.000026}$$

$E \approx 1141.2$ or $E \approx 2089.57$

You would expect to find 10 species of ants at elevations of 1141.2 meters and 2089.57 meters.

72. a. $4\ell + 3w = 900$

$3w = 900 - 4\ell$

$w = 300 - \frac{4}{3}\ell$

b. $\ell w = 12{,}000$

$\ell = \frac{12{,}000}{w}$

$$w = 300 - \frac{4}{3}\left(\frac{12{,}000}{w}\right)$$

$$w = 300 - \frac{16{,}000}{w}$$

$w^2 = 300w - 16{,}000$

$w^2 - 300w + 16{,}000 = 0$

$$w = \frac{-(-300) \pm \sqrt{(-300)^2 - 4(1)(16{,}000)}}{2(1)}$$

$$w = \frac{300 \pm \sqrt{26{,}000}}{2}$$

$$w = \frac{300 \pm 20\sqrt{65}}{2} = 150 \pm 10\sqrt{65}$$

$w \approx 230.62$ or $w \approx 69.38$

When $w \approx 230.62$: $\ell \approx \frac{12{,}000}{230.62} \approx 52.03$

When $w \approx 69.38$: $\ell \approx \frac{12{,}000}{69.38} \approx 172.96$

The possible dimensions of each section are 230.62 feet by 52.03 feet or 69.38 feet by 172.96 feet.

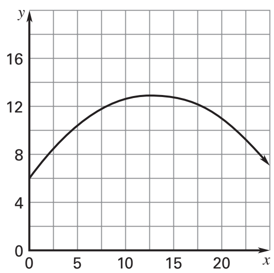
73. $x = 20t$

$y = -16t^2 + 21t + 6$

a.

t	0	0.25	0.5	0.75	1
x	0	5	10	15	20
y	6	10.25	12.5	12.75	11
(*x*, *y*)	(0, 6)	(5, 10.25)	(10, 12.5)	(15, 12.75)	(20, 11)

b.



c. No, the player does not make the free throw. The shot is too high. It goes over the backboard.

74. a. $h = -16t^2 + v_0t + 921$

The maximum height of 1081 feet occurs at the vertex.

$$t = -\frac{b}{2a} = -\frac{v_0}{2(-16)} = \frac{v_0}{32}$$

$$h = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) + 921 = 1081$$

$$-\frac{v_0^2}{64} + \frac{v_0^2}{32} + 921 = 1081$$

$$\frac{v_0^2}{64} = 160$$

$v_0^2 = 10{,}240$

$v_0 = \pm\sqrt{10{,}240} = \pm 101$

The initial velocity is about 101 feet per second.

b. When $v_0 = \sqrt{10{,}240}$:

$1081 = -16t^2 + \sqrt{10{,}240}\,t + 921$

$0 = -16t^2 + \sqrt{10{,}240}\,t - 160$

$$t = \frac{-\sqrt{10{,}240} \pm \sqrt{\left(\sqrt{10{,}240}\right)^2 - 4(-16)(-160)}}{2(-16)}$$

$$t = \frac{-\sqrt{10{,}240} \pm \sqrt{0}}{-32} = \frac{-32\sqrt{10}}{-32} = \sqrt{10} \approx 3.16$$

The time given by the model is longer than the time given in the brochure. The model is not extremely accurate.

Mixed Review

75. $(2, -7), (4, 9)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-7)}{4 - 2} = \frac{16}{2} = 8$$

The line rises from left to right.

76. $(-8, 3), (4, -5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 3}{4 - (-8)} = \frac{-8}{12} = -\frac{2}{3}$$

The line falls from left to right.

77. $(-3, -2), (6, -2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-2)}{6 - (-3)} = \frac{0}{9} = 0$$

The line is horizontal.

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78. $\left(\frac{3}{4}, 2\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{5}{4} - 2}{\frac{1}{2} - \frac{3}{4}} = 3$$

The line rises from left to right.

79. $(-1, 0), (-1, 5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{-1 - (-1)} = \frac{5}{0}; \text{ undefined}$$

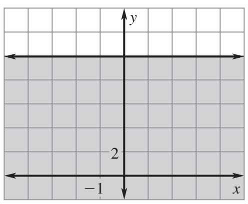
The line is vertical.

80. $\left(\frac{1}{3}, \frac{7}{3}\right), \left(4, \frac{2}{3}\right)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{3} - \frac{7}{3}}{4 - \frac{1}{3}} = -\frac{5}{11}$$

The line falls from left to right.

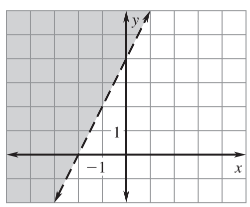
81.



82. $8x - 4y < -16$

$-4y < -8x - 16$

$y > 2x + 4$

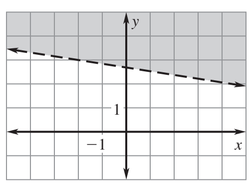


83. $\frac{1}{2}x + 3y > 8$

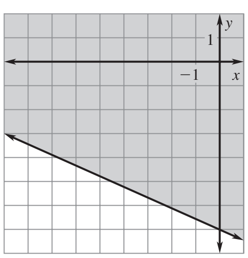
$x + 6y > 16$

$6y > -x + 16$

$y > -\frac{1}{6}x + \frac{8}{3}$



84.



85. $y = 3(x + 1)(x + 2)$

x-intercepts:

$p = -1$ and $q = -2$

$$x = \frac{p + q}{2} = \frac{-1 + (-2)}{2}$$

$$= -\frac{3}{2}$$

$$y = 3\left(-\frac{3}{2} + 1\right)\left(-\frac{3}{2} + 2\right) = -\frac{3}{4}$$

vertex: $\left(-\frac{3}{2}, -\frac{3}{4}\right)$

86. $y = -2(x - 3)(x - 1)$

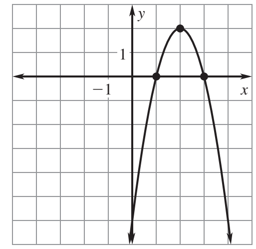
x-intercepts:

$p = 3$ and $q = 1$

$$x = \frac{p + q}{2} = \frac{3 + 1}{2} = 2$$

$y = -2(2 - 3)(2 - 1) = 2$

vertex: $(2, 2)$



87. $a = 2000 - 250t$

The height of the hang glider prior to the descent is 2000 feet.

$0 = 2000 - 250t$

$250t = 2000$

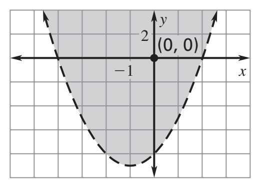
$t = 8$

It takes the hang glider 8 minutes to reach the ground.

Lesson 4.9

4.9 Guided Practice (pp. 301–303)

1. $y > x^2 + 2x - 8$



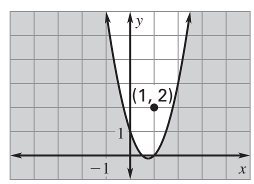
Test (0, 0):

$y > x^2 + 2x - 8$

$0 \overset{?}{>} 0^2 + 2(0) - 8$

$0 > -8$ ✓

2. $y \le 2x^2 - 3x + 1$



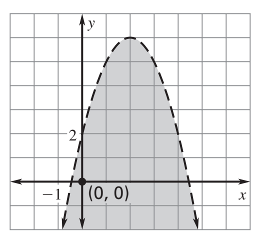
Test (1, 2):

$y \le 2x^2 - 3x + 1$

$2 \overset{?}{\le} 2(1)^2 - 3(1) + 1$

$2 \not\le 0$ ✗

3. $y < -x^2 + 4x + 2$



Test (0, 0):

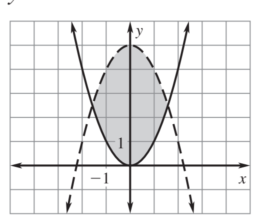
$y < -x^2 + 4x + 2$

$0 \overset{?}{<} -(0)^2 + 4(0) + 2$

$0 < 2$ ✓

4. $y \ge x^2$

$y < -x^2 + 5$



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5. $2x^2 + 2x \le 3$

$2x^2 + 2x - 3 \le 0$

$2x^2 + 2x - 3 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-3)}}{2(2)}$$

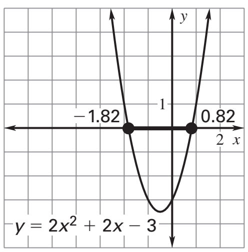
$$= \frac{-1 \pm \sqrt{7}}{2}$$

x	-3	-2	$\frac{-1-\sqrt{7}}{2}$	-1
$2x^2 + 2x - 3$	9	1	0	-3

x	0	$\frac{-1+\sqrt{7}}{2}$	1	2
$2x^2 + 2x - 3$	-3	0	1	9

The solution of the inequality is $\frac{-1-\sqrt{7}}{2} \le x \le \frac{-1+\sqrt{7}}{2}$.

$x \approx 0.82$ or $x \approx -1.82$



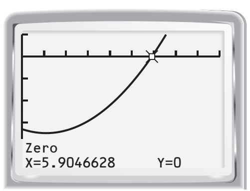
The solution of the inequality is approximately $-1.82 \le x \le 0.82$.

6. $T(x) = 7.51x^2 - 16.4x + 35.0, 0 \le x \le 9$

$T(x) \ge 200$

$7.51x^2 - 16.4x + 35.0 \ge 200$

$7.51x^2 - 16.4x - 165 \ge 0$



The graph's x-intercept is about 5.9. The graph lies on or above the x-axis when $5.9 \le x \le 9$. There were at least 200 teams participating in the years 1998–2001.

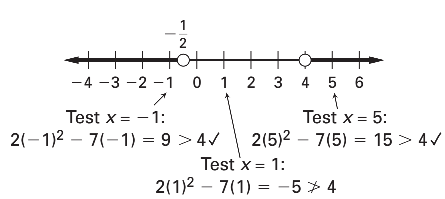
7. $2x^2 - 7x > 4$

$2x^2 - 7x = 4$

$2x^2 - 7x - 4 = 0$

$(2x + 1)(x - 4) = 0$

$$x = -\frac{1}{2} \text{ or } x = 4$$



The solution is $x < -\frac{1}{2}$ or $x > 4$.

4.9 Exercises (pp. 304–307)

Skill Practice

1. *Sample answer:*

Quadratic inequality in one variable:

$2x^2 + 7x - 1 > 0$

Quadratic inequality in two variables:

$y < 3x^2 + x - 4$

2. To solve $x^2 + 6x - 8 < 0$ using a table, make a table of values and notice which x-values satisfy the inequality. The table must include x-values for which the expression equals zero. To solve by graphing, find the x-intercepts, sketch the parabola, and find the x-values for which the graph lies below the x-axis. To solve algebraically, replace $<$ with $=$, solve the equation, plot the solutions on a number line, and test an x-value in each interval.

3. C; $y \le x^2 + 4x + 3$

Because the inequality symbol is $\le$, the parabola is solid.

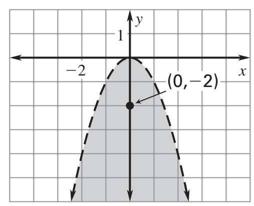
4. A; $y > -x^2 + 4x - 3$

Because $a < 0$, the parabola opens down.

5. B; $y < x^2 - 4x + 3$

Because $a > 0$, the parabola opens up.

6. $y < -x^2$



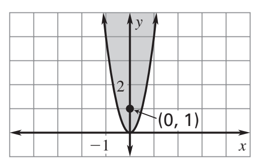
Test $(0, -2)$:

$y < -x^2$

$-2 < -(0)^2$

$-2 < 0$ ✓

7. $y \ge 4x^2$



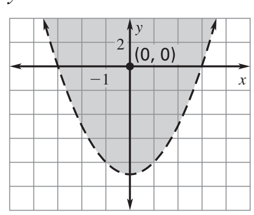
Test $(0, 1)$:

$y \ge 4x^2$

$1 \ge 4(0)^2$

$1 \ge 0$ ✓

8. $y > x^2 - 9$



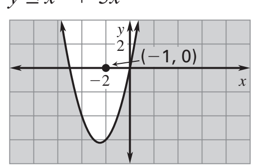
Test $(0, 0)$:

$y > x^2 - 9$

$0 > 0^2 - 9$

$0 > -9$ ✓

9. $y \le x^2 + 5x$



Test $(-1, 0)$:

$y \le x^2 + 5x$

$0 \le (-1)^2 + 5(-1)$

$0 \not\le -4$ ✗

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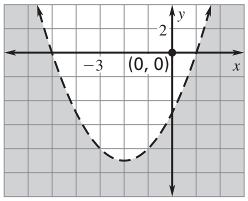
10. $y < x^2 + 4x - 5$



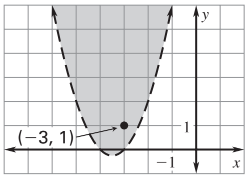
Test (0, 0):

$y < x^2 + 4x - 5$

$0 < 0^2 + 4(0) - 5$

$0 \not< -5$ ✗

11. $y > x^2 + 7x + 12$



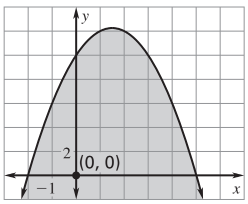
Test (−3, 1):

$y > x^2 + 7x + 12$

$1 > (-3)^2 + 7(-3) + 12$

$1 > 0$ ✓

12. $y \le -x^2 + 3x + 10$



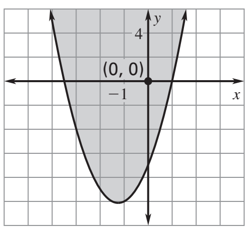
Test (0, 0):

$y \le -x^2 + 3x + 10$

$0 \le -(0)^2 + 3(0) + 10$

$0 \le 10$ ✓

13. $y \ge 2x^2 + 5x - 7$



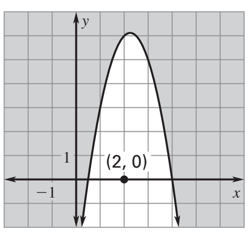
Test (0, 0):

$y \ge 2x^2 + 5x - 7$

$0 \ge 2(0)^2 + 5(0) - 7$

$0 \ge -7$ ✓

14. $y \ge -2x^2 + 9x - 4$



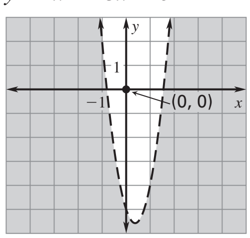
Test (2, 0):

$y \ge -2x^2 + 9x - 4$

$0 \ge -2(2)^2 + 9(2) - 4$

$0 \not\ge 6$ ✗

15. $y < 4x^2 - 3x - 5$



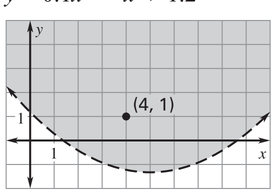
Test (0, 0):

$y < 4x^2 - 3x - 5$

$0 < 4(0)^2 - 3(0) - 5$

$0 \not< -5$ ✗

16. $y > 0.1x^2 - x + 1.2$



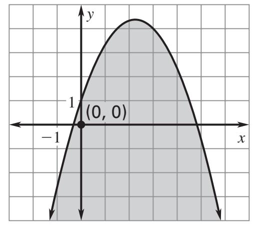
Test (4, 1):

$y > 0.1x^2 - x + 1.2$

$1 > 0.1(4)^2 - 4 + 1.2$

$1 > -1.2$ ✓

17. $y \le -\frac{2}{3}x^2 + 3x + 1$



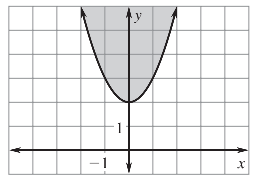
Test (0, 0):

$y \le -\frac{2}{3}x^2 + 3x + 1$

$0 \le -\frac{2}{3}(0)^2 + 3(0) + 1$

$0 \le 1$ ✓

18. Because the inequality symbol is $\ge$, the parabola should be solid.



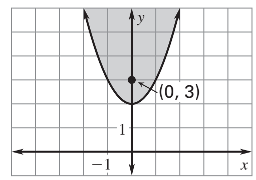
19. The wrong portion of the graph was shaded.

Test (0, 3):

$y \ge x^2 + 2$

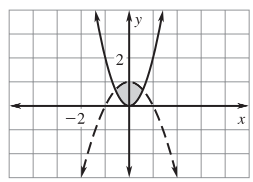
$3 \ge 0^2 + 2$

$3 \ge 2$ ✓



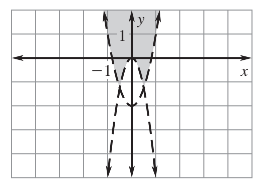
20. $y \ge 2x^2$

$y < -x^2 + 1$



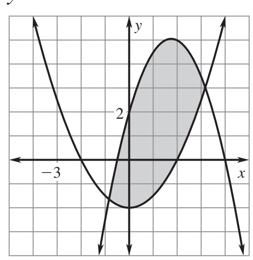
21. $y > -5x^2$

$y > 3x^2 - 2$



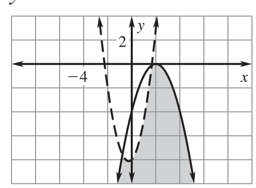
22. $y \ge x^2 - 4$

$y \le -2x^2 + 7x + 4$



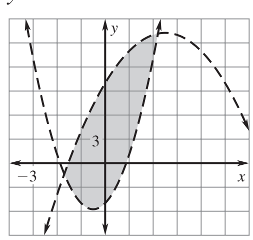
23. $y \le -x^2 + 4x - 4$

$y < 2x^2 + x - 8$



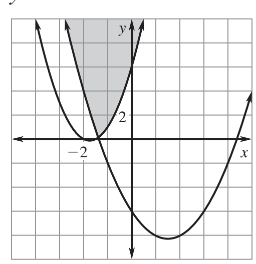
24. $y > 3x^2 + 3x - 5$

$y < -x^2 + 5x + 10$



25. $y \ge x^2 - 3x - 6$

$y \ge 2x^2 + 7x + 6$



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26. $x^2 - 5x < 0$

x	-2	-1	0	1	2
$x^2 - 5x$	14	6	0	-4	-6

x	3	4	5	6	7
$x^2 - 5x$	-6	-4	0	6	14

The solution of the inequality is $0 < x < 5$.

27. $x^2 + 2x - 3 > 0$

x	-5	-4	-3	-2	-1
$x^2 + 2x - 3$	12	5	0	-3	-4

x	0	1	2	3
$x^2 + 2x - 3$	-3	0	5	12

The solution of the inequality is $x < -3$ or $x > 1$.

28. $x^2 + 3x \le 10$

$x^2 + 3x - 10 \le 0$

x	-7	-6	-5	-4	-3	-2
$x^2 + 3x - 10$	18	8	0	-6	-10	-12

x	-1	0	1	2	3	4
$x^2 + 3x - 10$	-12	-10	-6	0	8	18

The solution of the inequality is $-5 \le x \le 2$.

29. $x^2 - 2x \ge 8$

$x^2 - 2x - 8 \ge 0$

x	-4	-3	-2	-1	0	1
$x^2 + 2x - 8$	16	7	0	-5	-8	-9

x	2	3	4	5	6
$x^2 + 2x - 8$	-8	-5	0	7	16

The solution of the inequality is $x \le -2$ or $x \ge 4$.

30. $-x^2 + 15x - 50 > 0$

$x^2 - 15x + 50 < 0$

x	3	4	5	6	7	8
$x^2 - 15x + 50$	14	6	0	-4	-6	-6

x	9	10	11	12
$x^2 - 15x + 50$	-4	0	6	14

The solution of the inequality is $5 < x < 10$.

31. $x^2 - 10x < -16$

$x^2 - 10x + 16 < 0$

x	0	1	2	3	4	5
$x^2 - 10x + 16$	16	7	0	-5	-8	-9

x	6	7	8	9	10
$x^2 - 10x + 16$	-8	-5	0	7	16

The solution of the inequality is $2 < x < 8$.

32. $x^2 - 4x > 12$

$x^2 - 4x - 12 > 0$

x	-4	-3	-2	-1	0	1
$x^2 - 4x - 12$	20	9	0	-7	-12	-15

x	2	3	4	5	6	7	8
$x^2 - 4x - 12$	-16	-15	-12	-7	0	9	20

The solution of the inequality is $x < -2$ or $x > 6$.

33. $3x^2 - 6x - 2 \le 7$

$3x^2 - 6x - 9 \le 0$

$x^2 - 2x - 3 \le 0$

x	-3	-2	-1	0	1	2
$x^2 - 2x - 3$	12	5	0	-3	-4	-3

x	3	4	5
$x^2 - 2x - 3$	0	5	12

The solution of the inequality is $-1 \le x \le 3$.

34. $2x^2 - 6x - 9 \ge 11$

$2x^2 - 6x - 20 \ge 0$

$x^2 - 3x - 10 \ge 0$

x	-4	-3	-2	-1	0	1
$x^2 - 3x - 10$	18	8	0	-6	-10	-12

x	2	3	4	5	6	7
$x^2 - 3x - 10$	-12	-10	-6	0	8	18

The solution of the inequality is $x \le -2$ or $x \ge 5$.

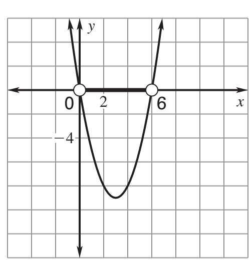
35. $x^2 - 6x < 0$

$x^2 - 6x = 0$

$x(x - 6) = 0$

$x = 0$ or $x = 6$

The solution of the inequality is $0 < x < 6$.



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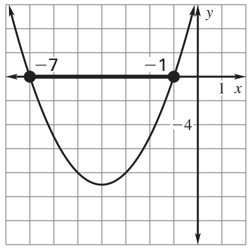
36. $x^2 + 8x \le -7$

$x^2 + 8x + 7 \le 0$

$x^2 + 8x + 7 = 0$

$(x + 7)(x + 1) = 0$

$x = -7$ or $x = -1$



The solution of the inequality is $-7 \le x \le -1$.

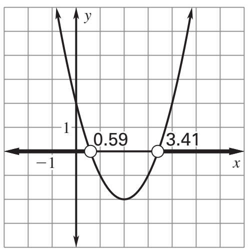
37. $x^2 - 4x + 2 > 0$

$x^2 - 4x + 2 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$x \approx 3.41$ or $x \approx 0.59$



The solution of the inequality is approximately $x < 0.59$ or $x > 3.41$.

38. $x^2 + 6x + 3 > 0$

$x^2 + 6x + 3 = 0$

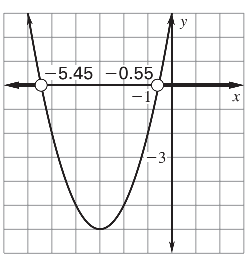
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{24}}{2}$$

$$= \frac{-6 \pm 2\sqrt{6}}{2}$$

$$= -3 \pm \sqrt{6}$$

$x \approx -0.55$ or $x \approx -5.45$



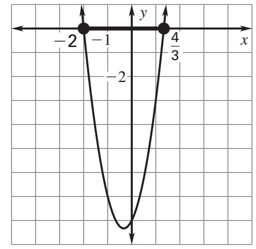
The solution of the inequality is approximately $x < -5.45$ or $x > -0.55$.

39. $3x^2 + 2x - 8 \le 0$

$3x^2 + 2x - 8 = 0$

$(3x - 4)(x + 2) = 0$

$x = \frac{4}{3}$ or $x = -2$



The solution of the inequality is $-2 \le x \le \frac{4}{3}$.

40. $3x^2 + 5x - 3 < 1$

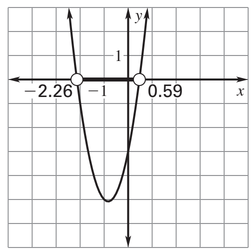
$3x^2 + 5x - 4 < 0$

$3x^2 + 5x - 4 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-4)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{73}}{6}$$

$x \approx 0.59$ or $x \approx -2.26$



The solution of the inequality is approximately $-2.26 < x < 0.59$.

41. $-6x^2 + 19x \ge 10$

$-6x^2 + 19x - 10 \ge 0$

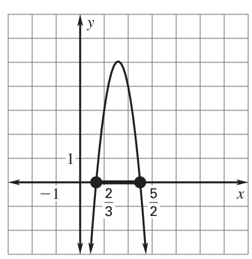
$-6x^2 + 19x - 10 = 0$

$$x = \frac{-19 \pm \sqrt{19^2 - 4(-6)(-10)}}{2(-6)}$$

$$= \frac{-19 \pm \sqrt{121}}{-12}$$

$$= \frac{-19 \pm 11}{-12}$$

$x = \frac{2}{3}$ or $x = \frac{5}{2}$



The solution of the inequality is $\frac{2}{3} \le x \le \frac{5}{2}$.

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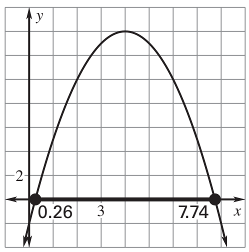
42.

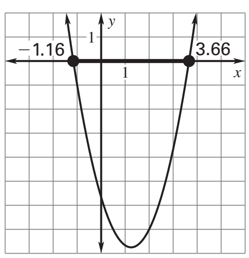
$$-\frac{1}{2}x^2 + 4x \geq 1$$
$$-x^2 + 8x \geq 2$$
$$-x^2 + 8x - 2 \geq 0$$
$$-x^2 + 8x - 2 = 0$$
$$x = \frac{-8 \pm \sqrt{8^2 - 4(-1)(-2)}}{2(-1)}$$
$$= \frac{-8 \pm \sqrt{56}}{-2}$$
$$= \frac{-8 \pm 2\sqrt{14}}{-2}$$
$$= 4 \pm \sqrt{14}$$
$$x \approx 7.74 \text{ or } x \approx 0.26$$



The solution of the inequality is approximately $0.26 \leq x \leq 7.74$.

43.

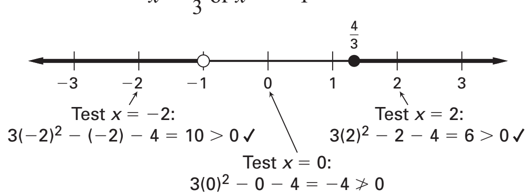
$$4x^2 - 10x - 7 < 10$$
$$4x^2 - 10x - 17 < 0$$
$$4x^2 - 10x - 17 = 0$$
$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(-17)}}{2(4)}$$
$$= \frac{10 \pm \sqrt{372}}{8} = \frac{10 \pm 2\sqrt{93}}{8} = \frac{5 \pm \sqrt{93}}{4}$$
$$x \approx 3.66 \text{ or } x \approx -1.16$$



The solution of the inequality is approximately $-1.16 < x < 3.66$

44. A;

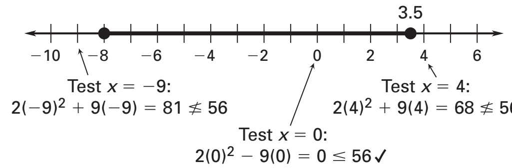
$$3x^2 - x - 4 > 0$$
$$3x^2 - x - 4 = 0$$
$$(3x - 4)(x + 1) = 0$$
$$x = \frac{4}{3} \text{ or } x = -1$$



The solution is $x < -1$ or $x > \frac{4}{3}$.

45. B;

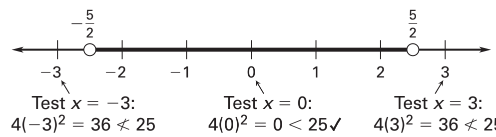
$$2x^2 + 9x \leq 56 \rightarrow 2x^2 + 9x - 56 \leq 0$$
$$2x^2 + 9x - 56 = 0$$
$$(2x - 7)(x + 8) = 0$$
$$x = 3.5 \text{ or } x = -8$$



The solution is $-8 \leq x \leq 3.5$.

46.

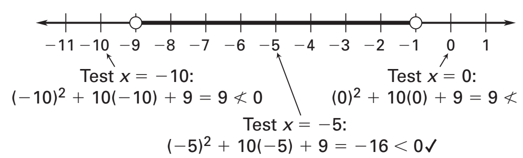
$$4x^2 < 25$$
$$4x^2 = 25$$
$$4x^2 - 25 = 0$$
$$(2x + 5)(2x - 5) = 0$$
$$x = -\frac{5}{2} \text{ or } x = \frac{5}{2}$$



The solution is $-\frac{5}{2} < x < \frac{5}{2}$.

47.

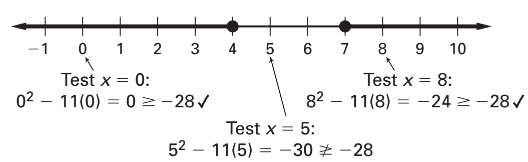
$$x^2 + 10x + 9 < 0$$
$$x^2 + 10x + 9 = 0$$
$$(x + 9)(x + 1) = 0$$
$$x = -9 \text{ or } x = -1$$



The solution is $-9 < x < -1$.

48.

$$x^2 - 11x \geq -28$$
$$x^2 - 11x = -28$$
$$x^2 - 11x + 28 = 0$$
$$(x - 7)(x - 4) = 0$$
$$x = 7 \text{ or } x = 4$$



The solution is $x \leq 4$ or $x \geq 7$.

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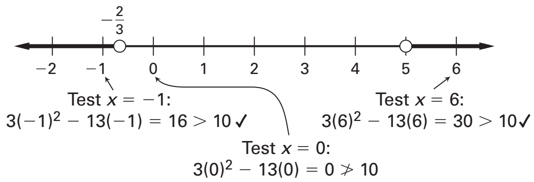
49. $3x^2 - 13x > 10$

$3x^2 - 13x = 10$

$3x^2 - 13x - 10 = 0$

$(3x + 2)(x - 5) = 0$

$x = -\frac{2}{3}$ or $x = 5$



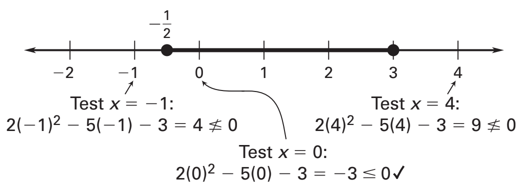
The solution is $x < -\frac{2}{3}$ or $x > 5$.

50. $2x^2 - 5x - 3 \le 0$

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}$ or $x = 3$



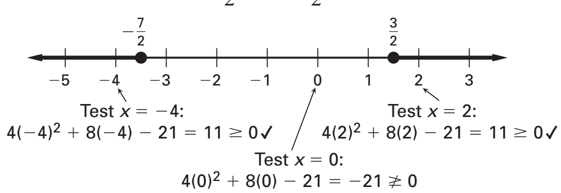
The solution is $-\frac{1}{2} \le x \le 3$.

51. $4x^2 + 8x - 21 \ge 0$

$4x^2 + 8x - 21 = 0$

$(2x + 7)(2x - 3) = 0$

$x = -\frac{7}{2}$ or $x = \frac{3}{2}$



The solution is $x \le -\frac{7}{2}$ or $x \ge \frac{3}{2}$.

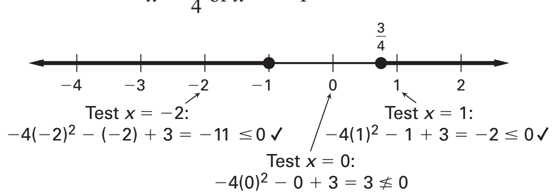
52. $-4x^2 - x + 3 \le 0$

$-4x^2 - x + 3 = 0$

$4x^2 + x - 3 = 0$

$(4x - 3)(x + 1) = 0$

$x = \frac{3}{4}$ or $x = -1$



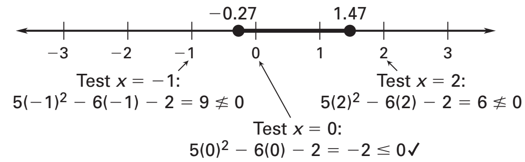
The solution is $x \le -1$ or $x \ge \frac{3}{4}$.

53. $5x^2 - 6x - 2 \le 0$

$5x^2 - 6x - 2 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)} = \frac{6 \pm \sqrt{76}}{10} = \frac{3 \pm \sqrt{19}}{5}$$

$x \approx 1.47$ or $x \approx -0.27$



The solution is approximately $-0.27 \le x \le 1.47$.

54. $-3x^2 + 10x > -2$

$-3x^2 + 10x = -2$

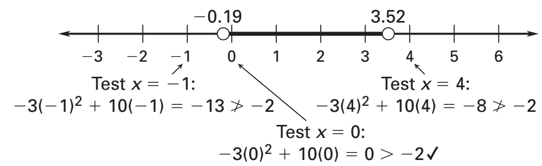
$-3x^2 + 10x + 2 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(-3)(2)}}{2(-3)}$$

$$= \frac{-10 \pm \sqrt{124}}{-6}$$

$$= \frac{5 \pm \sqrt{31}}{3}$$

$x \approx 3.52$ or $x \approx -0.19$



The solution is approximately $-0.19 < x < 3.52$.

55. $-2x^2 - 7x \ge 4$

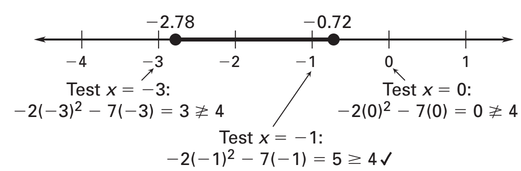
$-2x^2 - 7x = 4$

$-2x^2 - 7x - 4 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-2)(-4)}}{2(-2)}$$

$$= \frac{7 \pm \sqrt{17}}{-4}$$

$x \approx -2.78$ or $x \approx -0.72$



The solution is approximately $-2.78 \le x \le -0.72$.

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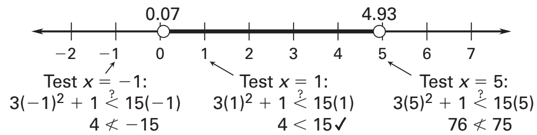
56.

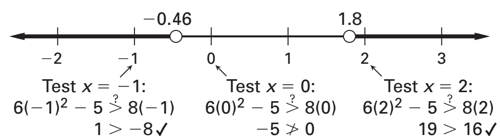
$$3x^2 + 1 < 15x$$
$$3x^2 + 1 = 15x$$
$$3x^2 - 15x + 1 = 0$$
$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(3)(1)}}{2(3)}$$
$$= \frac{15 \pm \sqrt{213}}{6}$$
$$x \approx 4.93 \text{ or } x \approx 0.07$$



The solution is approximately $0.07 < x < 4.93$.

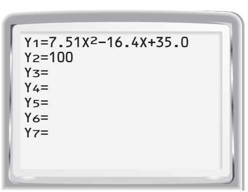
57.

$$6x^2 - 5 > 8x$$
$$6x^2 - 5 = 8x$$
$$6x^2 - 8x - 5 = 0$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(6)(-5)}}{2(6)}$$
$$= \frac{8 \pm \sqrt{184}}{12}$$
$$= \frac{4 \pm \sqrt{46}}{6}$$
$$x \approx 1.8 \text{ or } x \approx -0.46$$

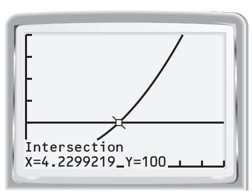


The solution is approximately $x < -0.46$ or $x > 1.8$.

58. a.



b–c.



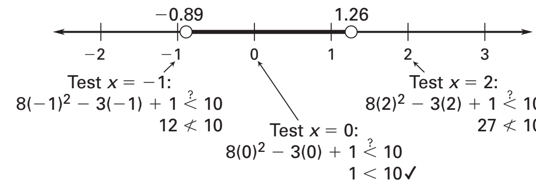
(4.2, 100)

d. There were more than 100 teams participating in the years 1997–2001.

The graph of $y = 7.51x^2 - 16.4x + 35.0$ lies above the graph of $y = 100$ when $4.2 < x \le 9$.

59.

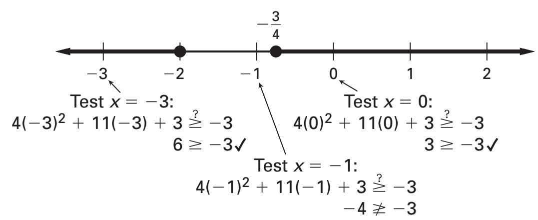
$$8x^2 - 3x + 1 < 10$$
$$8x^2 - 3x - 9 < 0$$
$$8x^2 - 3x - 9 = 0$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(8)(-9)}}{2(8)}$$
$$= \frac{3 \pm \sqrt{297}}{16}$$
$$= \frac{3 \pm 3\sqrt{33}}{16}$$
$$x \approx 1.26 \text{ or } x \approx -0.89$$



The solution is approximately $-0.89 < x < 1.26$.

60.

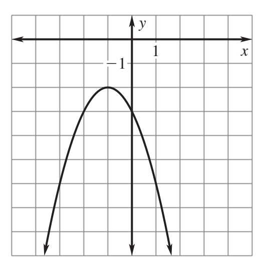
$$4x^2 + 11x + 3 \ge -3$$
$$4x^2 + 11x + 6 \ge 0$$
$$4x^2 + 11x + 6 = 0$$
$$(4x + 3)(x + 2) = 0$$
$$x = -\frac{3}{4} \text{ or } x = -2$$



The solution is $x \le -2$ or $x \ge -\frac{3}{4}$.

61.

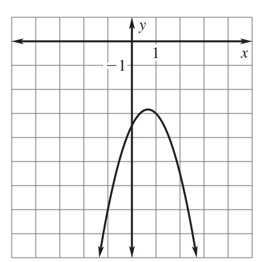
$$-x^2 - 2x - 1 > 2$$
$$-x^2 - 2x - 3 > 0$$
$$-x^2 - 2x - 3 = 0$$



There is no value of x for which $y > 0$, so there is no real solution to the inequality.

62.

$$-3x^2 + 4x - 5 \le 2$$
$$-3x^2 + 4x - 7 \le 0$$
$$-3x^2 + 4x - 7 = 0$$



Every value of x satisfies the inequality $y \le 0$, so the solution to the inequality is all real numbers.

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63. $x^2 - 7x + 4 > 5x - 2$

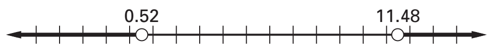
$x^2 - 12x + 6 > 0$

$x^2 - 12x + 6 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{12 \pm \sqrt{120}}{2} = \frac{12 \pm 2\sqrt{30}}{2} = 6 \pm \sqrt{30}$$

$x \approx 11.48$ or $x \approx 0.52$



The solution is approximately $x < 0.52$ or $x > 11.48$.

64. $2x^2 + 9x - 1 \geq -3x + 1$

$2x^2 + 12x - 2 \geq 0$

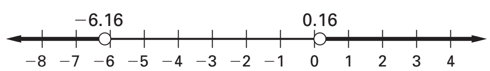
$2x^2 + 12x - 2 = 0$

$x^2 + 6x - 1 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{40}}{2} = \frac{-6 \pm 2\sqrt{10}}{2} = -3 \pm \sqrt{10}$$

$x \approx 0.16$ or $x \approx -6.16$



The solution is approximately $x \leq -6.16$ or $x \geq 0.16$.

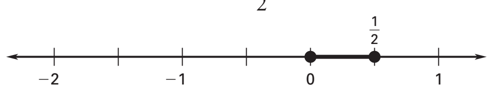
65. $3x^2 - 2x + 1 \leq -x^2 + 1$

$4x^2 - 2x \leq 0$

$4x^2 - 2x = 0$

$2x(2x - 1) = 0$

$$x = 0 \text{ or } x = \frac{1}{2}$$



The solution is approximately $0 \leq x \leq \frac{1}{2}$.

66. $5x^2 + x - 7 < 3x^2 - 4x$

$2x^2 + 5x - 7 < 0$

$2x^2 + 5x - 7 = 0$

$(2x + 7)(x - 1) = 0$

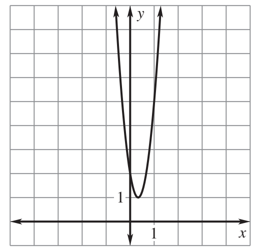
$$x = 1 \text{ or } x = -\frac{7}{2}$$

The solution is $-\frac{7}{2} < x < 1$.

67. $6x^2 - 5x + 2 < -3x^2 + x$

$9x^2 - 6x + 2 < 0$

$9x^2 - 6x + 2 = 0$



There is no value of x for which $y < 0$, so there is no real solution to the inequality.

68. *Sample answer:* $x^2 - 3x > 10$

69. $A = \frac{2}{3}bh$

a. $y \leq -x^2 + 4x,\ y \geq 0$

$y = -x^2 + 4x$

x-intercepts: $0 = -x^2 + 4x$

$0 = -x(x - 4)$

$x = 0$ or $x = 4$

Therefore, $b = 4 - 0 = 4$.

$$x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$

$y = -(2)^2 + 4(2) = 4$

Therefore, $h = 4$.

$A = \frac{2}{3}(4)(4) \approx 10.67$ square units

b. $y \geq x^2 - 4x - 5,\ y \leq 3$

Find the x-values for which $y = 3$.

$x^2 - 4x - 5 = 3$

$x^2 - 4x - 8 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{48}}{2} = 2 \pm 2\sqrt{3}$$

$x \approx 5.46$ or $x \approx -1.46$

Therefore, $b = 5.46 - (-1.46) = 6.92$.

$$x = -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2$$

$y = 2^2 - 4(2) - 8 = -12$

Therefore, $h = 12$.

$A = \frac{2}{3}(6.92)(12) = 55.36$ square units

Problem Solving

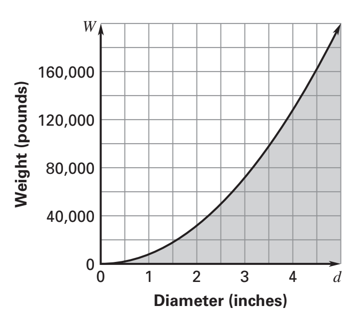
70. $W \leq 8000d^2$

Test (0, 4):

$W \leq 8000d^2$

$4 \stackrel{?}{\leq} 8000(0)^2$

$4 \nleq 0$ ✗



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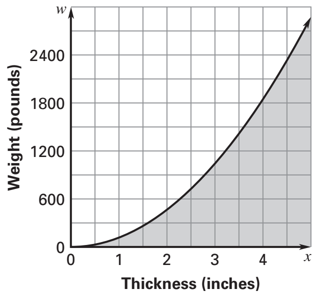
71. $W \le 115x^2$

Test (0, 2):

$W \le 115x^2$

$2 \overset{?}{\le} 115(0)^2$

$2 \not\le 0$ ✗

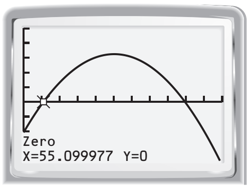


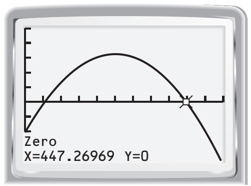
72. $y = -0.00211x^2 + 1.06x$

$$y > 52$$

$$-0.00211x^2 + 1.06x > 52$$

$$-0.00211x^2 + 1.06x - 52 > 0$$





The graph lies above the x-axis when $55.1 < x < 447.27$.

The arch is above the road between 55.1 meters and 447.27 meters.

73. $L(x) = 0.00170x^2 + 0.145x + 2.35,\ 0 \le x \le 40$

$$L(x) > 10$$

$$0.00170x^2 + 0.145x + 2.35 > 10$$

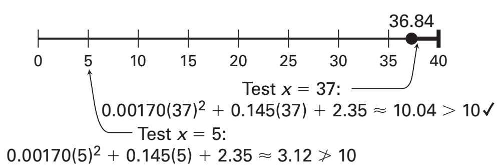
$$0.00170x^2 + 0.145x - 7.65 > 0$$

$$x = \frac{-0.145 \pm \sqrt{(0.145)^2 - 4(0.00170)(-7.65)}}{2(0.00170)}$$

$$= \frac{-0.145 \pm \sqrt{0.073045}}{0.0034}$$

$x \approx 36.84$ or $x \approx -122.14$

Reject the negative solution, 122.14.



The larvae's length tends to be greater than 10 millimeters between 37 and 40 days old. The domain restricts our solutions. Because the given domain is $0 \le x \le 40$, the solution cannot include ages beyond 40 days.

74. $A(x) = 0.0051x^2 - 0.319x + 15,\ 16 \le x \le 70$

$V(x) = 0.005x^2 - 0.23x + 22,\ 16 \le x \le 70$

a. $0.0051x^2 - 0.319x + 15 < 0.005x^2 - 0.23x + 22$

$0.0001x^2 - 0.089x - 7 < 0$

b.

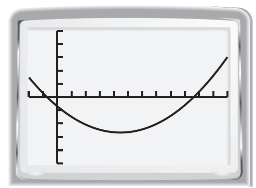
x	16	22	28	34
$0.0001x^2 - 0.089x - 7$	-8.4	-8.91	-9.41	-9.91

x	40	46	52
$0.0001x^2 - 0.089x - 7$	-10.4	-10.88	-11.36

x	58	64	70
$0.0001x^2 - 0.089x - 7$	-11.83	-12.29	-12.74

The solution of the inequality on the given domain is $16 \le x \le 70$.

c. The solution of the inequality is $-72.71 < x < 962.71$. This, however, is not a reasonable solution because it contains negative values and x-values that are too large. The driver's age cannot be represented by a negative number or a number as large as the graph indicates. Therefore, the domain restriction provides a reasonable solution.



d. Because a driver's reaction time to audio stimuli is less than his or her reaction time to visual stimuli, the driver would likely react more quickly to the siren of an approaching ambulance.

75. $y = -0.0540x^2 + 1.43x$

a. $-0.0540x^2 + 1.43x < 8$

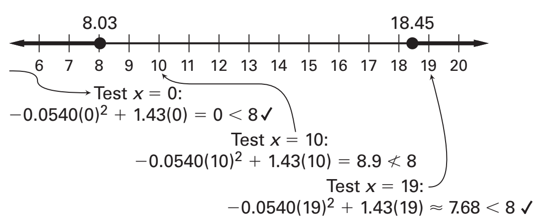
$-0.0540x^2 + 1.43x - 8 < 0$

$-0.0540x^2 + 1.43x - 8 = 0$

$$x = \frac{-1.43 \pm \sqrt{1.43^2 - 4(-0.0540)(-8)}}{2(-0.0540)}$$

$$= \frac{-1.43 \pm \sqrt{0.3169}}{-0.108}$$

$x \approx 8.03$ or $x \approx 18.45$



The ball is low enough to go into the goal if it is kicked from a distance less than 8.03 feet or more than 18.45 feet.

b. No, the player will not score a goal because the ball will be too high and will go over the goal.

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76. $y = -0.0625x^2 + 1.25x + 5.75$

a. $y = -0.0625x^2 + 1.25x + 5.75$

$x = -\frac{b}{2a} = -\frac{1.25}{2(-0.0625)} = 10$

$y = -0.0625(10)^2 + 1.245(10) + 5.75 = 12$

Vertex: (10, 12)

Assuming that the truck travels exactly through the middle of the arch, the top corners of the truck will be located at $x = 10 - \frac{7}{2} = 6.5$ and $x = 10 + \frac{7}{2} = 13.5$.

$x = 6.5$: $-0.0625(6.5)^2 + 1.25(6.5) + 5.75 \approx 11.23$

$x = 13.5$: $-0.0625(13.5)^2 + 1.25(13.5) + 5.75 \approx 11.23$

The truck will fit under the arch with about 0.23 foot, or 2.76 inches, of clearance on each side.

b. $-0.0625x^2 + 1.25x + 5.75 = 11$

$-0.0625x^2 + 1.25x - 5.25 = 0$

$x = \frac{-1.25 \pm \sqrt{1.25^2 - 4(-0.0625)(-5.25)}}{2(-0.0625)}$

$x = \frac{-1.25 \pm 0.5}{-0.125}$

$x = 6$ or $x = 14$

The maximum width that a truck 11 feet tall can have and still make it under the arch is $14 - 6 = 8$ feet.

c. The maximum height that a truck 7 feet wide can have and still make it under the arch is 11.23 feet, as shown in part (a).

77. $W(x) = 0.1x^2 - 0.5x - 5$

a. $0.1x^2 - 0.5x - 5 \geq 20$

$0.1x^2 - 0.5x - 25 \geq 0$

$0.1x^2 - 0.5x - 25 = 0$

$x = \frac{-(-0.5) \pm \sqrt{(-0.5)^2 - 4(0.1)(-25)}}{2(0.1)}$

$x = \frac{0.5 \pm \sqrt{10.25}}{0.2}$

$x \approx 18.51$ or $x \approx -13.51$

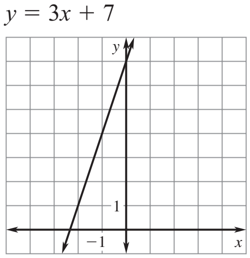
Reject the negative solution.

Ice that has a thickness of 18.51 inches or more can support a weight of 20 tons.

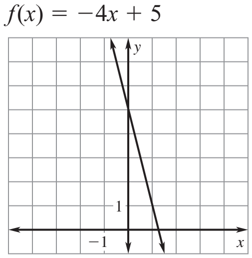
b. Because you cannot have a negative weight, look at the graph where the x-values correspond to a positive weight. Also, because you cannot have a negative thickness, look at the graph where the x-values are positive. Then you can determine the minimum x-value in the domain.

Mixed Review

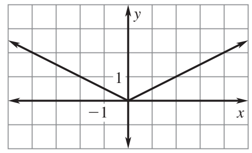
78. $y = 3x + 7$



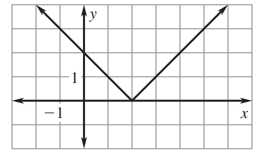
79. $f(x) = -4x + 5$



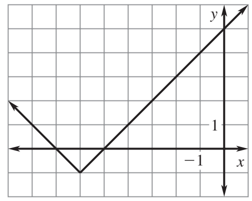
80. $y = \frac{1}{2}|x|$



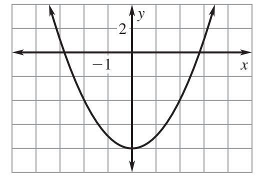
81. $y = |x - 2|$



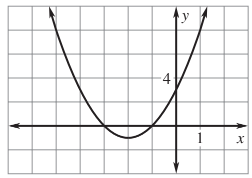
82. $y = |x + 6| - 1$



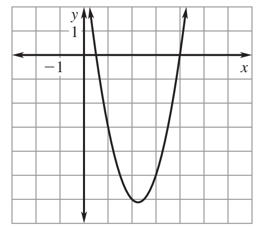
83. $g(x) = x^2 - 8$



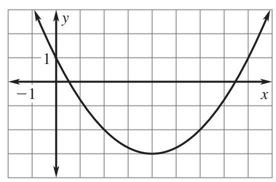
84. $f(x) = x^2 + 4x + 3$



85. $y = 2x^2 - 9x + 4$



86. $y = \frac{1}{4}x^2 - 2x + 1$



87. $x + y + z = -2$

$4x + 2y + z = 3$

$z = -3$

When $z = -3$:

$x + y + (-3) = -2 \rightarrow x + y = 1$

$4x + 2y + (-3) = 3 \rightarrow 4x + 2y = 6$

$$\begin{aligned} x + y &= 1 \xrightarrow{\times(-2)} & -2x - 2y &= -2 \\ 4x + 2y &= 6 \longrightarrow & 4x + 2y &= 6 \\ & & 2x &= 4 \\ & & x &= 2 \end{aligned}$$

When $x = 2$ and $z = -3$:

$2 + y + (-3) = -2$

$y = -1$

The solution is $x = 2$, $y = -1$, and $z = -3$, or the ordered triple $(2, -1, -3)$.

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88. $x + y + z = 3$

$2x + 3y - z = -8$

$z = 4$

When $z = 4$:

$x + y + 4 = 3 \rightarrow x + y = -1$

$2x + 3y - 4 = -8 \rightarrow 2x + 3y = -4$

$$\begin{array}{l} x + y = -1 \xrightarrow{\times(-3)} \\ 2x + 3y = -4 \longrightarrow \end{array} \begin{array}{r} -3x - 3y = 3 \\ \underline{2x + 3y = -4} \\ -x \quad = -1 \\ x = 1 \end{array}$$

When $x = -1$ and $z = 4$:

$1 + y + 4 = 3$

$y = -2$

The solution is $x = 1$, $y = -2$, and $z = 4$, or the ordered triple $(1, -2, 4)$.

89. $4x + 2y + z = -6$

$x + y + z = -3$

$16x + 4y + z = 0$

$$\begin{array}{r} 4x + 2y + z = -6 \\ \underline{-x - y - z = 3} \\ 3x + y \quad = -3 \end{array}$$

$$\begin{array}{r} -x - y - z = 3 \\ \underline{16x + 4y + z = 0} \\ 15x + 3y \quad = 3 \end{array}$$

$$\begin{array}{l} 3x + y = -3 \xrightarrow{\times(-3)} \\ 15x + 3y = 3 \longrightarrow \end{array} \begin{array}{r} -9x - 3y = 9 \\ \underline{15x + 3y = 3} \\ 6x \quad = 12 \\ x = 2 \end{array}$$

$3x + y = -3$

$3(2) + y = -3$

$6 + y = -3$

$y = -9$

$x + y + z = -3$

$2 + (-9) + 2 = -3$

$-7 + 2 = -3$

$z = 4$

The solution is $x = 2$, $y = -9$, and $z = 4$, or the ordered triple $(2, -9, 4)$.

90. $x + y + z = 8$

$9x - 3y + z = 0$

$4x - 2y + z = -1$

$$\begin{array}{r} -x - y - z = -8 \\ \underline{9x - 3y + z = 0} \\ 8x - 4y \quad = -8 \end{array}$$

$$\begin{array}{r} -9x + 3y - z = 0 \\ \underline{4x - 2y + z = -1} \\ -5x + y \quad = -1 \end{array}$$

$$\begin{array}{l} 8x - 4y = -8 \longrightarrow \\ -5x + y = -1 \xrightarrow{\times 4} \end{array} \begin{array}{r} 8x - 4y = -8 \\ \underline{-20x + 4y = -4} \\ -12x \quad = -12 \\ x = 1 \end{array}$$

$-5x + y = -1$

$-5(1) + y = -1$

$-5 + y = -1$

$y = 4$

$x + y + z = 8$

$1 + 4 + z = 8$

$5 + z = 8$

$z = 3$

The solution is $x = 1$, $y = 4$, and $z = 3$, or the ordered triple $(1, 4, 3)$.

91. $x + y + z = 5$

$2x - 3y + 3z = 9$

$-x + 7y - z = 11$

$$\begin{array}{r} x + y + z = 5 \\ \underline{-x + 7y - z = 11} \\ 8y \quad = 16 \\ y = 2 \end{array}$$

$2x - 3(2) + 3z = 9$

$-x + 7(2) - z = 11$

$$\begin{array}{l} \longrightarrow 2x + 3z = 15 \longrightarrow \\ \longrightarrow -x - z = -3 \xrightarrow{\times 2} \end{array} \begin{array}{r} 2x + 3z = 15 \\ \underline{-2x - 2z = -6} \\ z = 9 \end{array}$$

$x + y + z = 5$

$x + 2 + 9 = 5$

$x + 11 = 5$

$x = -6$

The solution is $x = -6$, $y = 2$, and $z = 9$, or the ordered triple $(-6, 2, 9)$.

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92. $x + y + z = 1$

$x - y + z = 1$

$3x + y + 3z = 3$

$$\begin{array}{r} x + y + z = 1 \\ \underline{x - y + z = 1} \\ 2x \quad + 2z = 2 \end{array}$$

$$\begin{array}{r} x - y + z = 1 \\ \underline{3x + y + 3z = 3} \\ 4x + \quad 4z = 4 \end{array}$$

$2x + 2z = 2 \xrightarrow{\times 2} -4x - 4z = -4$

$4x + 4z = 4 \longrightarrow \underline{4x + 4z = 4}$

$0 = 0$

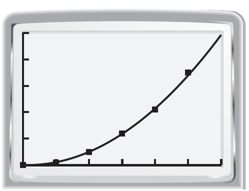
The solution has infinitely many solutions.

Lesson 4.10

Investigating Algebra Activity 4.10 (p. 308)

Diameter of circle (in.), x	Number of pennies, y
0	0
1	1
2	5
3	12
4	21
5	34

$y = 1.43x^2 - 0.37x$

1. 

The model appears to fit the data very well.

2. $y = 1.43x^2 - 0.37x$

$y = 1.43(6)^2 \bullet 0.37(6) \approx 49$

3. Because the area of a circle is given by $A = \pi r^2$, or $A = \frac{\pi d^2}{4}$, you would expect the number of pennies that fit inside a circle to be a quadratic function of the circle's diameter.

4. Area of circle $= \frac{\pi d^2}{4} = \frac{\pi x^2}{4}$

Area of penny $= \frac{\pi d^2}{4} = \frac{\pi\left(\frac{3}{4}\right)^2}{4} = \frac{9\pi}{64}$

Area of penny • Number of pennies ≤ Area of circle

$\frac{9\pi}{64} \quad \bullet \quad L \quad \leq \quad \frac{\pi x^2}{4}$

$L \leq \frac{\pi x^2}{4}\left(\frac{64}{9\pi}\right)$

$L \leq \frac{16}{9}x^2$

4.10 Guided Practice (pp. 310–311)

1. $y = a(x - h)^2 + k$

$y = a(x - 4)^2 - 5$

$-1 = a(2 - 4)^2 - 5$

$-1 = 4a - 5$

$1 = a$

A quadratic function is $y = (x - 4)^2 - 5$.

2. $y = a(x - h)^2 + k$

$y = a(x + 3)^2 + 1$

$-8 = a(0 + 3)^2 + 1$

$-8 = 9a + 1$

$-1 = a$

A quadratic function is $y = -(x + 3)^2 + 1$.

3. $y = a(x - p)(x - q)$

$y = a(x + 2)(x - 5)$

$2 = a(6 + 2)(6 - 5)$

$2 = 8a$

$\frac{1}{4} = a$

A quadratic function is $y = \frac{1}{4}(x + 2)(x - 5)$.

4. $y = ax^2 + bx + c$

$5 = a(-1)^2 + b(-1) + c \rightarrow \quad a - b + c = 5$

$-1 = a(0)^2 + b(0) + c \rightarrow \quad c = -1$

$11 = (a)^2 + b(2) + c \rightarrow 4a + 2b + c = 11$

$a - b + c = 5$

$a - b - 1 = 5$

$a - b = 6$

$4a + 2b + c = 11$

$4a + 2b - 1 = 11$

$4a + 2b = 12$

$a - b = 6 \xrightarrow{\times 2} 2a - 2b = 12$

$4a + 2b = 12 \longrightarrow \underline{4a + 2b = 12}$

$6a \quad = 24$

$a = 4$

So $4 - b = 6$, which means $b = -2$.

The solution is $a = 4$, $b = -2$, and $c = -1$. A quadratic function for the parabola is $y = 4x^2 - 2x - 1$.

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5. $y = ax^2 + bx + c$

$-1 = a(-2)^2 + b(-2) + c \rightarrow 4a - 2b + c = -1$

$3 = a(0)^2 + b(0) + c \rightarrow c = 3$

$1 = a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = 1$

$4a - 2b + c = -1$

$4a - 2b + 3 = -1$

$4a - 2b = -4$

$16a + 4b + c = 1$

$16a + 4b + 3 = 1$

$16a + 4b = -2$

$4a - 2b = -4 \xrightarrow{\times 2} 8a - 4b = -8$

$16a + 4b = -2 \longrightarrow 16a + 4b = -2$

$24a = -10$

$a = -\frac{5}{12}$

$4\left(-\frac{5}{12}\right) - 2b = -4$

$-\frac{5}{3} - 2b = -4$

$-2b = -\frac{7}{3}$

$b = \frac{7}{6}$

The solution is $a = -\frac{5}{12}$, $b = \frac{7}{6}$, and $c = -3$.

A quadratic function for the parabola is

$y = -\frac{5}{12}x^2 + \frac{7}{6}x + 3.$

6. $y = ax^2 + bx + c$

$0 = a(-1)^2 + b(-1) + c \rightarrow a - b + c = 0$

$-2 = a(1)^2 + b(1) + c \rightarrow a + b + c = -2$

$-15 = a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = -15$

$a - b + c = 0$

$c = b - a$

$a + b + c = -2$

$a + b + (b - a) = -2$

$2b = -2$

$b = -1$

$a + b + c = -2$

$a - 1 + c = -2$

$a + c = -1$

$4a + 2b + c = -15$

$4a + 2(-1) + c = -15$

$4a + c = -13$

$a + c = -1 \xrightarrow{\times (-1)} -a - c = 1$

$4a + c = -13 \longrightarrow 4a + c = -13$

$3a = -12$

$a = -4$

So $-4 + c = -1$, which means $c = 3$.

The solution is $a = -4$, $b = -1$, and $c = 3$. A quadratic function for the parabola is $y = -4x^2 - x + 3$.

7. At an angle of 43.3°, the pumpkin travels the farthest. The angle can be found by graphing the best-fitting quadratic model found in Example 4 on a graphing calculator and using the maximum feature.

4.10 Exercises (pp. 312–315)

Skill Practice

1. When you perform quadratic regression on a set of data, the quadratic model obtained is called the best-fitting quadratic model.

2. To write an equation of a parabola given three points on the parabola, first substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain a system of three linear equations. Then solve the system to find a, b, and c. Finally, substitute the values of a, b, and c into $y = ax^2 + bx + c$.

3. $y = a(x - h)^2 + k$

$y = a(x - 3)^2 + 2$

$6 = a(5 - 3)^2 + 2$

$6 = 4a + 2$

$1 = a$

A quadratic function for the parabola is $y = (x - 3)^2 + 2$.

4. $y = a(x - h)^2 + k$

$y = a(x + 2)^2 + 1$

$-1 = a(-1 + 2)^2 + 1$

$-1 = a + 1$

$-2 = a$

A quadratic function for the parabola is $y = -2(x + 2)^2 + 1$.

5. $y = a(x - h)^2 + k$

$y = a(x + 1)^2 - 3$

$-1 = a(1 + 1)^2 - 3$

$-1 = 4a - 3$

$\frac{1}{2} = a$

A quadratic function for the parabola is $y = \frac{1}{2}(x + 1)^2 - 3$.

6. $y = a(x - h)^2 + k$

$y = a(x + 4)^2 + 1$

$5 = a(-2 + 4)^2 + 1$

$5 = 4a + 1$

$1 = a$

A quadratic function is $y = (x + 4)^2 + 1$.

7. $y = a(x - h)^2 + k$

$y = a(x - 1)^2 + 6$

$2 = a(-1 - 1)^2 + 6$

$2 = 4a + 6$

$-1 = a$

A quadratic function is $y = -(x - 1)^2 + 6$.

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8. $y = a(x - h)^2 + k$

$y = a(x - 5)^2 - 4$

$20 = a(1 - 5)^2 - 4$

$20 = 16a - 4$

$24 = 16a$

$\frac{3}{2} = a$

A quadratic function is $y = \frac{3}{2}(x - 5)^2 - 4$.

9. $y = a(x - h)^2 + k$

$y = a(x + 3)^2 + 3$

$-1 = a(1 + 3)^2 + 3$

$-1 = 16a + 3$

$-\frac{1}{4} = a$

A quadratic function is $y = -\frac{1}{4}(x + 3)^2 + 3$.

10. $y = a(x - h)^2 + k$

$y = a(x - 5)^2$

$-27 = a(2 - 5)^2$

$-27 = 9a$

$-3 = a$

A quadratic function is $y = -3(x - 5)^2$.

11. $y = a(x - h)^2 + k$

$y = a(x + 4)^2 - 2$

$30 = a(0 + 4)^2 - 2$

$30 = 16a - 2$

$2 = a$

A quadratic function is $y = 2(x + 4)^2 - 2$.

12. $y = a(x - h)^2 + k$

$y = a(x - 2)^2 + 1$

$-2 = a(4 - 2)^2 + 1$

$-2 = 4a + 1$

$-\frac{3}{4} = a$

A quadratic function is $y = -\frac{3}{4}(x - 2)^2 + 1$.

13. $y = a(x - h)^2 + k$

$y = a(x + 1)^2 - 4$

$-1 = a(2 + 1)^2 - 4$

$-1 = 9a - 4$

$\frac{1}{3} = a$

A quadratic function is $y = \frac{1}{3}(x + 1)^2 - 4$.

14. $y = a(x - h)^2 + k$

$y = a(x - 3)^2 + 5$

$-3 = a(7 - 3)^2 + 5$

$-3 = 16a + 5$

$-\frac{1}{2} = a$

A quadratic function is $y = -\frac{1}{2}(x - 3)^2 + 5$.

15. C; $y = a(x - h)^2 + k$

$y = a(x - 5)^2 - 3$

$5 = a(1 - 5)^2 - 3$

$5 = 16a - 3$

$\frac{1}{2} = a$

A quadratic function for the parabola is

$y = \frac{1}{2}(x - 5)^2 - 3$.

$(-1, 15)$: $15 \stackrel{?}{=} \frac{1}{2}(-1 - 5)^2 - 3$

$15 = 15$ ✓

16. D; $y = a(x - p)(x - q)$

$y = a(x - 4)(x - 7)$

$-20 = a(2 - 4)(2 - 7)$

$-20 = 10a$

$-2 = a$

A quadratic function for the parabola is

$y = -2(x - 4)(x - 7)$.

$(5, 4)$: $4 \stackrel{?}{=} -2(5 - 4)(5 - 7)$

$4 = 4$ ✓

17. $y = a(x - p)(x - q)$

$y = a(x + 2)(x - 3)$

$6 = a(0 + 2)(0 - 3)$

$6 = -6a$

$-1 = a$

A quadratic function for the parabola is
$y = -(x + 2)(x - 3)$.

18. $y = a(x - p)(x - q)$

$y = a(x + 6)(x + 4)$

$3 = a(-3 + 6)(-3 + 4)$

$3 = 3a$

$1 = a$

A quadratic function for the parabola is
$y = (x + 6)(x + 4)$.

19. $y = a(x - p)(x - q)$

$y = a(x + 3)(x - 3)$

$-4 = a(1 + 3)(1 - 3)$

$-4 = -8a$

$\frac{1}{2} = a$

A quadratic function for parabola is

$y = \frac{1}{2}(x + 3)(x - 3)$.

20. $y = a(x - p)(x - q)$

$y = a(x - 2)(x - 5)$

$-2 = a(4 - 2)(4 - 5)$

$-2 = -2a$

$1 = a$

A quadratic function is $y = (x - 2)(x - 5)$.

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21. $y = a(x - p)(x - q)$

$y = a(x + 3)(x)$

$10 = a(2 + 3)(2)$

$10 = 10a$

$1 = a$

A quadratic function is $y = x(x + 3)$.

22. $y = a(x - p)(x - q)$

$y = a(x + 1)(x - 4)$

$4 = a(2 + 1)(2 - 4)$

$4 = -6a$

$-\frac{2}{3} = a$

A quadratic function is $y = -\frac{2}{3}(x + 1)(x - 4)$.

23. $y = a(x - p)(x - q)$

$y = a(x - 3)(x - 7)$

$-9 = a(6 - 3)(6 - 7)$

$-9 = -3a$

$3 = a$

A quadratic function is $y = 3(x - 3)(x - 7)$.

24. $y = a(x - p)(x - q)$

$y = a(x + 5)(x + 1)$

$-24 = a(-7 + 5)(-7 + 1)$

$-24 = 12a$

$-2 = a$

A quadratic function is $y = -2(x + 5)(x + 1)$.

25. $y = a(x - p)(x - q)$

$y = a(x + 6)(x - 3)$

$-9 = a(0 + 6)(0 - 3)$

$-9 = -18a$

$\frac{1}{2} = a$

A quadratic function is $y = \frac{1}{2}(x + 6)(x - 3)$.

26. The x- and y-values of the point were substituted for p and q, and the x-intercepts were substituted for the x- and y-values.

$y = a(x - 4)(x + 3)$

$-5 = a(5 - 4)(5 + 3)$

$-5 = 8a$

$-\frac{5}{8} = a$, so $y = -\frac{5}{8}(x - 4)(x + 3)$

27. Because the vertex and a point are given, the vertex form should be used, not the intercept form.

$y = a(x - 2)^2 + 3$

$5 = a(1 - 2)^2 + 3$

$5 = a + 3$

$2 = a$, so $y = 2(x - 2)^2 + 3$

28. $y = ax^2 + bx + c$

$-6 = a(1)^2 + b(1) + c \rightarrow a + b + c = -6$

$-1 = a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = -1$

$-3 = a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = -3$

$$\begin{array}{rcr} a + b + c = -6 & \xrightarrow{\times(-1)} & -a - b - c = 6 \\ 4a + 2b + c = -1 & \longrightarrow & \underline{4a + 2b + c = -1} \\ & & 3a + b = 5 \end{array}$$

$$\begin{array}{rcr} a + b + c = -6 & \xrightarrow{\times(-1)} & -a - b - c = 6 \\ 16a + 4b + c = -3 & \longrightarrow & \underline{16a + 4b + c = -3} \\ & & 15a + 3b = 3 \end{array}$$

$$\begin{array}{rcr} 3a + b = 5 & \xrightarrow{\times(-1)} & -9a - 3b = -15 \\ 15a + 3b = 3 & \longrightarrow & \underline{15a + 3b = 3} \\ & & 6a = -12 \\ & & a = -2 \end{array}$$

$3a + b = 5$

$3(-2) + b = 5$

$b = 11$

$a + b + c = -6$

$-2 + 11 + c = -6$

$c = -15$

The solution is $a = -2$, $b = 11$ and $c = -15$.
A quadratic function for the parabola is
$y = -2x^2 + 11x - 15$.

29. $y = ax^2 + bx + c$

$-2 = a(-6)^2 + b(-6) + c \rightarrow 36a - 6b + c = -2$

$-2 = a(-4)^2 + b(-4) + c \rightarrow 16a - 4b + c = -2$

$4 = a(-3)^2 + b(-3) + c \rightarrow 9a - 3b + c = 4$

$$\begin{array}{rcr} 36a - 6b + c = -2 & \xrightarrow{\times(-1)} & -36a + 6b - c = 2 \\ 16a - 4b + c = -2 & \longrightarrow & \underline{16a - 4b + c = -2} \\ & & -20a + 2b = 0 \end{array}$$

$$\begin{array}{rcr} 9a - 3b + c = 4 & \xrightarrow{\times(-1)} & -9a + 3b - c = -4 \\ 16a - 4b + c = -2 & \longrightarrow & \underline{16a - 4b + c = -2} \\ & & 7a - b = -6 \end{array}$$

$$\begin{array}{rcr} -20a + 2b = 0 & \longrightarrow & -20a + 2b = 0 \\ 7a - b = -6 & \xrightarrow{\times 2} & \underline{14a - 2b = -12} \\ & & -6a = -12 \\ & & a = 2 \end{array}$$

$7a - b = -6$

$7(2) - b = -6$

$14 - b = -6$

$b = 20$

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$$9a - 3b + c = 4$$
$$9(2) - 3(20) + c = 4$$
$$18 - 60 + c = 4$$
$$-42 + c = 4$$
$$c = 46$$

The solution is $a = 2$, $b = 20$, and $c = 46$. A quadratic function for the parabola is $y = 2x^2 + 20x + 46$.

30. $y = ax^2 + bx + c$

$$\begin{aligned} -6 &= a(-4)^2 + b(-4) + c \rightarrow 16a - 4b + c = -6 \\ -2 &= a(0)^2 + b(0) + c \rightarrow c = -2 \\ 6 &= a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = 6 \end{aligned}$$

$$16a - 4b + c = -6$$
$$16a - 4b + (-2) = -6$$
$$16a - 4b = -4$$
$$4a + 2b + c = 6$$
$$4a + 2b + (-2) = 6$$
$$4a + 2b = 8$$

$$\begin{array}{rcl} 16a - 4b = -4 & \longrightarrow & 16a - 4b = -4 \\ 4a + 2b = 8 & \xrightarrow{\times 2} & \underline{8a + 4b = 16} \\ & & 24a = 12 \\ & & a = \frac{1}{2} \end{array}$$

$$16\left(\frac{1}{2}\right) - 4b = -4$$
$$8 - 4b = -4$$
$$-4b = -12$$
$$b = 3$$

The solution is $a = \frac{1}{2}$, $b = 3$, and $c = -2$. A quadratic function for the parabola is $y = \frac{1}{2}x^2 + 3x - 2$.

31. $y = ax^2 + bx + c$

$$\begin{aligned} -3 &= a(-4)^2 + b(-4) + c \rightarrow 16a - 4b + c = -3 \\ -2 &= a(0)^2 + b(0) + c \rightarrow c = -2 \\ 7 &= a(1)^2 + b(1) + c \rightarrow a + b + c = 7 \end{aligned}$$

$$16a - 4b + c = -3$$
$$16a - 4b + (-2) = -3$$
$$16a - 4b = -1$$
$$a + b + c = 7$$
$$a + b + (-2) = 7$$
$$a + b = 9$$

$$\begin{array}{rcl} 16a - 4b = -1 & \longrightarrow & 16a - 4b = -1 \\ a + b = 9 & \xrightarrow{\times 4} & \underline{4a + 4b = 36} \\ & & 20a = 35 \\ & & a = \frac{7}{4} \end{array}$$

$$16\left(\frac{7}{4}\right) - 4b = -1$$
$$28 - 4b = -1$$
$$-4b = -29$$
$$b = \frac{29}{4}$$

The solution is $a = \frac{7}{4}$, $b = \frac{29}{4}$, and $c = -2$.
A quadratic function for the parabola is
$y = \frac{7}{4}x^2 + \frac{29}{4}x - 2$.

32. $y = ax^2 + bx + c$

$$\begin{aligned} -4 &= a(-2)^2 + b(-2) + c \rightarrow 4a - 2b + c = -4 \\ -10 &= a(0)^2 + b(0) + c \rightarrow c = -10 \\ -7 &= a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = -7 \end{aligned}$$

$$4a - 2b + c = -4$$
$$4a - 2b + (-10) = -4$$
$$4a - 2b = 6$$
$$9a + 3b + c = -7$$
$$9a + 3b + (-10) = -7$$
$$9a + 3b = 3$$

$$\begin{array}{rcl} 4a - 2b = 6 & \xrightarrow{\times 3} & 12a - 6b = 18 \\ 9a + 3b = 3 & \xrightarrow{\times 2} & \underline{18a + 6b = 6} \\ & & 30a = 24 \\ & & a = \frac{4}{5} \end{array}$$

$$4\left(\frac{4}{5}\right) - 2b = 6$$
$$\frac{16}{5} - 2b = 6$$
$$-2b = \frac{14}{5}$$
$$b = -\frac{7}{5}$$

The solution is $a = \frac{4}{5}$, $b = -\frac{7}{5}$, and $c = -10$. A quadratic function for the parabola is $y = \frac{4}{5}x^2 - \frac{7}{5}x - 10$.

33. $y = ax^2 + bx + c$

$$\begin{aligned} 4 &= a(-2)^2 + b(-2) + c \rightarrow 4a - 2b + c = 4 \\ 5 &= a(0)^2 + b(0) + c \rightarrow c = 5 \\ -11 &= a(1)^2 + b(1) + c \rightarrow a + b + c = -11 \end{aligned}$$

$$4a - 2b + c = 4$$
$$4a - 2b + 5 = 4$$
$$4a - 2b = -1$$
$$a + b + c = -11$$
$$a + b + 5 = -11$$
$$a + b = -16$$

$$\begin{array}{rcl} 4a - 2b = -1 & \longrightarrow & 4a - 2b = -1 \\ a + b = -16 & \xrightarrow{\times 2} & \underline{2a + 2b = -32} \\ & & 6a = -33 \\ & & a = -\frac{11}{2} \end{array}$$

$$4\left(-\frac{11}{2}\right) - 2b = -1$$
$$-22 - 2b = -1$$
$$-2b = 21$$
$$b = -\frac{21}{2}$$

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The solution is $a = -\frac{11}{2}$, $b = -\frac{21}{2}$, and $c = 5$.

A quadratic function for the parabola is

$y = -\frac{11}{2}x^2 - \frac{21}{2}x + 5.$

34. $y = ax^2 + bx + c$

$$\begin{aligned}
-1 &= a(-1)^2 + b(-1) + c \rightarrow a - b + c = -1\\
11 &= a(1)^2 + b(1) + c \rightarrow a + b + c = 11\\
7 &= a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = 7
\end{aligned}$$

$$\begin{array}{r}
a - b + c = -1\\
\underline{a + b + c = 11}\\
2a + 2c = 10
\end{array}$$

$$\begin{array}{rcr}
a + b + c = 11 & \xrightarrow{\times(-3)} & -3a - 3b - 3c = -33\\
9a + 3b + c = 7 & \longrightarrow & \underline{9a + 3b + c = 7}\\
 & & 6a - 2c = -26
\end{array}$$

$$\begin{array}{r}
2a + 2c = 10\\
\underline{6a - 2c = -26}\\
8a = -16\\
a = -2
\end{array}$$

$$\begin{aligned}
2(-2) + 2c &= 10\\
2c &= 14\\
c &= 7
\end{aligned}$$

$$\begin{aligned}
a + b + c &= 11\\
-2 + b + 7 &= 11\\
b + 5 &= 11\\
b &= 6
\end{aligned}$$

The solution is $a = -2$, $b = 6$, and $c = 7$. A quadratic function for the parabola is $y = -2x^2 + 6x + 7$.

35. $y = ax^2 + bx + c$

$$\begin{aligned}
9 &= a(-1)^2 + b(-1) + c \rightarrow a - b + c = 9\\
1 &= a(1)^2 + b(1) + c \rightarrow a + b + c = 1\\
17 &= a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = 17
\end{aligned}$$

$$\begin{array}{r}
a - b + c = 9\\
\underline{a + b + c = 1}\\
2a + 2c = 10
\end{array}$$

$$\begin{array}{rcr}
a - b + c = 9 & \xrightarrow{\times 3} & 3a - 3b + 3c = 27\\
9a + 3b + c = 17 & \longrightarrow & \underline{9a + 3b + c = 17}\\
 & & 12a + 4c = 44
\end{array}$$

$$\begin{array}{rcr}
2a + 2c = 10 & \xrightarrow{\times(-2)} & -4a - 4c = -20\\
12a + 4c = 44 & \longrightarrow & \underline{12a + 4c = 44}\\
 & & 8a = 24\\
 & & a = 3
\end{array}$$

$$\begin{aligned}
2a + 2c &= 10\\
2(3) + 2c &= 10\\
6 + 2c &= 10\\
2c &= 4\\
c &= 2
\end{aligned}$$

$$\begin{aligned}
a + b + c &= 1\\
3 + b + 2 &= 1\\
5 + b &= 1\\
b &= -4
\end{aligned}$$

The solution is $a = 3$, $b = -4$, and $c = 2$.

A quadratic function for the parabola is

$y = 3x^2 - 4x + 2.$

36. $y = ax^2 + bx + c$

$$\begin{aligned}
-1 &= a(-6)^2 + b(-6) + c \rightarrow 36a - 6b + c = -1\\
-4 &= a(-3)^2 + b(-3) + c \rightarrow 9a - 3b + c = -4\\
8 &= a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = 8
\end{aligned}$$

$$\begin{array}{r}
9a - 3b + c = -4\\
\underline{9a + 3b + c = 8}\\
18a + 2c = 4
\end{array}$$

$$\begin{array}{rcr}
36a - 6b + c = -1 & \longrightarrow & 36a - 6b + c = -1\\
9a - 3b + c = -40 & \xrightarrow{\times(-2)} & \underline{-18a + 6b - 2c = 8}\\
 & & 18a - c = 7
\end{array}$$

$$\begin{array}{rcr}
18a + 2c = 4 & \xrightarrow{\times(-1)} & -18a - 2c = -4\\
18a - c = 7 & \longrightarrow & \underline{18a - c = 7}\\
 & & -3c = 3\\
 & & c = -1
\end{array}$$

$$\begin{aligned}
18a + 2c &= 4\\
18a + 2(-1) &= 4\\
18a &= 6\\
a &= \frac{1}{3}
\end{aligned}$$

$$\begin{aligned}
9a - 3b + c &= -4\\
9\left(\frac{1}{3}\right) - 3b - 1 &= -4\\
3 - 3b - 1 &= -4\\
2 - 3b &= -4\\
-3b &= -6\\
b &= 2
\end{aligned}$$

The solution is $a = \frac{1}{3}$, $b = 2$, and $c = -1$.

A quadratic function for the parabola is $y = \frac{1}{3}x^2 + 2x - 1$.

37. $y = ax^2 + bx + c$

$$\begin{aligned}
-13 &= a(-2)^2 + b(-2) + c \rightarrow 4a - 2b + c = -13\\
3 &= a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = 3\\
5 &= a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = 5
\end{aligned}$$

$$\begin{array}{r}
4a - 2b + c = -13\\
\underline{4a + 2b + c = 3}\\
8a + 2c = -10
\end{array}$$

$$\begin{array}{rcr}
16a + 4b + c = 5 & \longrightarrow & 16a + 4b + c = 5\\
4a - 2b + c = -13 & \xrightarrow{\times 2} & \underline{8a - 4b + 2c = -26}\\
 & & 24a + 3c = -21
\end{array}$$

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$$\begin{aligned} 8a + 2c &= -10 \xrightarrow{\times(-3)} & -24a - 6c &= 30 \\ 24a + 3c &= -21 \longrightarrow & \underline{24a + 3c} &\underline{= -21} \\ & & -3c &= 9 \\ & & c &= -3 \end{aligned}$$

$$\begin{aligned} 8a + 2c &= -10 \\ 8a + 2(-3) &= -10 \\ 8a - 6 &= -10 \\ 8a &= -4 \\ a &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 4a + 2b + c &= 3 \\ 4\left(-\frac{1}{2}\right) + 2b - 3 &= 3 \\ -2 + 2b &= 6 \\ 2b &= 8 \\ b &= 4 \end{aligned}$$

The solution is $a = -\frac{1}{2}$, $b = 4$, and $c = -3$.

A quadratic function for the parabola is

$y = -\frac{1}{2}x^2 + 4x - 3$.

38. $y = ax^2 + bx + c$

$$\begin{aligned} 29 &= a(-6)^2 + b(-6) + c \rightarrow 36a - 6b + c = 29 \\ 12 &= a(-4)^2 + b(-4) + c \rightarrow 16a - 4b + c = 12 \\ -3 &= a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = -3 \end{aligned}$$

$$\begin{aligned} 36a - 6b + c &= 29 \longrightarrow & 36a - 6b + c &= 29 \\ 4a + 2b + c &= -3 \xrightarrow{\times 3} & \underline{12a + 6b + 3c} &\underline{= -9} \\ & & 48a + 4c &= 20 \end{aligned}$$

$$\begin{aligned} 16a - 4b + c &= 12 \longrightarrow & 16a - 4b + c &= 12 \\ 4a + 2b + c &= -3 \xrightarrow{\times 2} & \underline{8a + 4b + 2c} &\underline{= -6} \\ & & 24a + 3c &= 6 \end{aligned}$$

$$\begin{aligned} 48a + 4c &= 20 \longrightarrow & 48a + 4c &= 20 \\ 24a + 3c &= 6 \xrightarrow{\times(-2)} & \underline{-48a - 6c} &\underline{= -12} \\ & & -2c &= 8 \\ & & c &= -4 \end{aligned}$$

$$\begin{aligned} 24a + 3c &= 6 \\ 24a + 3(-4) &= 6 \\ 24a &= 18 \\ a &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} 4a + 2b + c &= -3 \\ 4\left(\frac{3}{4}\right) + 2b - 4 &= -3 \\ 3 + 2b - 4 &= -3 \\ 2b - 1 &= -3 \\ 2b &= -2 \\ b &= -1 \end{aligned}$$

The solution is $a = \frac{3}{4}$, $b = -1$, and $c = -4$.

A quadratic function for the parabola is

$y = \frac{3}{4}x^2 - x - 4$.

39. $y = ax^2 + bx + c$

$$\begin{aligned} -2 &= a(-3)^2 + b(-3) + c \rightarrow 9a - 3b + c = -2 \\ 10 &= a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = 10 \\ -2 &= a(6)^2 + b(6) + c \rightarrow 36a + 6b + c = -2 \end{aligned}$$

$$\begin{aligned} 9a - 3b + c &= -2 \\ \underline{9a + 3b + c} &\underline{= 10} \\ 18a + 2c &= 8 \end{aligned}$$

$$\begin{aligned} 9a - 3b + c &= -2 \xrightarrow{\times 2} & 18a - 6b + 2c &= -4 \\ 36a + 6b + c &= -2 \longrightarrow & \underline{36a + 6b + c} &\underline{= -2} \\ & & 54a + 3c &= -6 \end{aligned}$$

$$\begin{aligned} 18a + 2c &= 8 \xrightarrow{\times(-3)} & -54a - 6c &= -24 \\ 54a + 3c &= -6 \longrightarrow & \underline{54a + 3c} &\underline{= -6} \\ & & -3c &= -30 \\ & & c &= 10 \end{aligned}$$

$$\begin{aligned} 18a + 2c &= 8 \\ 18a + 2(10) &= 8 \\ 18a + 20 &= 8 \\ 18a &= -12 \\ a &= -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} 9a - 3b + c &= -2 \\ 9\left(-\frac{2}{3}\right) - 3b + 10 &= -2 \\ -6 - 3b + 10 &= -2 \\ -3b + 4 &= -2 \\ -3b &= -6 \\ b &= 2 \end{aligned}$$

The solution is $a = -\frac{2}{3}$, $b = 2$, and $c = 10$.

A quadratic function for the parabola is

$y = -\frac{2}{3}x^2 + 2x + 10$.

40. $y = ax^2 + bx + c$

$$\begin{aligned} -1 &= a(-0.5)^2 + b(-0.5) + c \rightarrow \frac{1}{4}a - \frac{1}{2}b + c = -1 \\ 8 &= a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = 8 \\ 25 &= a(11)^2 + b(11) + c \rightarrow 121a + 11b + c = 25 \end{aligned}$$

$$\begin{aligned} 0.25a - 0.5b + c &= -1 \xrightarrow{\times 4} & a - 2b + 4c &= -4 \\ 4a + 2b + c &= 8 \longrightarrow & \underline{4a + 2b + c} &\underline{= 8} \\ & & 5a + 5c &= 4 \end{aligned}$$

$$\begin{aligned} 0.25a - 0.5b + c &= -1 \\ &\xrightarrow{\times 22} & 5.5a - 11b + 22c &= -22 \\ 121a + 11b + c &= 25 \\ &\longrightarrow & \underline{121a + 11b + c} &\underline{= 25} \\ & & 126.5a + 23c &= 3 \end{aligned}$$

$$\begin{aligned} 5a + 5c &= 4 \xrightarrow{\times(-4.6)} & -23a - 23c &= -18.4 \\ 126.5a + 23c &= 3 \longrightarrow & \underline{126.5a + 23c} &\underline{= 3} \\ & & 103.5a &= -15.4 \\ & & a &\approx -0.15 \end{aligned}$$

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$$\begin{aligned} 5a + 5c &= 4 \\ 5(-0.15) + 5c &\approx 4 \\ -0.75 + 5c &\approx 4 \\ 5c &\approx 4.75 \\ c &\approx 0.95 \end{aligned}$$

$$\begin{aligned} 4a + 2b + c &= 8 \\ 4(-0.15) + 2b + 0.95 &\approx 8 \\ -0.6 + 2b + 0.95 &\approx 8 \\ 0.35 + 2b &\approx 8 \\ 2b &\approx 7.65 \\ b &\approx 3.83 \end{aligned}$$

The solution is $a \approx -0.15$, $b \approx 3.83$, and $c \approx 0.95$

A quadratic function is $y = -0.15x^2 + 3.83x + 0.95$

41.
$$\begin{aligned} y &= a(x - p)(x - q) \\ y &= a(x + 11)(x - 3) \\ -192 &= a(1 + 11)(1 - 3) \\ -192 &= -24a \\ 8 &= a \end{aligned}$$

A quadratic function is $y = 8(x + 11)(x - 3)$.

42.
$$\begin{aligned} y &= a(x - h)^2 + k \\ y &= a(x - 4.5)^2 + 7.25 \\ -3 &= a(7 - 4.5)^2 + 7.25 \\ -3 &= 6.25a + 7.25 \\ -10.25 &= 6.25a \\ -1.64 &= a \end{aligned}$$

A quadratic function is $y = -1.64(x - 4.5)^2 + 7.25$.

43. *Sample answer:*

$y = a(x - h)^2 + k$

$y = a(x - 0)^2 - 1$

$y = a(x)^2 - 1$

$3 = a(-2)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a$

Vertex and standard form:

$g = x^2 - 1$

Intercept form:

$g = (x + 1)(x - 1)$

44. For each data pair, find the ratio $\frac{x^2}{y}$. If the ratios are equivalent, the data can be modeled by a quadratic function of the form $y = ax^2$.

45. $y = ax^2 + bx + c$

$$\begin{aligned} -4 &= a(1)^2 + b(1) + c \to a + b + c = -4 \\ -16 &= a(-3)^2 + b(-3) + c \to 9a - 3b + c = -16 \\ 14 &= a(7)^2 + b(7) + c \to 49a + 7b + c = 14 \end{aligned}$$

$$\begin{array}{ll} a + b + c = -4 \xrightarrow{\times 3} & 3a + 3b + 3c = -12 \\ 9a - 3b + c = -16 \longrightarrow & \underline{9a - 3b + c = -16} \\ & 12a + 4c = -28 \end{array}$$

$$\begin{array}{ll} a + b + c = -4 \xrightarrow{\times (-7)} & -7a - 7b - 7c = 28 \\ 49a + 7b + c = 14 \longrightarrow & \underline{49a + 7b + c = 14} \\ & 42a - 6c = 42 \end{array}$$

$$\begin{array}{ll} 12a + 4c = -28 \xrightarrow{\times 3} & 36a + 12c = -84 \\ 42a - 6c = 42 \xrightarrow{\times 2} & \underline{84a - 12c = 84} \\ & 120a = 0 \\ & a = 0 \end{array}$$

$$\begin{aligned} 12a + 4c &= -28 \\ 12(0) + 4c &= -28 \\ 4c &= -28 \\ c &= -7 \end{aligned}$$

$$\begin{aligned} a + b + c &= -4 \\ 0 + b - 7 &= -4 \\ b &= 3 \end{aligned}$$

The solution is $a = 0$, $b = 3$, and $c = -7$. The function is $y = 0x^2 + 3x - 7$, or $y = 3x - 7$.

The model tells you that the three points lie on a line.

Problem Solving

46. $y = ax^2 + bx + c$

$$\begin{aligned} 4 &= a(0)^2 + b(0) + c \to c = 4 \\ 3.25 &= a(2)^2 + b(2) + c \to 4a + 2b + c = 3.25 \\ 3.0625 &= a(5)^2 + b(5) + c \to 25a + 5b + c = 3.0625 \end{aligned}$$

$$\begin{aligned} 4a + 2b + c &= 3.25 \\ 4a + 2b + 4 &= 3.25 \\ 4a + 2b &= -0.75 \end{aligned}$$

$$\begin{aligned} 25a + 5b + c &= 3.0625 \\ 25a + 5b + 4 &= 3.0625 \\ 25a + 5b &= -0.9375 \end{aligned}$$

$$\begin{array}{ll} 4a + 2b = -0.75 \xrightarrow{\times 5} & 20a + 10b = -3.75 \\ 25a + 5b = -0.9375 \xrightarrow{\times (-2)} & \underline{-50a - 10b = 1.875} \\ & -30a = -1.875 \\ & a = 0.0675 \end{array}$$

$$\begin{aligned} 4a + 2b &= -0.75 \\ 4(0.0625) + 2b &= -0.75 \\ 0.25 + 2b &= -0.75 \\ 2b &= -1 \\ b &= -0.5 \end{aligned}$$

A quadratic function that models the cross section is $y = 0.0625x^2 - 0.5x + 4$.

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47. $y = a(x - h)^2 + k$

$y = a(x - 20)^2 + 15$

$0 = a(0 - 20)^2 + 15$

$0 = 400a + 15$

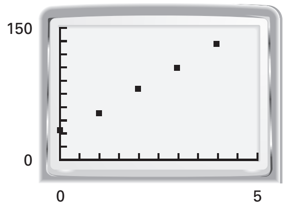
$-15 = 400a$

$-0.0375 = a$

A quadratic function that models the path is

$y = -0.0375(x - 20)^2 + 15.$

48. a.



b. The best fitting quadratic model is:

$y = 0.86x^2 + 21.77x + 33.31.$

c. When $x = 13$:

$y = 0.86(13)^2 + 21.77(13) + 33.31 = 461.66$

In 2010, the average number of hours per person spent on the Internet in the United States will be about 462.

49. a. The best-fitting quadratic model is

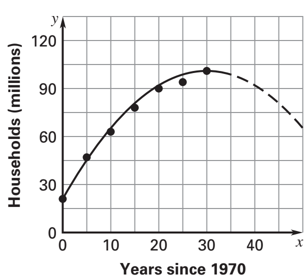
$t = 0.0119s^2 - 0.31s - 0.0005.$

b. $t = 0.0119(10)^2 - 0.31(10) - 0.0005$

$t \approx -1.91$

The change in finishing time when the wind speed is 10 m/sec is about 1.91 seconds.

50. a.



b. Points chosen will vary:

(0, 21), (10, 63), (20, 90)

$y = ax^2 + bx + c$

$21 = a(0)^2 + b(0) + c \rightarrow \quad c = 21$

$63 = a(10)^2 + b(10) + c \rightarrow 100a + 10b + c = 63$

$90 = a(20)^2 + b(20) + c \rightarrow 400a + 20b + c = 90$

$100a + 10b + c = 63$

$100a + 10b + 21 = 63$

$100a + 10b = 42$

$400a + 20b + c = 90$

$400a + 20b + 21 = 90$

$400a + 20b = 69$

$100a + 10b = 42 \xrightarrow{\times(-2)} -200a - 20b = -84$

$400a + 20b = 69 \longrightarrow \underline{400a + 20b = 69}$

$200a = -15$

$a = -0.075$

$100a + 10b = 42$

$100(-0.075) + 10b = 42$

$-7.5 + 10b = 42$

$10b = 49.5$

$b = 4.95$

A quadratic function for the data is

$y = -0.075x^2 + 4.95x + 21.$

c.

Years since 1970	0	5	10	15
Households with color TVs (millions)	21	43.9	63	78.4

Years since 1970	20	25	30
Households with color TVs (millions)	90	97.9	102

The numbers given by the function are a bit different than the numbers in the original table. The function from part (b) gives a slightly less accurate representation of the original data.

51. C; $y = ax^2 + bx + c$

$0 = a(0)^2 + b(0) + c \rightarrow \quad c = 0$

$38.2 = a(40)^2 + b(40) + c \rightarrow 1600a + 40b + c = 38.2$

$0 = a(165)^2 + b(165) + c \rightarrow 27{,}225a + 165b + c = 0$

$1600a + 40b + 0 = 38.2 \xrightarrow{\times(-4.125)} -6600a - 165b = -157.575$

$27{,}225a + 165b + 0 = 0 \longrightarrow \underline{27{,}225a + 165b = 0}$

$20{,}625a = -157.575$

$a = -0.00764$

$1600a + 40b = 38.2$

$1600(-0.00764) + 40b = 38.2$

$-12.224 + 40b = 38.2$

$40b = 50.424$

$b = 1.2606$

A quadratic function for the parabola is

$y = -0.00764x^2 + 1.2606x.$

(80, 51.95): $51.95 \stackrel{?}{=} -0.00764(80)^2 + 1.2606(80)$

$51.95 = 51.95$ ✓

52.

n	0	1	2	3	4	5	6
R	1	2	4	7	11	16	22

A quadratic model is $R = 0.5n^2 + 0.5n + 1.$

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Mixed Review

53. $x^2 - 3 = (5)^2 - 3 = 22$

54. $3a^5 - 10 = 3(-1)^5 - 10 = -13$

55. $x^4 = (-2)^4 = 16$

56. $4u^3 - 15 = 4(3)^3 - 15 = 93$

57. $v^2 + 3v - 5 = (5)^2 + 3(5) - 5 = 35$

58. $-y^3 + 2y + 5 = -2^3 + 2(2) + 5 = 1$

59.
$$\begin{array}{r} 4x + 5y = 18 \\ -x + 2y = 15 \end{array} \begin{array}{c} \longrightarrow \\ \xrightarrow{\times 4} \end{array} \begin{array}{r} 4x + 5y = 18 \\ \underline{-4x + 8y = 60} \\ 13y = 78 \\ y = 6 \end{array}$$

$$\begin{aligned} 4x + 5y &= 18 \\ 4x + 5(6) &= 18 \\ 4x + 30 &= 18 \\ 4x &= -12 \\ x &= -3 \end{aligned}$$

The solution is $(-3, 6)$.

60.
$$\begin{array}{r} 3x + 7y = 1 \\ 4x + 5y = 23 \end{array} \begin{array}{c} \xrightarrow{\times 5} \\ \xrightarrow{\times (-7)} \end{array} \begin{array}{r} 15x + 35y = 5 \\ \underline{-28x - 35y = -161} \\ -13x = -156 \\ x = 12 \end{array}$$

$$\begin{aligned} 3x + 7y &= 1 \\ 3(12) + 7y &= 1 \\ 36 + 7y &= 1 \\ 7y &= -35 \\ y &= -5 \end{aligned}$$

The solution is $(12, -5)$.

61.
$$\begin{array}{r} 3x + 4y = -1 \\ 2x + 6y = -31 \end{array} \begin{array}{c} \xrightarrow{\times 6} \\ \xrightarrow{\times (-4)} \end{array} \begin{array}{r} 18x + 24y = -6 \\ \underline{-8x - 24y = 124} \\ 10x = 118 \\ x = 11.8 \end{array}$$

$$\begin{aligned} 3x + 4y &= -1 \\ 3(11.8) + 4y &= -1 \\ 35.4 + 4y &= -1 \\ 4y &= -36.4 \\ y &= -9.1 \end{aligned}$$

The solution is $(11.8, -9.1)$.

62.
$$\begin{array}{r} 3x + y = 10 \\ -x + 2y = 20 \end{array} \begin{array}{c} \longrightarrow \\ \xrightarrow{\times 3} \end{array} \begin{array}{r} 3x + y = 10 \\ \underline{-3x + 6y = 60} \\ 7y = 70 \\ y = 10 \end{array}$$

$$\begin{aligned} 3x + y &= 10 \\ 3x + 10 &= 10 \\ 3x &= 0 \\ x &= 0 \end{aligned}$$

The solution is $(0, 10)$.

63.
$$\begin{array}{r} 4x + 5y = 2 \\ -3x + 2y = 33 \end{array} \begin{array}{c} \xrightarrow{\times 3} \\ \xrightarrow{\times 4} \end{array} \begin{array}{r} 12x + 15y = 6 \\ \underline{-12x + 8y = 132} \\ 23y = 138 \\ y = 6 \end{array}$$

$$\begin{aligned} 4x + 5y &= 2 \\ 4x + 5(6) &= 2 \\ 4x + 30 &= 2 \\ 4x &= -28 \\ x &= -7 \end{aligned}$$

The solution is $(-7, 6)$.

64.
$$\begin{array}{r} 2x + 3y = -1 \\ 10x + 7y = -1 \end{array} \begin{array}{c} \xrightarrow{\times (-5)} \\ \longrightarrow \end{array} \begin{array}{r} -10x - 15y = 5 \\ \underline{10x + 7y = -1} \\ -8y = 4 \\ y = -\frac{1}{2} \end{array}$$

$$\begin{aligned} 2x + 3y &= -1 \\ 2x + 3\left(-\frac{1}{2}\right) &= -1 \\ 2x - \frac{3}{2} &= -1 \\ 2x &= \frac{1}{2} \\ x &= \frac{1}{4} \end{aligned}$$

The solution is $\left(\frac{1}{4}, -\frac{1}{2}\right)$.

Quiz 4.8–4.10 (p. 315)

1. $x^2 - 4x + 5 = 0$

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \\ &= \frac{4 \pm \sqrt{-4}}{2} \\ &= 2 \pm i \end{aligned}$$

The solutions are $2 + i$ and $2 - i$.

2. $2x^2 - 8x + 1 = 0$

$$\begin{aligned} x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(1)}}{2(2)} \\ &= \frac{8 \pm \sqrt{56}}{4} = \frac{8 \pm 2\sqrt{14}}{4} = 2 \pm \frac{\sqrt{14}}{2} \end{aligned}$$

The solutions are $x = 2 + \frac{\sqrt{14}}{2} \approx 3.87$ and $x = 2 - \frac{\sqrt{14}}{2} \approx 0.13$.

3. $3x^2 + 5x + 4 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(4)}}{2(3)} = \frac{-5 \pm \sqrt{-23}}{6} = \frac{-5 \pm i\sqrt{23}}{6}$$

The solutions are $\frac{-5 + i\sqrt{23}}{6}$ and $\frac{-5 - i\sqrt{23}}{6}$.

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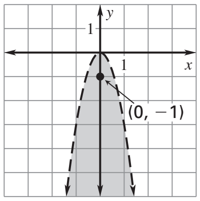
4. $y < -3x^2$

Test $(0, -1)$:

$y < -3x^2$

$-1 \overset{?}{<} -3(-1)^2$

$-1 < -3$ ✓



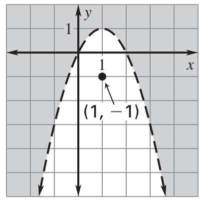
5. $y > -x^2 + 2x$

Test $(1, -1)$:

$y > -x^2 + 2x$

$-1 \overset{?}{>} -1^2 + 2(1)$

$-1 \not> 1$ ✗



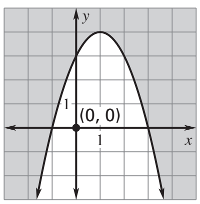
6. $y \geq -x^2 + 2x + 3$

Test $(0, 0)$:

$y \geq -x^2 + 2x + 3$

$0 \overset{?}{\geq} -0^2 + 2(0) + 3$

$0 \not\geq 3$ ✗



7. $0 \geq x^2 + 5$

$x^2 + 5 = 0$

$x^2 = -5$

$x = \pm\sqrt{-5}$

$x = \pm 5i$

No real solutions

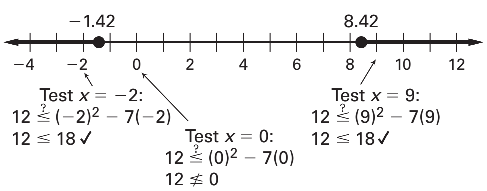
8. $12 \leq x^2 - 7x$

$0 \leq x^2 - 7x - 12$

$x^2 - 7x - 12 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-12)}}{2(1)} = \frac{7 \pm \sqrt{97}}{2}$$

$x \approx 8.42$ or $x \approx -1.42$



The solution is approximately $x \leq -1.42$ or $x \geq 8.42$.

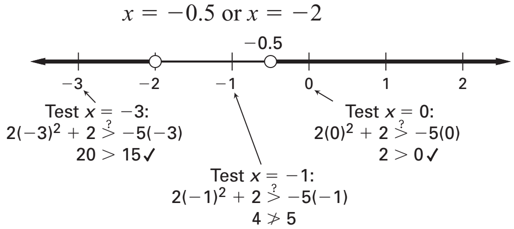
9. $2x^2 + 2 > -5x$

$2x^2 + 5x + 2 > 0$

$2x^2 + 5x + 2 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(2)}}{2(2)} = \frac{-5 \pm \sqrt{9}}{4} = \frac{-5 \pm 3}{4}$$

$x = -0.5$ or $x = -2$



The solution is $x < -2$ or $x > -0.5$.

10.
$$\begin{aligned} y &= a(x-h)^2 + k \\ y &= a(x-5)^2 + 7 \\ 11 &= a(3-5)^2 + 7 \\ 11 &= 4a + 7 \\ 4 &= 4a \\ 1 &= a \end{aligned}$$

A quadratic function is $y = (x-5)^2 + 7$.

11.
$$\begin{aligned} y &= a(x-p)(x-q) \\ y &= a(x+3)(x-5) \\ -40 &= a(7+3)(7-5) \\ -40 &= 20a \\ -2 &= a \end{aligned}$$

A quadratic function is $y = -2(x+3)(x-5)$.

12. $y = ax^2 + bx + c$

$$\begin{aligned} 2 &= a(-1)^2 + b(-1) + c \rightarrow a - b + c = 2 \\ -23 &= a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = -23 \\ -7 &= a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = -7 \end{aligned}$$

$$\begin{array}{lcl} a - b + c = 2 & \xrightarrow{\times 2} & 2a - 2b + 2c = 4 \\ 4a + 2b + c = -7 & \longrightarrow & \underline{4a + 2b + c = -7} \\ & & 6a + 3c = -3 \end{array}$$

$$\begin{array}{lcl} a - b + c = 2 & \xrightarrow{\times 4} & 4a - 4b + 4c = 8 \\ 16a + 4b + c = -23 & \longrightarrow & \underline{16a + 4b + c = -23} \\ & & 20a + 5c = -15 \end{array}$$

$$\begin{array}{lcl} 6a + 3c = -3 & \xrightarrow{\times(-5)} & -30a - 15c = 15 \\ 20a + 5c = -15 & \xrightarrow{\times 3} & \underline{60a + 15c = -45} \\ & & 30a = -30 \\ & & a = -1 \end{array}$$

$$\begin{aligned} 6a + 3c &= -3 \\ 6(-1) + 3c &= -3 \\ -6 + 3c &= -3 \\ 3c &= 3 \\ c &= 1 \\ a - b + c &= 2 \\ -1 - b + 1 &= 2 \\ -b &= 2 \\ b &= -2 \end{aligned}$$

The solution is $a = -1$, $b = -2$, and $c = 1$. A quadratic function is $y = -x^2 - 2x + 1$.

13. $h = -16t^2 + v_o + h_0$

$0 = -16t^2 + 30t + 5$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(5)}}{2(-16)}$$

$$= \frac{-30 \pm \sqrt{1220}}{-32}$$

$t \approx -0.15$ or $t \approx 2.03$

Reject the solution -0.15 because the ball's time in the air cannot be negative. So, the ball is in the air for about 2.03 seconds.

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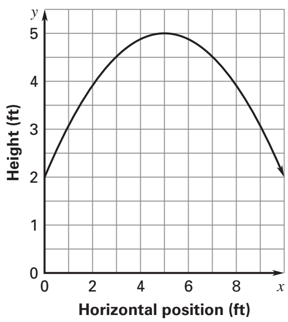
Mixed Review of Problem Solving (p. 316)

1. a.

$$\begin{aligned} y &= -0.12x^2 + 1.2x + 2 \\ y &= -0.12(x^2 - 10x) + 2 \\ y + (-0.12)(25) &= -0.12(x^2 - 10x + 25) + 2 \\ y - 3 &= -0.12(x - 5)^2 + 2 \\ y &= -0.12(x - 5)^2 + 5 \end{aligned}$$

b. Vertex: (5, 5)

$x = 0$: $y = -0.12(0 - 5)^2 + 5 = 2$; (0, 2)



c. Because the y-coordinate of the vertex is 5, the bean bag's maximum height is 5 feet.

2. a. $\begin{matrix}\text{Monthly} \\ \text{revenue} \\ \text{(dollars)}\end{matrix} = \begin{matrix}\text{Price} \\ \text{(dollars/} \\ \text{drum)}\end{matrix} \cdot \begin{matrix}\text{Sales} \\ \text{(drums)}\end{matrix}$

$$R(x) = (120 - 5x) \cdot (50 + 4x)$$

b. $R(x) = (-5x + 120)(4x + 50)$

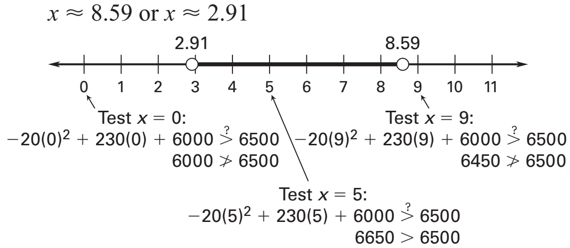
$R(x) = -20x^2 + 230x + 6000 > 6500$

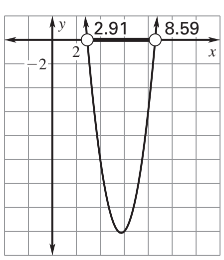
c.

$$\begin{aligned} -20x^2 + 230x + 6000 &> 6500 \\ -20x^2 + 230x - 500 &> 0 \\ 2x^2 - 23x + 50 &< 0 \\ 2x^2 - 23x + 50 &= 0 \end{aligned}$$

$$x = \frac{-(-23) \pm \sqrt{(-23)^2 - 4(2)(50)}}{2(2)} = \frac{23 \pm \sqrt{129}}{4}$$

$x \approx 8.59$ or $x \approx 2.91$





The solution is approximately $2.91 < x < 8.59$.

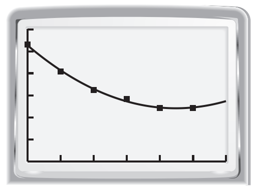
$120 - 5(2.91) = 105.45$

$120 - 5(8.59) = 77.05$

The prices that result in revenues over \$6500 are between \$77.05 and \$105.45.

3. a. The best-fitting quadratic model is $p = 3.55t^2 - 31.85t + 131.89$.

b.



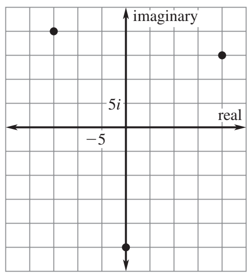
c. No; as the years increase past 2003, the model predicts that the price of a VCR will increase. This is not likely, because the popularity of VCRs will continue to decline as DVD technology becomes more popular and affordable.

4. *Sample answers:*

$-25i$

$-15 + 20i$

$20 + 15i$



5. $(5 - 9i)(5 + 9i) = 25 + 45i - 45i - 81i^2$

$= 25 - 81(-1) = 106$

6. $y = ax^2 + bx + c$

$0 = a(0)^2 + b(0) + c \rightarrow \qquad c = 0$

$3.8 = a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = 3.8$

$0 = a(5)^2 + b(5) + c \rightarrow 25a + 5b + c = 0$

$9a + 3b + 0 = 3.8 \xrightarrow{\times 5} 45a + 15b = 19$

$25a + 5b + 0 = 0 \xrightarrow{\times (-3)} -75a - 15b = 0$

$-30a = 19$

$a = -\frac{19}{30}$

$$\begin{aligned} 9a + 3b &= 3.8 \\ 9\left(-\frac{19}{30}\right) + 3b &= 3.8 \\ -5.7 + 3b &= 3.8 \\ 3b &= 9.5 \\ b &= \frac{19}{6} \end{aligned}$$

The solution is $a = -\frac{19}{30}$, $b = \frac{19}{6}$, and $c = 0$. A quadratic function that models the parabolic cross section of the lamp is $y = -\frac{19}{30}x^2 + \frac{19}{6}x$. You can verify that your model is correct by substituting the x- and y-values from the three points into the equation to see if they check.

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7. a. $h = -16t^2 + V_0 t + h_0$

$h = -16t^2 + 50t + 5$

b. $3 = -16t^2 + 50t + 5$

$0 = -16t^2 + 50t + 2$

$$t = \frac{-50 \pm \sqrt{50^2 - 4(-16)(2)}}{2(-16)} = \frac{-50 \pm \sqrt{2628}}{-32}$$

$t \approx -0.04$ or $t \approx 3.16$

Reject the negative solution. The ball is in the air for about 3.16 seconds.

c. 1. Complete the square to rewrite the function in vertex form.

$$h = -16t^2 + 50t + 5$$

$$h = -16\left(t^2 - \frac{25}{8}t\right) + 5$$

$$h + (-16)\left(\frac{625}{256}\right) = -16\left(t^2 - \frac{25}{8}t + \frac{625}{256}\right) + 5$$

$$h - \frac{625}{16} = -16\left(t - \frac{25}{16}\right)^2 + 5$$

$$h = -16\left(t - \frac{25}{16}\right)^2 + \frac{705}{16}$$

The maximum height of the ball is $\frac{705}{16} = 44.1$ feet.

2. Find the t-coordinate of the vertex and use it to find the h-coordinate of the vertex.

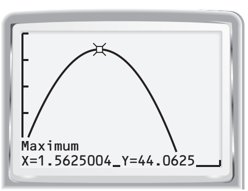
$$t = -\frac{b}{2a} = -\frac{50}{2(-16)} \approx 1.56$$

$h = -16(1.56)^2 + 50(1.56) + 5 \approx 44.1$

The maximum height of the ball is about 44.1 feet.

3. Use the maximum feature of a graphing calculator.

The maximum height of the ball is about 44.1 feet.



8.

Area of paper (square inches)	−	Length of white part (inches)	•	Width of white part (inches)	=	Area of stripes (square inches)
5(8)	−	(8 − x)	•	(5 − x)	=	$\frac{1}{3}$(5 • 8)

$$40 - (8 - x)(5 - x) = \frac{1}{3}(40)$$

$$40 - (8 - x)(5 - x) = \frac{40}{3}$$

$$40 - (40 - 8x - 5x + x^2) = \frac{40}{3}$$

$$x^2 - 13x + \frac{40}{3} = 0$$

$$3x^2 - 39x + 40 = 0$$

$$x = \frac{-(-39) \pm \sqrt{(-39)^2 - 4(3)(40)}}{2(3)} = \frac{39 \pm \sqrt{1041}}{6}$$

$x \approx 11.88$ or $x \approx 1.12$

You must reject the solution 11.88 because 11.88 inches exceeds the dimensions of the paper. The width x of the stripes will be 1.12 inches.

9. $3x^2 + 5x - 2 = 0$

$b^2 - 4ac = 5^2 - 4(3)(-2) = 49$

Chapter 4 Review (pp. 318–322)

1. To determine whether a function has a maximum value or a minimum value, look at the coefficient a of the x^2 term. If $a < 0$, the function has a maximum value. If $a > 0$, the function has a minimum value.

2. A pure imaginary number is a complex number $a + bi$ where $a = 0$ and $b \neq 0$.

3. A function of the form $y = a(x - h)^2 + k$ is written in vertex form.

4. *Sample answer:* $y = 4x^2 - 2x + 7$

$b^2 - 4ac = (-2)^2 - 4(4)(7) = -108$

5. $y = x^2 + 2x - 3$

$$x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$$

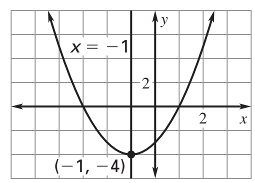
$y = (-1)^2 + 2(-1) - 3 = -4$

Vertex: $(-1, -4)$

Axis of symmetry : $x = -1$

$x = 1$:

$y = 1^2 + 2(1) - 3 = 0$; $(1, 0)$



6. $y = -3x^2 + 12x - 7$

$$x = -\frac{b}{2a} = \frac{-12}{2(-3)} = 2$$

$y = -3(2)^2 + 12(2) - 7 = 5$

Vertex: $(2, 5)$

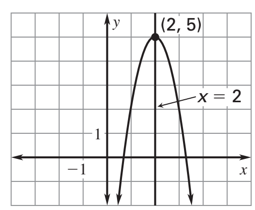
Axis of symmetry: $x = 2$

y-intercept: -7; $(0, -7)$

$x = 1$:

$y = -3(1)^2 + 12(1) - 7$

$= 2$; $(1, 2)$



7. $f(x) = -x^2 - 2x - 6$

$$x = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$$

$f(-1) = -(-1)^2 - 2(-1) - 6 = -5$

Vertex: $(-1, -5)$

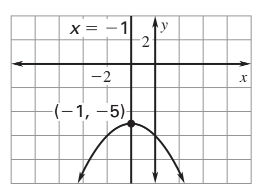
Axis of symmetry: $x = -1$

y-intercept: -6; $(0, -6)$

$x = 1$:

$f(1) = -1^2 - 2(1) - 6$

$= -9$; $(1, -9)$



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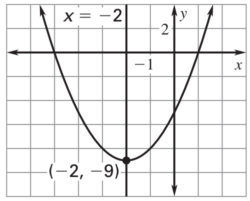
8. $y = (x - 1)(x + 5)$



x-intercepts: $p = 1$ and $q = -5$

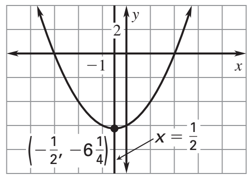
$x = \frac{p + q}{2} = \frac{1 + (-5)}{2} = -2$

$y = (-2, -1)(-2 + 5) = -9$

Vertex: $(-2, -9)$

Axis of symmetry: $x = -2$

9. $g(x) = (x + 3)(x - 2)$



x-intercepts: $p = -3$ and $q = 2$

$x = \frac{p + q}{2} = \frac{-3 + 2}{2} = -\frac{1}{2}$

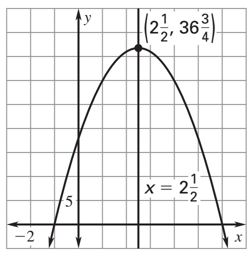
$g\left(-\frac{1}{2}\right) = \left(-\frac{1}{2} + 3\right)\left(-\frac{1}{2} - 2\right)$

$= -\frac{25}{4}$

Vertex: $\left(-\frac{1}{2}, -\frac{25}{4}\right)$

Axis of symmetry: $x = -\frac{1}{2}$

10. $y = -3(x + 1)(x - 6)$



x-intercepts: $p = -1$ and $q = 6$

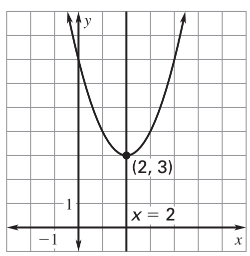
$x = \frac{p + q}{2} = \frac{-1 + 6}{2} = \frac{5}{2}$

$y = -3\left(\frac{5}{2} + 1\right)\left(\frac{5}{2} - 6\right) = \frac{147}{4}$

Vertex: $\left(\frac{5}{2}, \frac{147}{4}\right)$

Axis of symmetry: $x = \frac{5}{2}$

11. $y = (x - 2)^2 + 3$



Vertex: $(2, 3)$

Axis of symmetry: $x = 2$

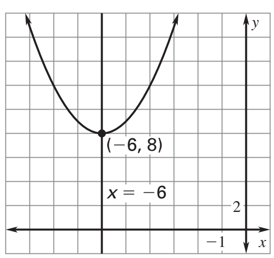
$x = 0$:

$y = (0 - 2)^2 + 3 = 7; (0, 7)$

$x = 1$:

$y = (1 - 2)^2 + 3 = 4; (1, 4)$

12. $f(x) = (x + 6)^2 + 8$



Vertex: $(-6, 8)$

Axis of symmetry: $x = -6$

$x = -2$:

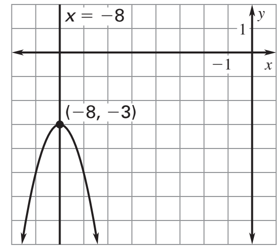
$f(-2) = (-2 + 6)^2 + 8$

$= 24; (-2, 24)$

$x = -4$:

$f(-4) = (-4 + 6)^2 + 8 = 12; (-4, 12)$

13. $y = -2(x + 8)^2 - 3$



Vertex: $(-8, -3)$

Axis of symmetry: $x = -8$

$x = -6$:

$y = -2(-6 + 8)^2 - 3$

$= -11; (-6, -11)$

$x = -5$:

$y = -2(-5 + 8)^2 - 3 = -21; (-5, -21)$

14. $y = -0.073x(x - 33)$

$y = -0.073(x - 0)(x - 33)$

Because the x-intercepts are 0 and 33, the flea jumped a distance of 33 centimeters.

$x = \frac{p + q}{2} = \frac{0 + 33}{2} = 16.5$

$y = -0.073(16.5)(16.5 - 33) \approx 19.9$

The flea's maximum height was about 19.9 centimeters.

15. $x^2 + 5x = 0$

$x(x + 5) = 0$

$x = 0$ or $x + 5 = 0$

$x = 0$ or $x = -5$

16. $z^2 = 63z$

$z^2 - 63z = 0$

$z(z - 63) = 0$

$z = 0$ or $z - 63 = 0$

$z = 0$ or $z = 63$

17. $s^2 - 6s - 27 = 0$

$(s + 3)(s - 9) = 0$

$s + 3 = 0$ or $s - 9 = 0$

$s = -3$ or $s = 9$

18. $k^2 + 12k - 45 = 0$

$(k + 15)(k - 3) = 0$

$k + 15 = 0$ or $k - 3 = 0$

$k = -15$ or $k = 3$

19. $x^2 + 18x = -81$

$x^2 + 18x + 81 = 0$

$(x + 9)^2 = 0$

$x + 9 = 0$

$x = -9$

20. $n^2 + 5n = 24$

$n^2 + 5n - 24 = 0$

$(n + 8)(n - 3) = 0$

$n + 8 = 0$ or $n - 3 = 0$

$n = -8$ or $n = 3$

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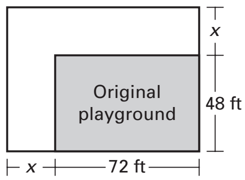
21.



$$\underset{\text{(square feet)}}{\text{New area}} = \underset{\text{(feet)}}{\text{New length}} \cdot \underset{\text{(feet)}}{\text{New width}}$$

$$2(72)(48) = (72 + x) \cdot (48 + x)$$
$$6912 = 3456 + 120x + x^2$$
$$0 = x^2 + 120x - 3456$$
$$0 = (x + 144)(x - 24)$$
$$x + 144 = 0 \quad \text{or} \quad x - 24 = 0$$
$$x = -144 \quad \text{or} \quad x = 24$$

Reject the negative value, -144. The playground's length and width should each be increased by 24 feet.

22. $16 = 38r - 12r^2$

$$12r^2 - 38r + 16 = 0$$
$$6r^2 - 19r + 8 = 0$$
$$(3r - 8)(2r - 1) = 0$$
$$3r - 8 = 0 \quad \text{or} \quad 2r - 1 = 0$$
$$r = \frac{8}{3} \quad \text{or} \quad r = \frac{1}{2}$$

23. $3x^2 - 24x - 48 = 0$

$$x^2 - 8x - 16 = 0$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-16)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{128}}{2}$$
$$= \frac{8 \pm 8\sqrt{2}}{2}$$
$$= 4 \pm 4\sqrt{2}$$
$$x = 4 - \sqrt{2} \quad \text{or} \quad x = 4 + \sqrt{2}$$

24. $20a^2 - 13a - 21 = 0$

$$(4a + 3)(5a - 7) = 0$$
$$4a + 3 = 0 \quad \text{or} \quad 5a - 7 = 0$$
$$a = -\frac{3}{4} \quad \text{or} \quad a = \frac{7}{5}$$

25. $3x^2 = 108$

$$x^2 = 36$$
$$x = \pm\sqrt{36}$$
$$x = \pm 6$$

26. $5y^2 + 4 = 14$

$$5y^2 = 10$$
$$y^2 = 2$$
$$y = \pm\sqrt{2}$$

27. $3(p + 1)^2 = 81$

$$(p + 1)^3 = 27$$
$$p + 1 = \pm\sqrt{27}$$
$$p + 1 = \pm 3\sqrt{3}$$
$$p = -1 \pm 3\sqrt{3}$$

28. $s = 4\pi r^2$

$$510{,}000{,}000 = 4\pi r^2$$
$$\frac{127{,}500{,}000}{\pi} = r^2$$
$$\pm\frac{\sqrt{127{,}500{,}000}}{\pi} = r$$
$$\pm 6370.6 = r$$

The radius of Earth is about 6370.6 kilometers.

29. $-9i(2 - i) = -18i + 9i^2 = -18i + 9(-1) = -9 - 18i$

30. $(5 + i)(4 - 2i) = 20 - 10i + 4i - 2i^2$

$$= 20 - 6i - 2(-1) = 22 - 6i$$

31. $(2 - 5i)(2 + 5i) = 4 + 10i - 10i - 25i^2 = 4 - 25(-1)$

$$= 4 + 25 = 29$$

32. $(8 - 6i) + (7 + 4i) = (8 + 7) + (-6 + 4)i = 15 - 2i$

33. $(2 - 3i) - (6 - 5i) = (2 - 6) + (-3 + 5)i = -4 + 2i$

34. $\dfrac{4i}{-3 + 6i} = \dfrac{4i}{-3 + 6i} \cdot \dfrac{-3 - 6i}{-3 - 6i} = \dfrac{-12i - 24i^2}{9 + 18i - 18i - 36i^2}$

$$= \frac{-12i - 24(-1)}{9 - 36(-1)} = \frac{24 - 12i}{45} = \frac{8}{15} - \frac{4}{15}i$$

35. $x^2 - 6x - 15 = 0$

$$x^2 - 6x = 15$$
$$x^2 - 6x + 9 = 15 + 9$$
$$(x - 3)^2 = 24$$
$$x - 3 = \pm\sqrt{24}$$
$$x = 3 \pm 2\sqrt{6}$$

36. $3x^2 - 12x + 1 = 0$

$$3x^2 - 12x = -1$$
$$3(x^2 - 4x) = -1$$
$$3(x^2 - 4x + 4) = -1 + 3(4)$$
$$3(x - 2)^2 = 11$$
$$(x - 2)^2 = \frac{11}{3}$$
$$x - 2 = \pm\sqrt{\frac{11}{3}}$$
$$x = 2 \pm \sqrt{\frac{11}{3}}$$
$$x = 2 \pm \frac{\sqrt{33}}{3}$$

37. $x^2 + 3x - 1 = 0$

$$x^2 + 3x = 1$$
$$x^2 + 3x + \frac{9}{4} = 1 + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = \frac{13}{4}$$
$$x + \frac{3}{2} = \pm\sqrt{\frac{13}{4}}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

38. $x^2 + 4x - 3 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{28}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$$

The solutions are $x = -2 + \sqrt{7} \approx 0.65$ and $x = -2 - \sqrt{7} \approx -4.65.0$

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39. $9x^2 = -6x - 1$

$9x^2 + 6x + 1 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(9)(1)}}{2(9)}$$

$$x = \frac{-6 \pm \sqrt{0}}{18} = \frac{-6}{18} = -\frac{1}{3}$$

The solution is $-\frac{1}{3}$.

40. $6x^2 - 8x = -3$

$6x^2 - 8x + 3 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(6)(3)}}{2(6)}$$

$$x = \frac{8 \pm \sqrt{-8}}{12} = \frac{8 \pm 2\sqrt{2}i}{12} = \frac{4 \pm i\sqrt{2}}{6}$$

The solutions are $\frac{4 + i\sqrt{2}}{6}$ and $\frac{4 - i\sqrt{2}}{6}$.

41. $h = -16t^2 + v_0 t + h_0$

$0 = -16t^2 - 40t + 9$

$$t = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(-16)(9)}}{2(-16)} = \frac{40 \pm \sqrt{2176}}{-32}$$

$t \approx -2.71$ or $t \approx 0.21$

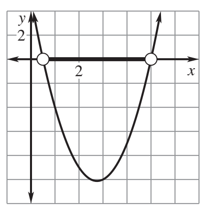
Reject the negative solution. The ball is in the air for about 0.21 second after it is spiked.

42. $2x^2 - 11x + 5 < 0$

$2x^2 - 11x + 5 = 0$

$(2x - 1)(x - 5) = 0$

$x = \frac{1}{2}$ or $x = 5$



The solution of the inequality is $\frac{1}{2} < x < 5$.

43. $-x^2 + 4x + 3 \geq 0$

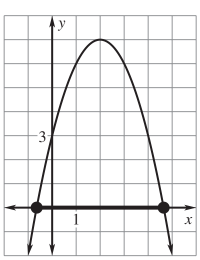
$-x^2 + 4x + 3 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(3)}}{2(-1)}$$

$$x = \frac{-4 \pm \sqrt{28}}{-2} = \frac{-4 \pm 2\sqrt{7}}{-2}$$

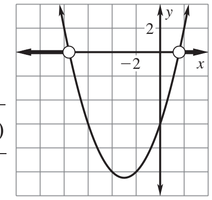
$$= 2 \pm \sqrt{7}$$

$x \approx 4.65$ or $x \approx -0.65$



The solution of the inequality is approximately $-0.65 \leq x \leq 4.65$.

44. $\frac{1}{2}x^2 + 3x - 6 > 0$



$\frac{1}{2}x^2 + 3x - 6 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4\left(\frac{1}{2}\right)(-6)}}{2\left(\frac{1}{2}\right)}$$

$x = -3 \pm \sqrt{21}$

$x \approx 1.58$ or $x \approx -7.58$

The solution of the inequality is approximately $x < -7.58$ or $x > 1.58$.

45. $y = a(x - p)(x - q)$

$y = a(x + 3)(x - 2)$

$12 = a(3 + 3)(3 - 2)$

$12 = 6a$

$2 = a$

A quadratic function is $y = 2(x + 3)(x - 2)$.

46. $y = ax^2 + bx + c$

$2 = a(5)^2 + b(5) + c \rightarrow 25a + 5b + c = 2$

$2 = a(0)^2 + b(0) + c \rightarrow \quad c = 2$

$-6 = a(8)^2 + b(8) + c \rightarrow 64a + 8b + c = -6$

$25a + 5b + c = 2$

$25a + 5b + 2 = 2$

$25a + 5b = 0$

$64a + 8b + c = -6$

$64a + 8b + 2 = -6$

$64a + 8b = -8$

$25a + 5b = 0 \xrightarrow{\times 8} 200a + 40b = 0$

$64a + 8b = -8 \xrightarrow{\times (-5)} -320a - 40b = 40$

$-120a = 40$

$a = -\frac{1}{3}$

$25a + 5b = 0$

$25\left(-\frac{1}{3}\right) + 5b = 0$

$-\frac{25}{3} + 5b = 0$

$5b = \frac{25}{3}$

$b = \frac{5}{3}$

The solution is $a = -\frac{1}{3}$, $b = \frac{5}{3}$, and $c = 2$. A quadratic function is $y = -\frac{1}{3}x^2 + \frac{5}{3}x + 2$.

47. $y = a(x - h)^2 + k$

$y = a(x - 2)^2 + 7$

$2 = a(4 - 2)^2 + 7$

$2 = 4a + 7$

$-5 = 4a$

$-\frac{5}{4} = a$

A quadratic function is $y = -\frac{5}{4}(x - 2)^2 + 7$.

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48. $y = a(x - h)^2 + k$

$y = a(x - 12)^2 + 7$

$0 = a(0 - 12)^2 + 7$

$0 = 144a + 7$

$-7 = 144a$

$-\frac{7}{144} = a$

A quadratic function that models the soccer ball's path is $y = -\frac{7}{144}(x - 12)^2 + 7.$

Chapter 4 Test (p. 323)

1. $y = x^2 - 8x - 20$

$x = -\frac{b}{2a} = -\frac{(-8)}{2(1)} = 4$

$y = 4^2 - 8(4) - 20 = -36$

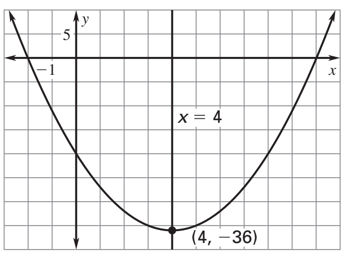
Vertex: $(4, -36)$

Axis of symmetry: $x = 4$

y-intercept: -20; $(0, -20)$

$x = -2$:

$y = (-2)^2 - 8(-2) - 20 = 0$; $(-2, 0)$



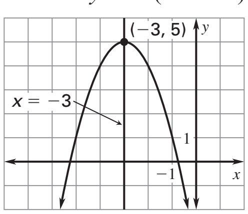
2. $y = -(x + 3)^2 + 5$

Vertex: $(-3, 5)$

Axis of symmetry: $x = -3$

$x = -2$: $y = -(-2 + 3)^2 + 5 = 4$; $(-2, 4)$

$x = -1$: $y = -(-1 + 3)^2 + 5 = 1$; $(-1, 1)$



3. $f(x) = 2(x + 4)(x - 2)$

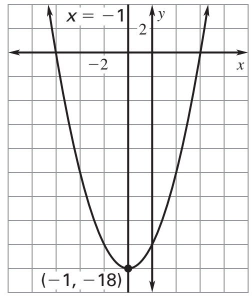
x-intercepts: $p = -4$ and $q = 2$

$x = \frac{p + q}{2} = \frac{-4 + 2}{2} = -1$

$f(-1) = 2(-1 + 4)(-1 - 2) = -18$

Vertex: $(-1, -18)$

Axis of symmetry: $x = -1$



4. $x^2 - 11x + 30 = (x - 6)(x - 5)$

5. $z^2 + 2z - 15 = (z + 5)(z - 3)$

6. $n^2 - 64 = n^2 - 8^2 = (n + 8)(n - 8)$

7. $2s^2 + 7s - 15 = (2s - 3)(s + 5)$

8. $9x^2 + 30x + 25 = 3x^2 + 2(3x)(5) + 5^2 = (3x + 5)^2$

9. $6t^2 + 23t + 20 = (3t + 4)(2t + 5)$

10. $x^2 - 3x - 40 = 0$

$(x + 5)(x - 8) = 0$

$x + 5 = 0$ or $x - 8 = 0$

$x = -5$ or $x = 8$

11. $r^2 - 13r + 42 = 0$

$(r - 7)(r - 6) = 0$

$r - 7 = 0$ or $r - 6 = 0$

$r = 7$ or $r = 6$

12. $2w^2 + 13w - 7 = 0$

$(2w - 1)(w + 7) = 0$

$2w - 1 = 0$ or $w + 7 = 0$

$w = \frac{1}{2}$ or $w = -7$

13. $10y^2 + 11y - 6 = 0$

$(5y - 2)(2y + 3) = 0$

$5y - 2 = 0$ or $2y + 3 = 0$

$y = \frac{2}{5}$ or $y = -\frac{3}{2}$

14. $2(m - 7)^2 = 16$

$(m - 7)^2 = 8$

$m - 7 = \pm\sqrt{8}$

$m = 7 \pm 2\sqrt{2}$

15. $(x + 2)^2 - 12 = 36$

$(x + 2)^2 = 48$

$x + 2 = \pm\sqrt{48}$

$x = -2 \pm 4\sqrt{3}$

16. $(3 + 4i) - (2 - 5i) = (3 - 2) + (4 + 5)i = 1 + 9i$

17. $(2 - 7i)(1 + 2i) = 2 + 4i - 7i - 14i^2$

$= 2 - 3i - 14(-1) = 16 - 3i$

18. $\frac{3 + i}{2 - 3i} = \frac{3 + i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} = \frac{6 + 9i + 2i + 3i^2}{4 + 6i - 6i - 9i^2}$

$= \frac{6 + 11i + 3(-1)}{4 - 9(-1)} = \frac{3 + 11i}{13} = \frac{3}{13} + \frac{11}{13}i$

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19. $x^2 + 4x - 14 = 0$

$$x^2 + 4x = 14$$
$$x^2 + 4x + 4 = 14 + 4$$
$$(x + 2)^2 = 18$$
$$x + 2 = \pm\sqrt{18}$$
$$x + 2 = \pm 3\sqrt{2}$$
$$x = -2 \pm 3\sqrt{2}$$

The solutions are $-2 + 3\sqrt{2}$ and $-2 - 3\sqrt{2}$.

20. $x^2 - 10x - 7 = 0$

$$x^2 - 10x = 7$$
$$x^2 - 10x + 25 = 7 + 25$$
$$(x - 5)^2 = 32$$
$$x - 5 = \pm\sqrt{32}$$
$$x - 5 = \pm 4\sqrt{2}$$
$$x = 5 \pm 4\sqrt{2}$$

The solutions are $5 + 4\sqrt{2}$ and $5 - 4\sqrt{2}$.

21. $4x^2 + 8x + 3 = 0$

$$4x^2 + 8x = -3$$
$$4(x^2 + 2x) = -3$$
$$4(x^2 + 2x + 1) = -3 + (4)(1)$$
$$4(x + 1)^2 = 1$$
$$(x + 1)^2 = \frac{1}{4}$$
$$x + 1 = \pm\sqrt{\frac{1}{4}}$$
$$x + 1 = \pm\frac{1}{2}$$
$$x = -1 \pm \frac{1}{2}$$

The solutions are $-\frac{1}{2}$ and $-\frac{3}{2}$.

22. $3x^2 + 10x - 5 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-5)}}{2(3)}$$
$$x = \frac{-10 \pm \sqrt{160}}{6} = \frac{-10 \pm 4\sqrt{10}}{6} = \frac{-5 \pm 2\sqrt{10}}{3}$$

The solutions are $x = \frac{-5 + 2\sqrt{10}}{3} \approx 0.44$ and

$$x = \frac{-5 - 2\sqrt{10}}{3} \approx -3.77.$$

23. $2x^2 - x + 6 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(6)}}{2(2)} = \frac{1 \pm \sqrt{-47}}{4} = \frac{1 \pm i\sqrt{47}}{4}$$

The solutions are $\frac{1 + i\sqrt{47}}{4}$ and $\frac{1 - i\sqrt{47}}{4}$.

24. $5x^2 + 2x + 5 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(5)(5)}}{2(5)} = \frac{-2 \pm \sqrt{-96}}{10} = \frac{-2 \pm 4i\sqrt{6}}{10}$$
$$x = \frac{-1 \pm 2i\sqrt{6}}{5}$$

The solutions are $\frac{-1 \pm 2i\sqrt{6}}{5}$ and $\frac{-1 - 2i\sqrt{6}}{5}$.

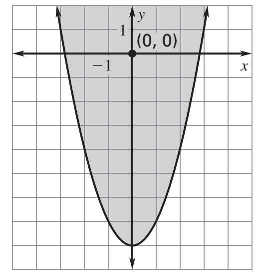
25. $y \geq x^2 - 8$

Test (0, 0).

$y \geq x^2 - 8$

$0 \overset{?}{\geq} 0^2 - 8$

$0 \geq -8$ ✓



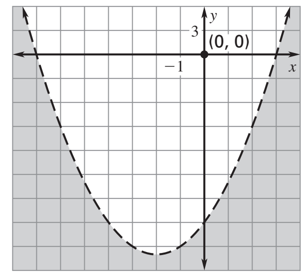
26. $y < x^2 + 4x - 21$

Test (0, 0).

$y < x^2 + 4x - 21$

$0 \overset{?}{<} 0^2 + 4(0) - 21$

$0 \not< -21$ ✗



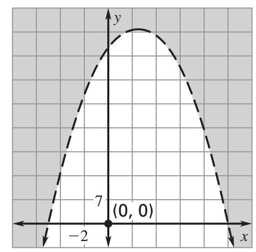
27. $y > -x^2 + 5x + 50$

Test (0, 0).

$y > -x^2 + 5x + 50$

$0 \overset{?}{>} -0^2 + 5(0) + 50$

$0 \not> 50$ ✗



28.
$$y = a(x - p)(x - q)$$
$$y = a(x + 7)(x + 3)$$
$$12 = a(-1 + 7)(-1 + 3)$$
$$12 = 12a$$
$$1 = a$$

A quadratic function is $y = (x + 7)(x + 3)$.

29.
$$y = a(x - h)^2 + k$$
$$y = a(x + 3)^2 - 2$$
$$-10 = a(1 + 3)^2 - 2$$
$$-10 = 16a - 2$$
$$-8 = 16a$$
$$-\frac{1}{2} = a$$

A quadratic function is $y = -\frac{1}{2}(x + 3)^2 - 2$.

30. $y = ax^2 + bx + c$

$8 = a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = 8$

$-4 = a(7)^2 + b(7) + c \rightarrow 49a + 7b + c = -4$

$0 = a(8)^2 + b(8) + c \rightarrow 64a + 8b + c = 0$

$16a + 4b + c = 8 \xrightarrow{\times(-2)} -32a - 8b - 2c = -16$

$64a + 8b + c = 0 \longrightarrow \underline{64a + 8b + c = 0}$

$32a - c = -16$

$16a + 4b + c = 8 \xrightarrow{\times(-7)} -112a - 28b - 7c = -56$

$49a + 7b + c = -4 \xrightarrow{\times 4} \underline{196a + 28b + 4c = -16}$

$84a - 3c = -72$

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$$\begin{aligned}32a - c &= -16 \xrightarrow{\times(-3)} & -96a + 3c &= 48\\ 84a - 3c &= -72 \longrightarrow & 84a - 3c &= -72\\ & & -12a &= -24\\ & & a &= 2\end{aligned}$$

$$\begin{aligned}32a - c &= -16\\ 32(2) - c &= -16\\ 64 - c &= -16\\ -c &= -80\\ c &= 80\end{aligned}$$

$$\begin{aligned}16a + 4b + c &= 8\\ 16(2) + 4b + 80 &= 8\\ 32 + 4b + 80 &= 8\\ 112 + 4b &= 8\\ 4b &= -104\\ b &= -26\end{aligned}$$

The solution is $a = 2$, $b = -26$, and $c = 80$. A quadratic function is $y = 2x^2 - 26x + 80$.

31. $(16x)^2 + (9x)^2 = 32^2$

$$\begin{aligned}256x^2 + 81x^2 &= 1024\\ 337x^2 &= 1024\\ x^2 &= \frac{1024}{337}\\ x &= \pm\sqrt{\frac{1024}{337}}\\ x &\approx \pm 1.74\end{aligned}$$

Reject the negative solution. The widescreen TV has a width of $(16)(1.74) = 27.84$ inches and a height of $(9)(1.74) = 15.66$ inches.

32. The best-fitting quadratic model is $s = 0.0008m^2 - 0.048m + 1.12$.

Standardized Test Preparation (p. 325)

1. C; $h = -16t^2 + 47t + 3$

$$t = -\frac{b}{2a} = \frac{-47}{2(-16)} = \frac{47}{32}$$

$$h = -16\left(\frac{47}{32}\right)^2 + 47\left(\frac{47}{32}\right) + 3 = \frac{2401}{64} \approx 37.5$$

The maximum height of the tennis ball is about 37.5 feet.

2. C; The x-coordinate of the vertex of the graph represents the number of seconds it takes the ball to reach its maximum height.

Standardized Test Practice (pp. 326–327)

1. B; The statement "The y-intercept is -2" is not true.

2. B; $y = a(x - p)(x - q)$

$$\begin{aligned}y &= a(x + 2)(x - 4)\\ 3 &= a(1 + 2)(1 - 4)\\ 3 &= -9a\\ -\frac{1}{3} &= a\\ y &= -\frac{1}{3}(x + 2)(x - 4)\end{aligned}$$

3. B;

$$\underset{\text{(square inches)}}{\text{Area of border}} = \underset{\text{(square inches)}}{\text{Area of photograph and border}} - \underset{\text{(square inches)}}{\text{Area of photograph}}$$

$$\begin{aligned}300 &= (10 + 2x)(10 + 2x) - (10)(10)\\ 300 &= 100 + 40x + 4x^2 - 100\\ 0 &= 4x^2 + 40x - 300\\ 0 &= x^2 + 10x - 75\\ 0 &= (x + 15)(x - 5)\end{aligned}$$

$x + 15 = 0$ or $x - 5 = 0$

$x = -15$ or $x = 5$

Reject the negative value, -15. The largest possible frame width is 5 inches.

4. C;

$$\underset{\text{(dollars)}}{\text{Revenue}} = \underset{\text{(dollars/pair)}}{\text{Price}} \cdot \underset{\text{(pairs)}}{\text{Sales}}$$

$$\begin{aligned}R(x) &= (15 - x) \cdot (50 + 10x)\\ R(x) &= -(x - 15)(10x + 50)\\ R(x) &= -10(x - 15)(x + 5)\end{aligned}$$

$$x = \frac{p + q}{2} = \frac{15 - 5}{2} = 5$$

The artist should charge $15 - x = 15 - 5 = \$10$ to maximize revenue.

5. B; x-intercepts: $p = -3$ and $q = 2$

$$\begin{aligned}y &= a(x - p)(x - q)\\ y &= a(x + 3)(x - 2)\\ -6 &= a(0 + 3)(0 - 2)\\ -6 &= -6a\\ 1 &= a\\ y &= (x + 3)(x - 2)\\ y &= x^2 + x - 6\end{aligned}$$

Test (0, 0).

$y \geq x^2 + x - 6$

$0 \overset{?}{\geq} 0^2 + 0 - 6$

$0 \geq -6$ ✓

6. B; $h = -16t^2 + v_0t + h_0$

$$0 = -16t^2 + 30t + 2$$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(2)}}{2(-16)} = \frac{-30 \pm \sqrt{1028}}{-32}$$

$t \approx -0.1$ or $t \approx 1.9$

Reject the negative solution. The horseshoe is in the air about 1.9 seconds.

7. B; $h = -16t^2 + 30t + 2$

$$t = -\frac{b}{2a} = -\frac{30}{2(-16)} \approx \frac{15}{16}$$

$$h = -16\left(\frac{15}{16}\right)^2 + 30\left(\frac{15}{16}\right) + 2 \approx 16.1$$

The maximum height of the horseshoe is about 16.1 feet.

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8. D; Area of square: $A = (2r)(2r) = 4r^2$

Area of circle: $A = \pi r^2$

$$\text{Area of square} - \text{Area of circle} = \text{Area of shaded region}$$

$$4r^2 - \pi r^2 = 21.5$$

$$r^2(4 - \pi) = 21.5$$

$$r^2 = \frac{21.5}{4 - \pi}$$

$$r = \pm\sqrt{\frac{21.5}{4 - \pi}} \approx \pm 5$$

Reject the negative solution. A side of the square is $(2)(5) = 10$ inches.

9. $6x^2 - 11x - 10 = (3x + 2)(2x - k)$

$$= 6x^2 - 3kx + 4x - 2k$$

$$= 6x^2 - (3k - 4)x - 2k$$

$2k = 10 \qquad 3k - 4 = 11$

$k = 5 \qquad 3k = 15$

$k = 5$

10. $(5 + i)(10 - i) = 50 - 5i + 10i - i^2$

$$= 50 + 5i - (-1)$$

$$= 51 + 5i$$

The real part is 51.

11. $x^2 - 7x + c$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$(x)^2 - 2(x)\left(\frac{7}{2}\right) + \left(\frac{7}{2}\right)^2 = \left(x - \frac{7}{2}\right)^2$$

$$x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$$

The expression is a perfect square trinomial when $c = \frac{49}{4}$.

12. $y = -3(x - 2)^2 + 6$

Vertex: (2, 6)

The maximum value of the function is 6.

13. $y = x^2 - 25x + 66$

$0 = x^2 - 25x + 66$

$0 = (x - 22)(x - 3)$

$x - 22 = 0$ or $x - 3 = 0$

$x = 22$ or $x = 3$

The greatest zero of the function is 22.

14. $|-5 + 12i| = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$

15. $y = ax^2 + bx + c$

$-22 = a(0)^2 + b(0) + c \rightarrow \qquad c = -22$

$-6 = a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = -6$

$-12 = a(5)^2 + b(5) + c \rightarrow 25a + 5b + c = -12$

$4a + 2b + c = -6$

$4a + 2b - 22 = -6$

$4a + 2b = 16$

$25a + 5b + c = -12$

$25a + 5b - 22 = -12$

$25a + 5b = 10$

$4a + 2b = 16 \xrightarrow{\times 5} 20a + 10b = 80$

$25a + 5b = 10 \xrightarrow{\times(-2)} -50a - 10b = -20$

$-30a = 60$

$a = -2$

$4a + 2b = 16$

$4(-2) + 2b = 16$

$-8 + 2b = 16$

$2b = 24$

$b = 12$

A quadratic equation for the parabola is $y = -2x^2 + 12x - 22$.

$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = 3$$

The x-coordinate of the vertex is 3.

16. $f(x) = 4x^2 + 24x + 39$

$$x = -\frac{b}{2a} = -\frac{24}{2(4)} = -3$$

$f(-3) = 4(-3)^2 + 24(-3) + 39 = 3$

The minimum value of the function is 3.

17. $3x^2 + 6x + 14 = 0$ Write original equation

$3(x^2 + 2x) + 14 = 0$ Factor 3 from first two terms.

$3(x^2 + 2x) = -14$ Subtract 14 from each side.

$3(x^2 + 2x + 1) = -14 + 3(1)$

Add 3(1) to each side.

$3(x + 1)^2 = -11$ Write $x^2 + 2x + 1$ as a binomial squared.

$(x + 1)^2 = -\frac{11}{3}$ Divide each side by 3.

$x + 1 = \pm\sqrt{-\frac{11}{3}}$

Take square roots of each side.

$x = -1 \pm \sqrt{-\frac{11}{3}}$

Solve for x.

$x = -1 \pm i\frac{\sqrt{33}}{3}$

Write in terms of the imaginary unit i.

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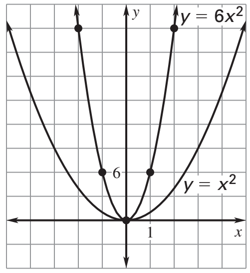
18. Both graphs open up and have the same vertex and axis of symmetry. The graph of $y = 6x^2$ is narrower than the graph of $y = x^2$.

$y = 6x^2$

x	-2	-1	0	1	2
y	24	6	0	6	24



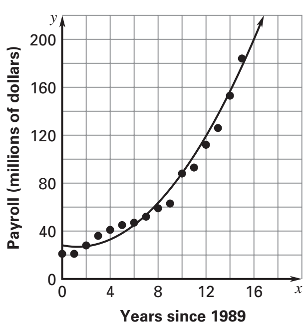
19. For an object that is launched or thrown, an extra term v_0t must be added to the function for a dropped object, to account for the object's initial vertical velocity.

20. The graphs of $y = 2x^2 - 5x - 12$ and $y = \frac{1}{2}x^2 - 3x + 4$ intersect at the points that make the two expressions for y equal.

$$2x^2 - 5x - 12 = \frac{1}{2}x^2 - 3x + 4$$
$$4x^2 - 10x - 24 = x^2 - 6x + 8$$
$$3x^2 - 4x - 32 = 0$$
$$(3x + 8)(x - 4) = 0$$
$$3x + 8 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -\frac{8}{3} \quad \text{or} \quad x = 4$$
$$x = -\frac{8}{3}\text{: } y = 2\left(-\frac{8}{3}\right) - 5\left(-\frac{8}{3}\right) - 12 = \frac{140}{9}$$
$$x = 4\text{: } y = 2(4)^2 - 5(4) - 12 = 0$$

The graphs intersect at $\left(-\frac{8}{3}, \frac{140}{9}\right)$ and $(4, 0)$.

21. a–b.



c. Points chosen will vary: (0, 21), (6, 47), (15, 184)

$$y = ax^2 + bx + c$$
$$21 = a(0)^2 + b(0) + c \rightarrow \quad c = 21$$
$$47 = a(6)^2 + b(6) + c \rightarrow 36a + 6b + c = 47$$
$$184 = a(15)^2 + b(15) + c \rightarrow 225a + 15b + c = 184$$
$$36a + 6b + c = 47$$
$$36a + 6b + 21 = 47$$
$$36a + 6b = 26$$
$$225a + 15b + c = 184$$
$$225a + 15b + 21 = 184$$
$$225a + 15b = 163$$

$$36a + 6b = 26 \xrightarrow{\times 15} 540a + 90b = 390$$
$$225a + 15b = 163 \xrightarrow{\times(-6)} -1350a - 90b = -978$$
$$-810a = -588$$
$$a \approx 0.726$$

$$36a + 6b = 26$$
$$36(0.726) + 6b \approx 26$$
$$26.136 + 6b \approx 26$$
$$6b \approx -0.136$$
$$b \approx -0.023$$

A quadratic model for the data is $y = 0.726x^2 - 0.023x + 21$.

d.

Years since 1989	0	1	2	3	4	5	6	7
Payroll	21	22	24	27	33	39	47	56
Years since 1989	8	9	10	11	12	13	14	15
Payroll	67	80	93	109	125	143	163	184

Some of the numbers given by this model are smaller than the actual data values, and some are larger. Overall, this model seems to generally follow the trend of the data.

22. $h = -16t^2 + 30t + 6$

a. $t = -\frac{b}{2a} = -\frac{30}{2(-16)} \approx 0.94$

$h = -16(0.94)^2 + 30(0.94) + 6 \approx 20.1$

The maximum height of the volleyball is about 20.1 feet. This was found by first finding the time when the volleyball was at its maximum height and then substituting this time into the function to find the corresponding height.

b. After about 0.94 second, the volleyball reaches its maximum height.

c. $0 = -16t^2 + 30t + 6$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(6)}}{2(-16)} = \frac{-30 \pm \sqrt{1284}}{-32}$$

$t \approx -0.18$ or $t \approx 2.06$

Reject the negative solution. After about 2.06 seconds, the volleyball hits the ground.

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